Physics Department
Kuwait University EXP. FOUR
Course 127

February, 2001

Inductance

Objective

- To get familiar with the behavior of an inductor in an ac circuit.
- To determine the inductance of an inductor.

Theory

According to Faraday's law, if the current in a coil changes with time, an induced electromotive force (emf) is produce in the coil itself. This is called self-induction. Consider a "close-packed" coil of N number of turns carrying an alternating current i. A magnetic flux Φ_B is set up in each turn. The value of the emf , ε, induced in the coil is given as

$$\varepsilon = \frac{d(N\Phi_B)}{dt}, \tag{1}$$

where $N\Phi_B$, which is called the flux linkages, is proportional to the current i, or

$$N\Phi_B = Li. \tag{2}$$

where L is called the inductance of the inductor. Therefore,

$$\varepsilon = \frac{d(N\Phi_B)}{dt} = L\frac{di}{dt}, \tag{3}$$

from which

$$\frac{di}{dt} = \frac{\varepsilon}{L}. \tag{4}$$

For a circuit containing an inductive (coil) element only, acted on by an alternating emf of the form

$$\varepsilon\,(t) = \varepsilon_m \sin\omega t, \tag{5}$$

where ε_m, and ω are the maximum induced emf and the angular frequency, respectively ($\omega=2\pi f$, where f is the frequency of the emf). The voltage, V_L, across the inductor, is the same as the emf, therefore,

$$V_L = V_{L,m} \sin\omega t. \tag{6}$$

Substituting V_L for ε in Eq.(4) gives

$$\frac{di}{dt} = \frac{V_L}{L} = \left(\frac{V_{L,m}}{L}\right)\sin\omega t. \tag{7}$$

Solving for i yields

$$i_L = \left(\frac{V_{L,m}}{X_L}\right)\sin\left(\omega t - 90^o\right) = i_{L,m}\sin\left(\omega t - 90^o\right), \tag{8}$$

where, $X_L=\omega L$, and we used $-\cos\omega t = \sin(\omega t\text{-}90)$. The term X_L, is called the inductive reactance of an inductor. It represents the opposition that the inductor shows to the flow of an alternating current (ac), and is measured in units of ohm. We can see from Eq.(8) that i lags V_L by $\frac{\pi}{2}$.

Now, consider an ac circuit (Fig. 1) consisting of a inductor L and a resistor R

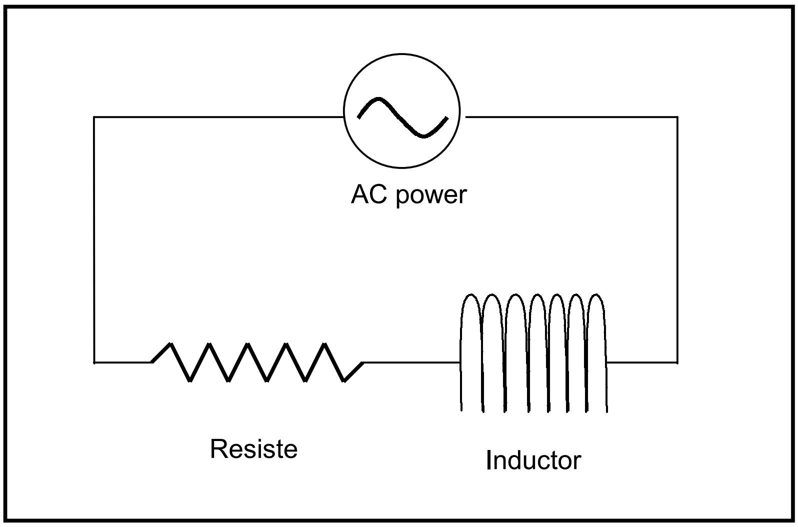

Figure 1: AC circuit

connected in series to an ac power source of emf, $\mathcal{E}$ as given by Eq.(5), the voltage across the inductor, V_L, leads the current i by $\pi/2$, whereas the voltage across the resistor, V_R, is in phase with the current, i , therefore, and using a phasor diagram (Fig. 2) , we can write

$$\boldsymbol{\mathcal{E}_m} = \boldsymbol{V}_{R,m}+\boldsymbol{V}_{L,m}. \tag{9}$$

