Kuwait University **Physics Department**

Physics 122

Second Midterm Examination-Solutions
Summer Semester 2009

August 8, 2009 (9.00 – 10.30 a.m.)

Name:....................................... **Student No. ....**

Instructor: Prof. M.M. Sharma

<u>Instructions to the students</u>:

1. Solve all the problems. Points for each problem are indicated.
2. The solution should be given explicitly for each problem.

(Fundamental Constants)

$k = \frac{1}{4\pi\varepsilon_0} = 9.0\times10^9 \text{ N.m}^2/\text{C}^2$	(Coulomb's constant)
$\varepsilon_o = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$	(Permittivity of free space)
$e = 1.60 \times 10^{-19}$ C	(Elementary unit of charge)
$N_A = 6.02 \times 10^{23}$	(Avogadro's number)
$g = 9.8$ m/s^2	(Acceleration due to gravity)
$m_e = 9.1 \times 10^{-31}$ kg	(Electron mass)
$m_p = 1.67 \times 10^{-27}$ kg	(Proton mass)

<u>Prefixes of units</u>

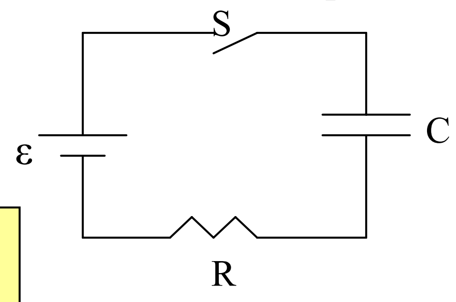
m = 10^{-3}	μ = 10^{-6}	n = 10^{-9}	p = 10^{-12}
k = 10^{3}	M = 10^{6}	G = 10^{9}	T = 10^{12}

For use by Instructors only

Problem	1	2	3	4	5	6	7	8	Total
Marks									

1. In the RC-circuit shown, how much energy is stored on the capacitor, when the current in the circuit is 3.0 mA? Given, R = 12 kΩ, C = 25 μF, and ε = 50 V.

[3 points]



At time t,

$I = 3.0\text{ mA} = 3.0\times10^{-3}\text{ A}$

$V_R = IR = 3.0\times10^{-3}\text{ A}\times12\times10^{3}\ \Omega = 36.0\text{ V}$

$\varepsilon = V_C + V_R \quad\rightarrow\quad V_C = \varepsilon - V_R = 50-36 = 14\text{ V}$

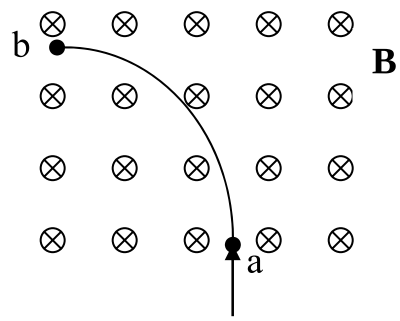
Energy stored:

$$U = \frac{1}{2}CV^2 = 0.5\times25\mu\text{F}\times(14\text{ V})^2 = 2450\ \mu\text{J}$$

$$= 2.45\times10^{-3}\text{ J} = 2.45\text{ mJ}$$

2. A proton enters perpendicular to a magnetic field B = 1.2 T with a speed 5.0 × 10^5 m/s as shown in the figure. How much time does it take to travel from the point a to the point b?

[3 points]



Circular motion perpendicular to a magnetic field: $qvB = \frac{mv^2}{r} \quad\rightarrow\quad r = \frac{mv}{qB}$

Radius of the circle: $r = \frac{1.67\times10^{-27}\times5.0\times10^{5}}{1.6\times10^{-19}\times1.2}\text{ m} = 0.0043\text{ m}$

Time period: (time to travel one full-circle)

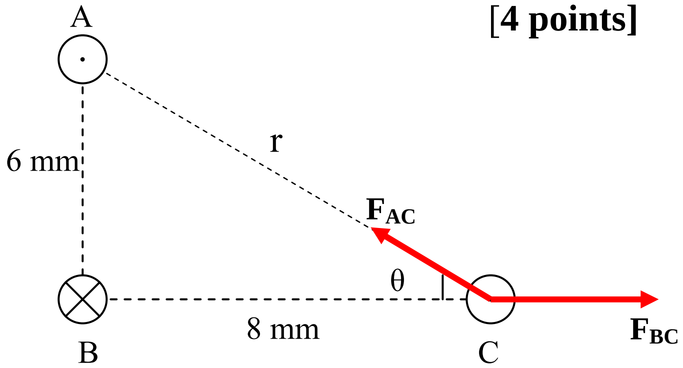
$$T = \frac{2\pi r}{v} = \frac{2\pi\times0.0043}{5.0\times10^{5}}\text{ s} = 5.4\times10^{-8}\text{ s}$$

$$\text{OR}:\ T = \frac{2\pi r}{v} = \frac{2\pi}{v}.\frac{mv}{qB} = \frac{2\pi m}{qB} = \frac{2\pi\times1.67\times10^{-27}}{1.6\times10^{-19}\times1.2} = 5.4\times10^{-8}\text{ s}$$

