

1. A ball is thrown upward from the top of a building that has a height of 15 m, as shown in the figure. If the ball hits the ground with a speed of 20 m/s, what was its maximum height from the top of the building? [2 points]

At the highest point, $v_o = 0$.

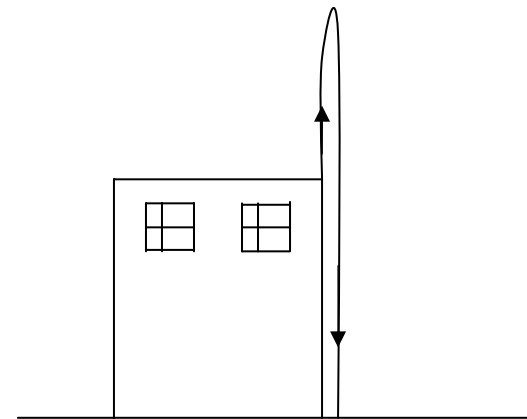
$$v^2 = v_o^2 + 2gh$$

$$20^2 = 0 + 2 \times 9.8 h$$

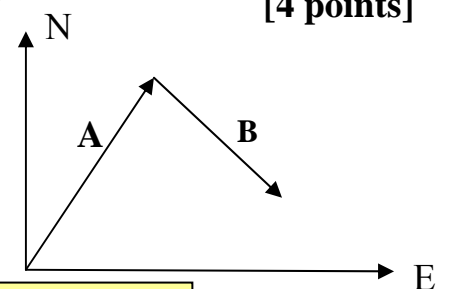
$$h = \frac{20^2}{2 \times 9.8} = 20.4 \text{ m}$$

Thus, the distance of the highest point from the top of the building :

$$h' = h - 15 = 20.4 - 15 = 5.4 \text{ m}$$



2. A person travels a distance $A = 3$ km in the direction 60° north of east and then travels $B = 2$ km in the direction 45° south of east, as shown. What is the magnitude and direction of the resultant displacement of the person? [4 points]



$$\vec{D} = \vec{A} + \vec{B}$$

$$A_x = A \cos 60^\circ = 3 \cos 60^\circ \text{ km} = 1.5 \text{ km}$$

$$A_y = A \sin 60^\circ = 3 \sin 60^\circ \text{ km} = 2.6 \text{ km}$$

$$B_x = B \cos 45^\circ = 2 \cos 45^\circ \text{ km} = 1.4 \text{ km}$$

$$B_y = -B \sin 45^\circ = -2 \sin 45^\circ \text{ km} = -1.4 \text{ km}$$

$$D_x = A_x + B_x = 1.5 + 1.4 = 2.9 \text{ km}$$

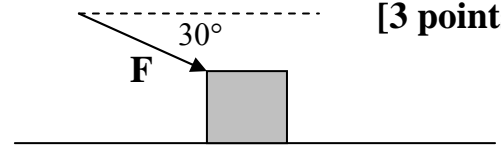
$$D_y = A_y + B_y = 2.6 - 1.4 = 1.2 \text{ km}$$

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(2.9)^2 + (1.2)^2} = 3.14 \text{ km}$$

$$\theta = \tan^{-1} \left(\frac{D_y}{D_x} \right) = \tan^{-1} \left(\frac{1.2}{2.9} \right) = 22.5^\circ$$

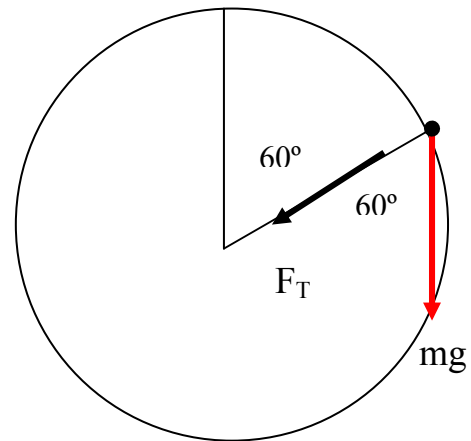
Thus, the direction is 22.5° from + x axis, or 22.5° north of east.