Dividing all terms by $\sqrt{2}$ yields

$$\boldsymbol{\mathcal{E}} = \boldsymbol{V}_R+\boldsymbol{V}_L, \tag{10}$$

where all terms are in root mean square (rms) values. Also we can write

$$\mathcal{E}^2 = V_R^2+V_L^2, \tag{11}$$

which is expanded as

$$i^2.Z^2 = i^2.R^2+ i^2. X_L^2, \tag{12}$$

where Z is called the impedence of the circuit. Then by eliminating i^2, we get

$$Z^2 = R^2+ X_L^2. \tag{13}$$

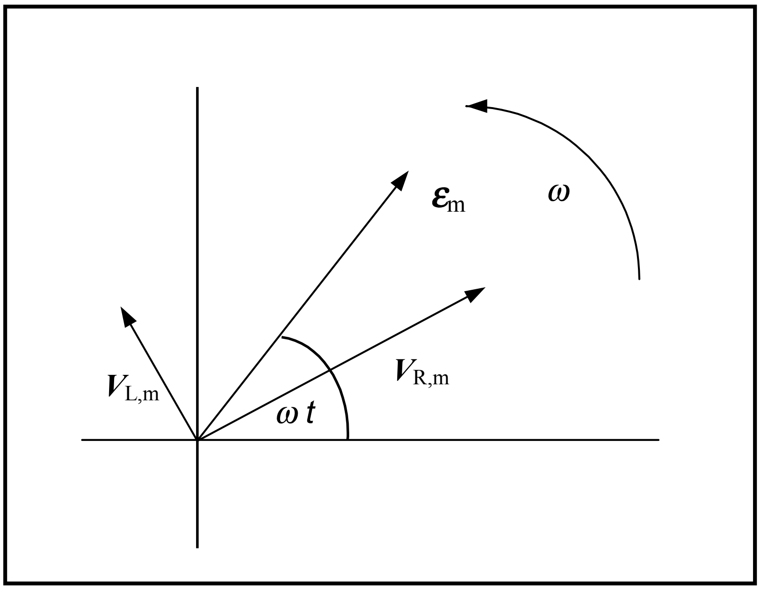

Figure 2: Phasor diagram

In practice, a coil is not a pure inductor, since it has an internal ohmic resistance. For that reason, the measured potential difference across the inductor doesn't correspond to V_L, since it is coupled with V_r, therefore it is denoted by V_L', and since the voltage V_r across the internal resistance, r, is in phase with V_R, Eq.(13) is replaced by

$$Z^2 = (R+r)^2 + X_L^2. \tag{14}$$

For a given value of R, r, and f, and by measuring the current, i, Z can be calculated as $Z = \frac{\varepsilon}{i}$. Therefore, a plot of Z^2 versus $(R+r)^2$ will result in a straight line the slope of which equals 1, and the y-intercept equals X_L^2. The inductance, L, then can be calculated as $L = X_L/2\pi f$. Also, L can be determined using the phasor diagram. Therefore, if ε, V_R, and V_L' are measured at a specific R value, then by using a suitable scale, a vector diagram is drawn as shown in Fig. 3.