(then you do not need to calculate r)

Time t to travel $\frac{1}{4}$.circle: $t = \frac{T}{4} = \frac{5.4\times10^{-8}}{4}\text{ s} = 1.35\times10^{-8}\text{ s} \approx 1.4\times10^{-8}\text{ s}$

3. Three parallel wires are placed perpendicular to the page, each carrying a current of 32 A as shown. The current in wires A and C is out of the page and in B it is into the page. What is the magnitude of the resultant force on 2 m length of the wire C? **[4 points]**



$$r = \sqrt{(6)^2 + (8)^2}\ \text{mm} = 10\ \text{mm}$$

$$F_{AC} = \frac{\mu_0 I_A I_C \ell}{2\pi d} = \frac{4\pi \times 10^{-7} \times 32 \times 32 \times 2}{2\pi \times 10 \times 10^{-3}} = 4.1 \times 10^{-2}\ \text{N}$$

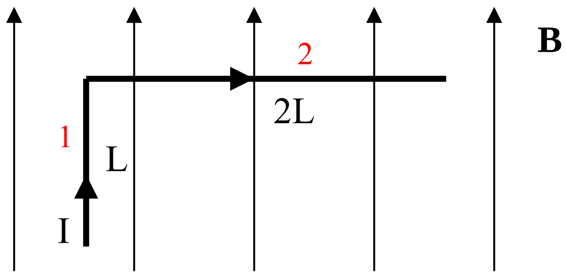
$$F_{BC} = \frac{\mu_0 I_B I_C \ell}{2\pi d} = \frac{4\pi \times 10^{-7} \times 32 \times 32 \times 2}{2\pi \times 8 \times 10^{-3}} = 5.1 \times 10^{-2}\ \text{N}$$

$$F_x = F_{BC} - F_{AC}\cos\theta = 5.1 \times 10^{-2} - 4.1 \times 10^{-2} \times \left(\frac{8\ \text{mm}}{10\ \text{mm}}\right) = 1.82 \times 10^{-2}\ \text{N}$$

$$F_y = F_{AC}\sin\theta = 4.1 \times 10^{-2} \times \left(\frac{6\ \text{mm}}{10\ \text{mm}}\right) = 2.46 \times 10^{-2}\ \text{N}$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(1.82)^2 + (2.46)^2} \times 10^{-2}\ \text{N} = 3.06 \times 10^{-2}\ \text{N} = 0.03\ \text{N}$$

4. A straight wire is bent into the shape as shown. It carries a current I = 20 A and is placed in a uniform magnetic field B = 30 mT. If L = 20 cm, what is the magnitude and direction of the net magnetic force on the wire? **[3 points]**



Force on segment 1 of the wire:

$$F_1 = I\ell B \sin 0^\circ = 0 \ \text{(the wire is parallel to the field B)}$$

Force on segment 2 of the wire:

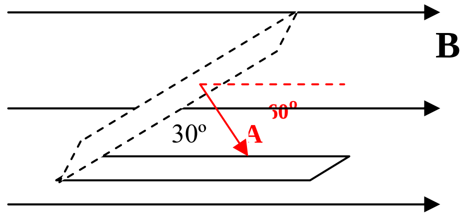
$$F_2 = I\ell B \sin 90^\circ = I.2LB = 20 \times 2 \times 0.20 \times 30 \times 10^{-3}\ \text{N}$$

$$= 0.24\ \text{N out of page (using the right hand thumb rule)}$$

$$\text{Total force } \vec{F} = \vec{F_1} + \vec{F_2} = \vec{F_2}$$

$$\rightarrow F = F_2 = 0.24\ \text{N out of page}$$

5. A rectangular loop with length ℓ = 80 cm and width w = 20 cm lies with its plane parallel to a uniform magnetic field B = 0.30 T. The loop is then rotated 30º from its initial orientation in 30 ms. If the resistance of the loop is 0.5 Ω, what is the current induced in the loop? **[3 points]**



$$\text{Faraday's Law of Induction}: \varepsilon = -N\frac{\Delta\Phi_B}{\Delta t};\ \Phi_B = BA\cos\theta$$

$$\Delta\Phi = \Phi_f - \Phi_i$$

$$\Phi_i = BA\cos 90^\circ = 0$$

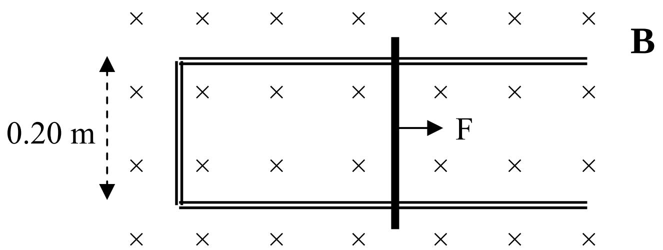
$$\Phi_f = BA\cos 60^\circ = B\ell w\cos 60^\circ = 0.30\times 0.80\times 0.20\times 0.5 = 0.024\ \text{Wb}$$

