Kuwait University Physics Department

Physics 107

Charging and Discharging of a Capacitor

Introduction

In this experiment, we will study charging a capacitor by connecting it to an emf source through a resistor. The experiment also includes the study of discharging phenomena of a capacitor through a resistor.

Objectives

- To study charging and discharging process through capacitors.
- To determine the time constant τ of an RC-circuit.

Equipment to be Used:

- Electronic design experimenter.
- 220 kΩ resistor.
- 470 μF capacitor.
- Multimeter.
- Stopwatch.
- Connection wires.

Theory

Charging a Capacitor:

Consider a circuit as shown in **Figure 1**. Capacitor C is initially uncharged, by closing the switch S a current i is setup in the loop and the capacitor begins to charge. Applying Kirchoffs loop rule, we get

$$\varepsilon - iR - \frac{Q}{C} = 0, \tag{1}$$

Where ε is the electromotive force (dc voltage supply), R is the resistor, Q is the charge of the capacitor and C is the capacitance. Substituting $\frac{dQ}{dt}$ for the current i, Equation (1) becomes

$$\varepsilon - \frac{dQ}{dt}R - \frac{Q}{C} = 0, \tag{2}$$

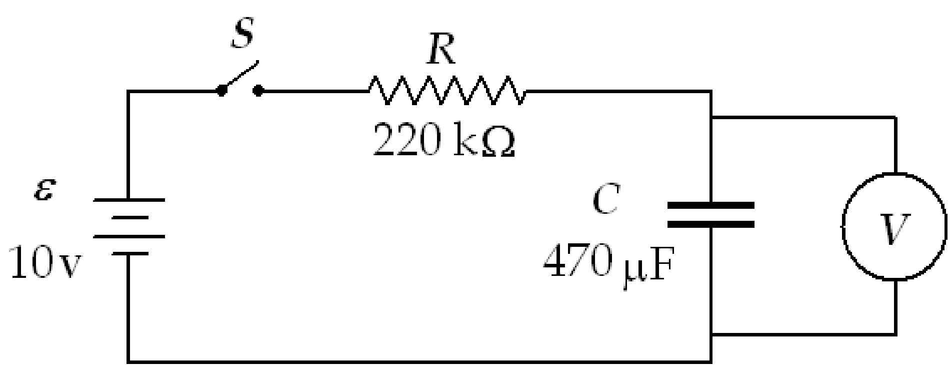

Figure 1: Charging circuit

Rearranging the terms, Equation (2) becomes

$$\frac{dQ}{dt} = \frac{\varepsilon}{R} - \frac{Q}{RC}. \tag{3}$$

The solution of Equation (3) is given as

$$Q = C\varepsilon\left(1 - e^{-t/RC}\right), \tag{4}$$

which determines the charge on the capacitor as a function of time t.
$Ce = Q_o$, which represents the maximum charge the capacitor can hold for a given emf. The voltage across the capacitor V_c is given as

$$V_c = \frac{Q}{C}. \tag{5}$$

Dividing Equation (4) by C yields

$$V_c = \varepsilon\left(1 - e^{-t/RC}\right). \tag{6}$$

At a specific value of time $t = \tau = RC$ (called the time constant of the R-C circuit),

$$V_c = \varepsilon\left(1 - e^{-1}\right). \tag{7}$$

or

$$V_c = 0.63\,\varepsilon. \tag{8}$$

Therefore, by plotting V_c versus t, the time constant τ may be determined, and hence, the value of C can be calculated, provided R is known.

Equation (6) shows that the growth of the capacitors voltage is not linear, but rather grows exponentially reaching a saturation value which equals the voltage of the emf source. The capacitor is considered to be fully charged after a period of about five time constants.
The current i in the circuit at a given time t is given as

$$i = \frac{dQ}{dt} = \frac{\varepsilon}{R}e^{-t/RC}. \tag{9}$$

where $\frac{\varepsilon}{R} = i_o$ represents the initial current in the circuit.
Therefore, we can write

$$i = i_o\, e^{-t/RC}. \tag{10}$$

At a time $t = \tau$, $i = i_o\, e^{-1}$, or

$$i = 0.37\, i_o. \tag{11}$$

Power relation:

Energy considerations give us additional insight into the behavior of an R-C circuit. While the capacitor is charging, the instantaneous rate at which the battery delivers energy to the circuit is

$$P = i\,\varepsilon. \tag{12}$$

The instantaneous rate at which electrical energy is dissipated in the resistor is $P_R = i^2 R$. On the other hand the rate at which energy is stored in the capacitor is $P_C = i\,V_c = i\frac{Q}{C}$. According to energy conservation principle, the power supplied by the battery equals the sum of the power dissipated in the resistor and the power stored in the capacitor, i.e.: $P = P_R + P_C$.