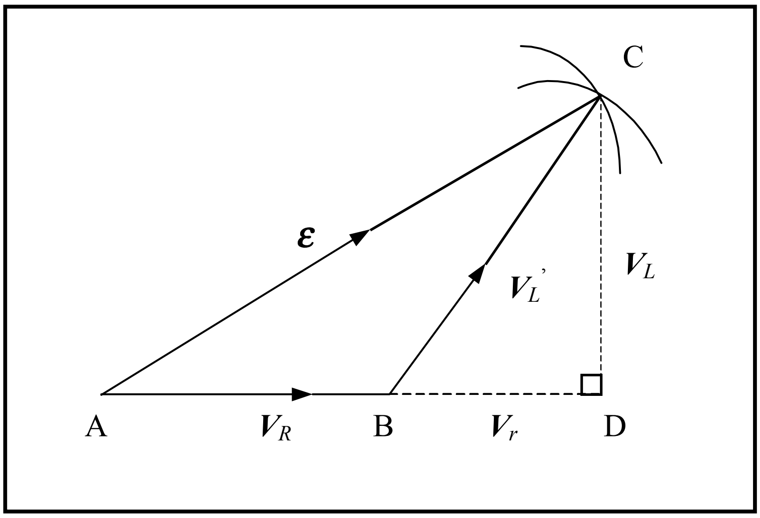

Figure 3: Vector diagram

From the diagram

$$\frac{V_L}{V_R} = \frac{iX_L}{iR} = \frac{\overline{CD}}{\overline{AB}}. \tag{15}$$

Therefore,

$$X_L = R \cdot \frac{\overline{CD}}{\overline{AB}}. \tag{16}$$

Following the same procedure, r can be determined from

$$r = R \cdot \frac{\overline{BD}}{\overline{AB}}. \tag{17}$$

Procedure

1. Connect the circuit as shown in Fig. 4, using the red coil (labeled 1600).
2. Measure the internal resistance of the coil, *r*, using the ohmmeter.
3. Set the frequency of the signal generator to 250 Hz, and the amplitude to maximum.
4. Set the resistance, *R*, of the decade box to 50 Ω.
5. Measure the total voltage, ε, by connecting the voltmeter in parallel to both *R* & *L*.

   Record in Table I
6. Measure the current, *i*, by connecting the ammeter in series with the components.

   Record in the table.
7. Calculate $Z = \varepsilon / i$, Z^2, and $(R+r)^2$. Record in Table I.
8. Repeat steps 5-7 varying *R* from 75-150 Ω. Fill the table.
9. Plot Z^2 versus $(R+r)^2$, from which, determine the slope, and X_L, then calculate *L*.
10. Set *R* to 100 Ω, measure ε, V_R, and V_L', then draw a vector diagram using a suitable

    scale. Calculate X_L, *L*, and *r*.

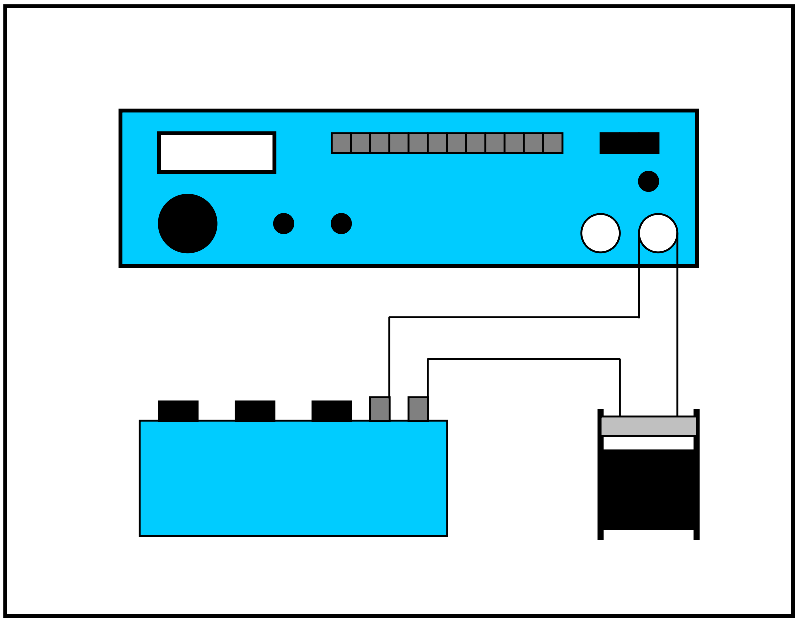

Figure 4: Connection diagram

Table I

$r =$, $f = 250$ Hz

R (Ω)	$(R+r)^2$ (Ω^2)	ε (V)	i (A)	Z (Ω)	Z^2 (Ω^2)
50					
75					
100					
125					
150					

Questions

1. Explain what is meant by the root mean square value of an alternating current.
2. What is meant by self-induction?
3. Define the impedance of an ac circuit.
4. Why $\varepsilon \neq V_R + V_L$ as a scalar sum?