3. The force F acting on the box is half of its weight mg . If the box is moving with a constant speed, what is the coefficient of the kinetic friction between the box and the surface? [3 points]



$$\begin{aligned} \text{Given, } F &= \frac{mg}{2} \\ \Sigma F_y &= 0 \Rightarrow F \sin 30^\circ + mg = F_N \\ F_N &= F \sin 30^\circ + mg = \frac{mg}{2} \sin 30^\circ + mg \\ &= 1.25 mg \\ \Sigma F_x &= 0 \text{ (constant speed)} \\ F \cos 30^\circ - F_{fr} &= 0 \\ F_{fr} &= F \cos 30^\circ \\ \mu_k F_N &= F \cos 30^\circ \\ \mu_k \times 1.25 mg &= \frac{mg}{2} \cos 30^\circ \\ \mu_k &= \frac{\cos 30^\circ}{2 \times 1.25} = 0.35 \end{aligned}$$

4. A 2 kg heavy object connected to a 40 cm long rope is making a circular motion in a vertical plane. If the tension in the rope is 80 N at the point shown in the figure, what is the speed of the circular motion at this point? [2 points]



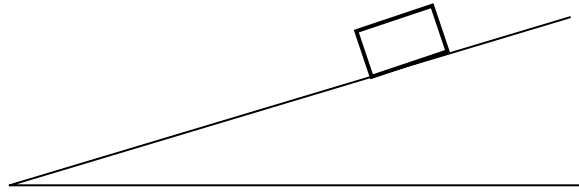
$$\begin{aligned} \Sigma F_c &= 0 \Rightarrow \\ mg \cos 60^\circ + F_T &= \frac{mv^2}{r} \\ 2 \times 9.8 \cos 60^\circ + 80 &= \frac{2v^2}{0.4} \\ 5v^2 &= 89.8 \\ v &= 4.2 \text{ m/s} \end{aligned}$$

5. A 5 kg box moving on a *frictionless* surface with a constant speed of 4 m/s hits a spring and stops. If the spring constant is 500 N/m, how much is the spring compressed? Neglect the mass of the spring. [2 points]

$$\begin{aligned} \frac{1}{2} mv^2 &= \frac{1}{2} kx^2 \\ x &= \sqrt{\frac{m}{k}} \cdot v = \sqrt{\frac{5}{500}} \times 4 \\ &= 0.40 \text{ m} \end{aligned}$$



6. A box ($m = 3 \text{ kg}$) is released from rest on a 25° slope and achieves a speed of 6 m/s after 10 s . What is the work done by the friction? **[4 points]**



$$v = v_o + at \Rightarrow 6 = 0 + a \times 10 \Rightarrow a = 0.6 \text{ m/s}^2$$

For downward motion: $ma = mg \sin \theta - F_{\text{fr}}$

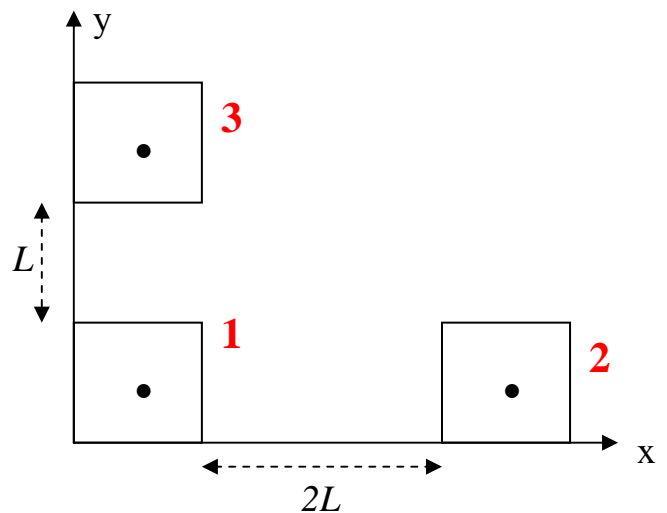
$$\begin{aligned} F_{\text{fr}} &= mg \sin \theta - ma \\ &= 3 \times 9.8 \sin 25^\circ - 3 \times 0.6 \\ &= 10.6 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{Distance travelled: } d &= v_o t + \frac{1}{2} at^2 \\ &= 0 + 0 + \frac{1}{2} \times 0.6 \times 10^2 = 30 \text{ m} \end{aligned}$$

$$\begin{aligned} W_{\text{fr}} &= F_{\text{fr}} d \cos 180^\circ = -F_{\text{fr}} d \\ &= -10.6 \times 30 = -319 \text{ J} \end{aligned}$$

7. Three identical square plates of side L and mass m are placed on the x-y plane as shown. What are the coordinates of the centre-of-mass of the plates? **[4 points]**

$$\begin{aligned} x_1 &= 0.5L \\ x_2 &= L + 2L + 0.5L = 3.5L \\ x_3 &= 0.5L \\ y_1 &= 0.5L \\ y_2 &= 0.5L \\ y_3 &= L + L + 0.5L = 2.5L \\ x_{cm} &= \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} \\ &= \frac{m \times 0.5L + m \times 3.5L + m \times 0.5L}{m + m + m} \\ &= \frac{4.5mL}{3m} = 1.5L \\ y_{cm} &= \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} \\ &= \frac{m \times 0.5L + m \times 0.5L + m \times 2.5L}{m + m + m} \\ &= \frac{3.5mL}{3m} = 1.16L \end{aligned}$$



8. Three forces $F_1 = 20 \text{ N}$, $F_2 = 40 \text{ N}$ and $F_3 = 30 \text{ N}$ are applied on a wheel as shown in the figure. What is the magnitude and direction of the net torque about the axle of the wheel, if the inner and outer radii of the wheel are 15 cm and 30 cm, respectively? [4 points]

Definition $\tau = rF_{\perp} = r_{\perp}F = rF \sin\theta$

Given, $a = 15 \text{ cm}$, $b = 30 \text{ cm}$

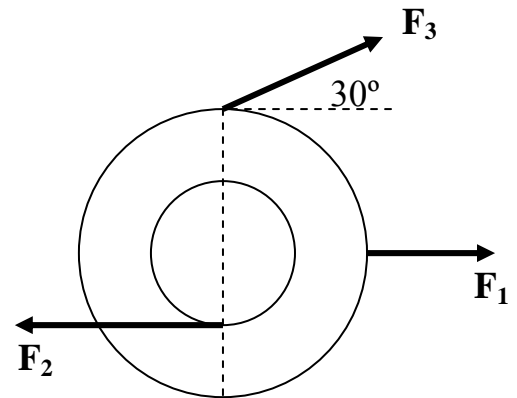
$$\tau_1 = rF \sin 0^\circ = 0$$

$$\tau_2 = -rF \sin 90^\circ = -aF_2 = -0.15 \times 40 = -6.0 \text{ m.N}$$

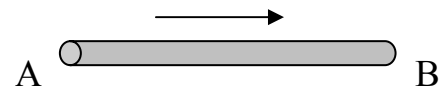
$$\begin{aligned} \tau_3 &= -rF \sin 60^\circ = -bF_3 \sin 60^\circ \\ &= -0.30 \times 30 \sin 60^\circ = -7.8 \text{ m.N} \end{aligned}$$

$$\tau = \tau_1 + \tau_2 + \tau_3 = 0 - 6.0 - 7.8 = -13.8 \text{ m.N}$$

$$\tau_{net} = 13.8 \text{ m.N clockwise}$$



9. Blood is supplied to an artery by heart. The coefficient of viscosity of blood flowing through the 7.5 cm long artery is $4 \times 10^{-3} \text{ Pa.s}$. Its radius is 2.0 mm and the rate of flow of the blood from the end A to the end B of the artery is $6000 \text{ cm}^3/\text{min}$. If the pressure at the end B is $1.0 \times 10^4 \text{ N/m}^2$, what is the force acting on the end A? Given, $Q = \pi r^4 (P_1 - P_2) / (8\eta L)$. [3 points]