Discharging a Capacitor:

For the discharging process, consider the circuit shown in **Figure 2**. After closing the switch S for a long time (compared to the circuits time constant), the capacitor will be fully charged to a value of $Q = C\,\varepsilon$. As the switch is opened, the power supply is disconnected from the circuit and the capacitor starts to discharge through the resistor.

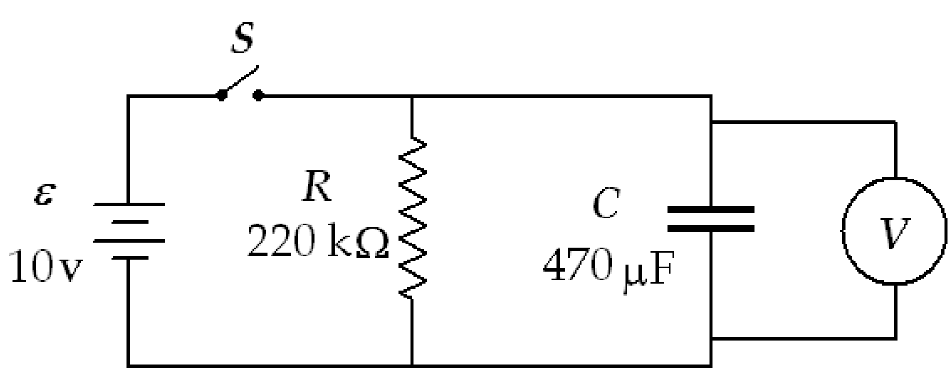

Figure 2: discharging circuit

Following the same procedure as for the charging analysis, the differential equation that characterizes the discharging process is given as

$$-\frac{dQ}{dt}R - \frac{Q}{C} = 0. \tag{13}$$

The solution to Equation (13) is given as

$$Q = C\varepsilon e^{-t/RC}. \tag{14}$$

Equation (14) determines the charge on the capacitor at as a function of time t. The voltage across the capacitor V_c :

$$V_c = \varepsilon e^{-t/RC}. \tag{15}$$

At time $t = \tau = RC$ (the time constant), V_c becomes

$$V_c = \varepsilon e^{-1}, \tag{16}$$

or

$$V_c = 0.37\varepsilon. \tag{17}$$

The current i in the circuit is given as

$$i = \frac{dQ}{dt} = -\frac{\varepsilon}{R}e^{-t/RC}, \tag{18}$$

where $\frac{\varepsilon}{R} = i_o$ represents the initial current in the discharging circuit. Therefore, we can write

$$i = -i_o\, e^{-t/RC}, \tag{19}$$

At time $t = \tau$, the current $i = -i_o\, e^{-1}$, or

$$i = -0.37\, i_o \tag{20}$$

The minus sign may be ignored and it implies that the charge is decreasing with time. Therefore by plotting i versus t, the time constant τ can be determined, and hence, the value of C can be calculated, if R is known.

The capacitor can be considered to be fully discharged, during a time lapse of five time constants.

Procedure

Part One: Charging a capacitor (Voltage vs time)

1) **Connect** the circuit as shown in **Figure 1** (make sure that the lead of the capacitor at the arrow head side is connected to the ground).

2) **Turn on** the power supply, and **set** the output of the power supply to 10 V.

3) **Short out** the capacitor, temporarily, by connecting a wire parallel to it, so that it is completely discharged.

4) **Close** the switch S, and **reset** the stopwatch.

5) **Record** the time, t_1, corresponding to V_c reaching values indicated in **Table 1**.

6) **Reset** the stopwatch.

7) **Repeat** steps above for time corresponding to the same values of V_c and **record** your results as t_2 in Table 1.

8) **Calculate** the average time t_{avg}.

9) **Plot** a graph for V_c versus t_{avg} from which determine τ and calculate C.

Table 1.