Induced e.m.f.:

$$|\varepsilon| = \left|-N\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{|\Phi_f - \Phi_i|}{\Delta t} = \frac{0.024 - 0}{30\times 10^{-3}} = 0.8\ \text{V}$$

$$\text{Induced current}: \quad I = \frac{\varepsilon}{R} = \frac{0.8\ \text{V}}{0.5\ \Omega} = 1.6\ \text{A}$$

6. A conducting rod is moved to the right on U-shaped conducting rails in a uniform magnetic field B = 0.60 T by a force F = 30 mN. The conducting rails have negligible resistance, whereas the resistance of the rod is 1.2 Ω. What is the rate of the energy dissipated in the loop? **[3 points]**



Magnetic force:

$$F_M = F_{applied} = 30\ \text{mN} = 30\times 10^{-3}\ \text{N} = 0.030\ \text{N}$$

$$F_M = IBl\sin 90^\circ = 0.030\ \text{N}$$

$$I = \frac{0.030}{B\ell} = \frac{0.030}{0.60\times 0.20} = 0.25\ \text{A}$$

$$\text{Rate of energy dissipated} = \text{Power } P = I^2R$$

$$P = (0.25)^2\times 1.2 = 0.075\ \text{W} = 75\ \text{mW}$$

7. The isotope ^{3}H (tritium) is formed in the atmosphere by the cosmic rays. It is radioactive and decays as: $^3_1\text{H} \rightarrow \, ^3_2\text{He} + e^- + \bar{\upsilon}$. What is the maximum velocity of the emitted electron assuming the recoil energy of the daughter nucleus is negligible. Given, M(^{3}H) = 3.016049 u, M(^{3}He) = 3.016029 u, and m(e^-) = 0.00054858 u. **[3 points]**

$$\text{Q - value for } \beta^- \text{decay:} \quad Q = (M_P - M_D)c^2$$

$$Q = \left[M(^3\text{H}) - M(^3\text{He})\right] c^2$$

$$= (3.016049 - 3.016029) \times 931.5 \text{ MeV} = 18.63 \times 10^{-3} \text{MeV}$$

$$Q = K_{e^-} + K_\upsilon = K_{e-}^{max} + 0 = 18.63 \times 10^{-3} \text{MeV}$$

$$K_{e-}^{max} = \frac{1}{2} mv^2 = 18.63 \times 10^{-3} \times 10^6 \times 1.6 \times 10^{-19} \text{J} = 29.8 \times 10^{-16} \text{J}$$

$$v_{max} = \sqrt{\frac{2 \times 29.8 \times 10^{-16}}{9.1 \times 10^{-31}}} \text{ m/s} = 8.09 \times 10^7 \text{ m/s} \approx 8.1 \times 10^7 \text{ m/s}$$

8. The short-lived radioisotope ^{99}Tc (Technetium, half-life = 6.01 h) is used in nuclear medicine for diagnostic imaging of thyroid gland. How much material of ^{99}Tc should be injected to a patient so that the decay rate becomes about 1 per second after 1 week? **[4 points]**

Decay - rate equation:

$$R = R_0 e^{-\lambda t} = R_0 e^{-\frac{0.693 t}{T_{1/2}}}$$

$$1 = R_0 e^{-\frac{0.693 \times 7 \times 24 \text{ h}}{6.01 \text{ h}}} = R_0 \times 3.86 \times 10^{-9} \rightarrow R_0 = 2.59 \times 10^8 \text{ /s}$$

Number of nuclei present initially (t = 0):

$$R_0 = \lambda N_0 \rightarrow N_0 = \frac{R_0}{\lambda} = \frac{2.59 \times 10^8 \text{ /s} \times 6.01 \times 60 \times 60 \text{ s}}{0.693} = 8.1 \times 10^{12} \text{nuclei}$$

Use the formula: (for number of atoms N in mass m)

$N = \frac{N_A m}{M}$, where M is the molar mass and N_A is the Avogadro number.

$$\rightarrow m = \frac{NM}{N_A} = \frac{8.1 \times 10^{12} \times 99 \text{ gram}}{6.02 \times 10^{23}} = 1.33 \times 10^{-9} \text{g} = 1.33 \text{ ng}$$