$$Q = \frac{\pi r^4 (P_1 - P_2)}{8\eta L}$$

Given, $Q = 6000 \text{ cm}^3/\text{min}$

$$= \frac{6000 \times (10^{-2} \text{ m})^3}{60 \text{ s}} = 1.0 \times 10^{-4} \text{ m}^3/\text{s}$$

$$r = 2.0 \text{ mm} = 2.0 \times 10^{-3} \text{ m}$$

$$P_2 = P_B = 1.0 \times 10^4 \text{ N/m}^2$$

$$P_1 - P_2 = \frac{Q \cdot 8\eta L}{\pi r^4} = \frac{1.0 \times 10^{-4} \times 8 \times 4 \times 10^{-3} \times 7.5 \times 10^{-2}}{\pi \times (2.0 \times 10^{-3})^4} = 4.8 \times 10^3 \text{ Pa}$$

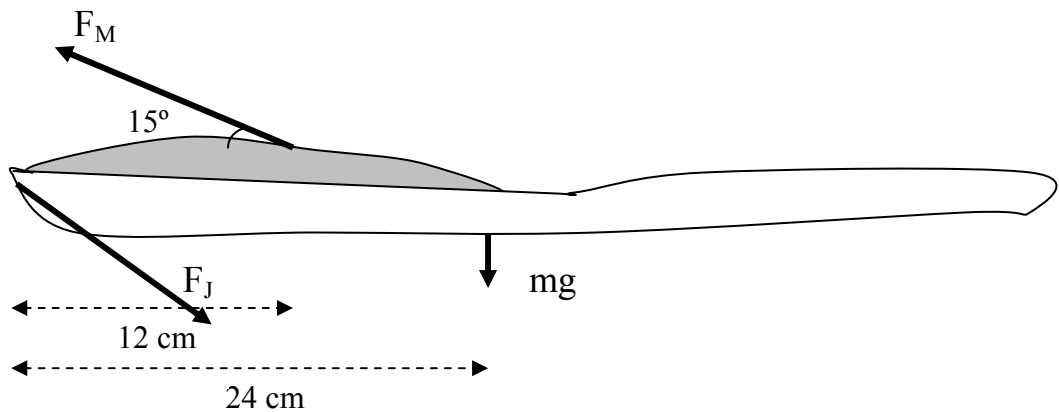
$$P_1 = P_2 + 4.8 \times 10^3 = 1.0 \times 10^4 + 4.8 \times 10^3 = 1.48 \times 10^4 \text{ Pa}$$

$$P_A = P_1 = 1.48 \times 10^4 \text{ Pa}$$

$$F_A = P_A \cdot A = 1.48 \times 10^4 \times \pi r^2$$

$$= 1.48 \times 10^4 \times \pi \times (2.0 \times 10^{-3})^2 = 0.186 \text{ N} \approx 0.19 \text{ N}$$

10. An outstretched arm (with the total mass $m = 3.5$ kg) is shown in the figure. Here F_M is the force exerted by the “deltoid” muscle to hold the arm in the horizontal position and F_J is the force exerted by the shoulder at the joint. What is the horizontal component of the force F_J ? **[3 points]**



$$\Sigma \tau = 0 \Rightarrow +0.12 \times F_M \sin 15^\circ - 0.24 \times mg = 0$$

$$F_M = \frac{0.24 \times 3.5 \times 9.8}{0.12 \sin 15^\circ} = 265.0 \text{ N}$$

$$\text{For the horizontal component of } F_J : \Sigma F_x = 0 \Rightarrow$$

$$F_{Jx} - F_M \cos 15^\circ = 0$$

$$F_{Jx} = 265 \cos 15^\circ = 256 \text{ N}$$