V_c (V)	t_1 (s)	t_2 (s)	t_{avg} (s)
0			
1			
2			
3			
4			
5			
6			
7			
8			

Part Two: Charging a capacitor (Current vs time)

1) **Connect** the circuit as shown in **Figure 1** (make sure that the lead of the capacitor at the arrow head side is connected to the ground).

2) **Turn on** the power supply, and **set** the output of the power supply to 10V.

3) **Short out** the capacitor, temporarily, by connecting a wire parallel to it, so that it is completely discharged.

4) **Close** the switch S and **reset** the stopwatch.

5) **Record** the time, t_1, corresponding to V_R reaching values indicated in Table 2.

6) **Reset** the stopwatch.

7) **Repeat** steps above for time corresponding to the same values of V_R and **record** your results as t_2 in Table 2.

8) **Calculate** the average time t_{avg}.

9) For each value of V_R **calculate** the current i in the circuit as $\frac{V_R}{R_{measured}}$ and **record** it in Table 2.

10) **Plot** a graph for i versus t_{avg} from which determine τ and calculate C.

Table 2. $R_{measured} =$ ....

V_R (V)	t_1 (s)	t_2 (s)	t_{avg} (s)	i (mA)
10				
9				
8				
7				
6				
5				
4				
3				
2				

Power relation:

1) **Refer to** Table 3 and the graph of V_c versus t_{avg} obtained from the first part of the experiment. For each value of time t in Table 3 **find** the corresponding value of V_c from the graph and **record** it in Table 3.

2) Similarly, **Refer to** Table 3 and the graph of i versus t_{avg} obtained from the second part of the experiment. For each value of time t in Table 3 **find** the corresponding value of i from the graph and **record** it in Table 3.

3) **Calculate** P, P_R, and P_C, and **Comment** on the relation between the three quantities.

Table 3.

t (s)	V_c (V)	i (A)	P (W)	P_R (W)	P_C (W)
20					
40					
60					
80					
100					
120					
140					
160					
180					

Comment on the relation between P, P_R and P_C :

....

Part Three: Discharging a capacitor (Voltage vs. time)

1) **Connect** the circuit as shown in Figure 2 (make sure that the lead of the capacitor at the arrow head side is connected to the ground).

2) **Turn on** the power supply and **set** the voltage to 10 V.

3) **Close** the switch S. This will cause the capacitor to charge up immediately.

4) **Start** the stopwatch and **open** the switch S simultaneously.

5) Corresponding to integer values of V_c according to Table 4, **pause** the display of the stopwatch and **record** the time as t_1 in Table 4 and **remove** the pause of the stopwatch.

6) **Reset** the stopwatch.

7) **Repeat** steps above for time corresponding to the same values of V_c and **record** your results as t_2 in Table 4.

8) **Calculate** the average time t_{avg}.

9) **Plot** a graph for V_c versus t_{avg} from which bf determine τ and calculate C.

Table 4.

V_c (V)	t_1 (s)	t_2 (s)	t_{avg} (s)
10			
9			
8			
7			
6			
5			
4			
3			
2			

Part Four: Discharging a capacitor (Current vs time)

1) **Connect** the circuit as shown in Figure 2 (make sure that the lead of the capacitor at the arrow head side is connected to the ground).

2) **Turn on** the power supply and **set** the voltage to 10 V.

3) **Close** the switch S. This will cause the capacitor to charge up immediately.

4) **Start** the stopwatch and **open** the switch S simultaneously.

5) Corresponding to integer values of V_R , according to Table 5, **pause** the display of the stopwatch and **record** the time as t_1 in Table 5 and **remove** the pause of the stopwatch.

6) **Reset** the stopwatch.

7) **Repeat** steps above for time corresponding to the same values of V_R and **record** your results as t_2 in Table 5.

8) **Calculate** the average time t_{avg}.

9) For each value of V_R **calculate** the current i in the circuit as $\frac{V_R}{R_{measured}}$ and **write** it in Table 5.

10) **Plot** a graph of i versus t_{avg} from which **determine** τ and **calculate** C.

Table 5.

V_R (V)	t_1 (s)	t_2 (s)	t_{avg} (s)	i (mA)
10				
9				
8				
7				
6				
5				
4				
3				
2				

The average value of the capacitance C_{avg} = ...

The percentage error in the capacitance value

$$\left(\frac{C_{theoretical} - C_{experimental}}{C_{theoretical}}\right) \times 100 \ = \ \text{..}$$

Conclusion: