Kuwait University   Physics Department

Physics 105

Rotational motion

Introduction

In this experiment the moment of inertia of a solid disk is determined by studying the dynamics of the rotating disk when a constant torque is applied to it. We should be able to verify that the value of moment of inertia found experimentally is equal to the theoretical value within experimental error. Furthermore the conservation of mechanical energy in rotational motion is studied and also the effect of any non-conservative forces on the motion will be investigated, minimized and compensated.

Objectives

- To study rotational kinematics (uniformly accelerated rotational motion) and to learn how to calculate moment of inertia of a rotating disk (about 1350g).

- To verify that the experimental value of moment of inertia is consistent with the corresponding theoretical value.

- To study rotational kinetic energy and to verify the conservation of mechanical energy.

Equipment to be used:

- Setup for Circular Motion (Cast Iron "A" base with rotating shaft, aluminum track, a 300g square mass with thumbscrew, rotating disk, mass hanger, mass pieces, string with hook, mounting rod with pulley).

- Measuring devices:

    Photogate timer.

    Ruler (For the dimensions of the rotating disk).

    Vernier caliper (For the radius of the vertical pulley).

Theory

In pure rotational motion, the motion with a *constant angular acceleration* about a fixed axis is an important special case. There is a set of equations of motion relating the angular quantities analogues to equations of motion for the motion with *constant linear acceleration.*

Table 1 below shows the analogy between the equations for motion with constant angular acceleration about a fixed axis and the corresponding equations for motion with constant linear acceleration.

Table 1. Equations of motion.

Translational motion	missing	variables	Rotational motion
$v = v_o + at$	x	θ	$\omega = \omega_o + \alpha t$
$\Delta x = v_o t + \frac{1}{2}at^2$	v	ω	$\Delta\theta = \omega_o t + \frac{1}{2}\alpha t^2$
$v^2 = v_o^2 + 2a\Delta x$	t	t	$\omega^2 = \omega_o^2 + 2\alpha\Delta\theta$
$\Delta x = \frac{1}{2}(v_o + v)t$	a	α	$\Delta\theta = \frac{1}{2}(\omega_o + \omega)t$
$\Delta x = vt - \frac{1}{2}at^2$	v_o	ω_o	$\Delta\theta = \omega t - \frac{1}{2}\alpha t^2$

Where θ is the angular position, $\omega = \frac{d\theta}{dt}$ is the angular velocity, and $\alpha = \frac{d^2\theta}{dt^2}$ is the angular acceleration.

Part one: Moment of Inertia of a Solid Disk

Newton's second law for uniformly accelerated rotational motion is stated as

$$\tau = I\alpha \tag{1}$$

where I is the *moment of inertia* of the rotating solid disk about the axis of rotation, α is the angular acceleration of the disk, and τ is the net *torque* applied to it. In order to compute the experimental value of the moment of inertia I of the solid disk, then we should determine first the experimental value of both its angular acceleration α and the amount of torque τ applied to it.

- To determine the angular acceleration α of the disk we will consider the following equation

$$\Delta\theta = \omega_o t + \frac{1}{2}\alpha t^2 \tag{2}$$

  with initial conditions $\theta_o = 0$ and $\omega_o = 0$. Hence, Equation (2) is reduced to

$$\theta = \frac{1}{2}\alpha t^2 \tag{3}$$

  which implies the fact the α is determined by studying the relation between the angular position (θ) and the time of revolution (t).

- To determine the torque τ applied to the rotating disk refer to Figure 1. the torque is due to the tension of the string wrapped around the vertical pulley.
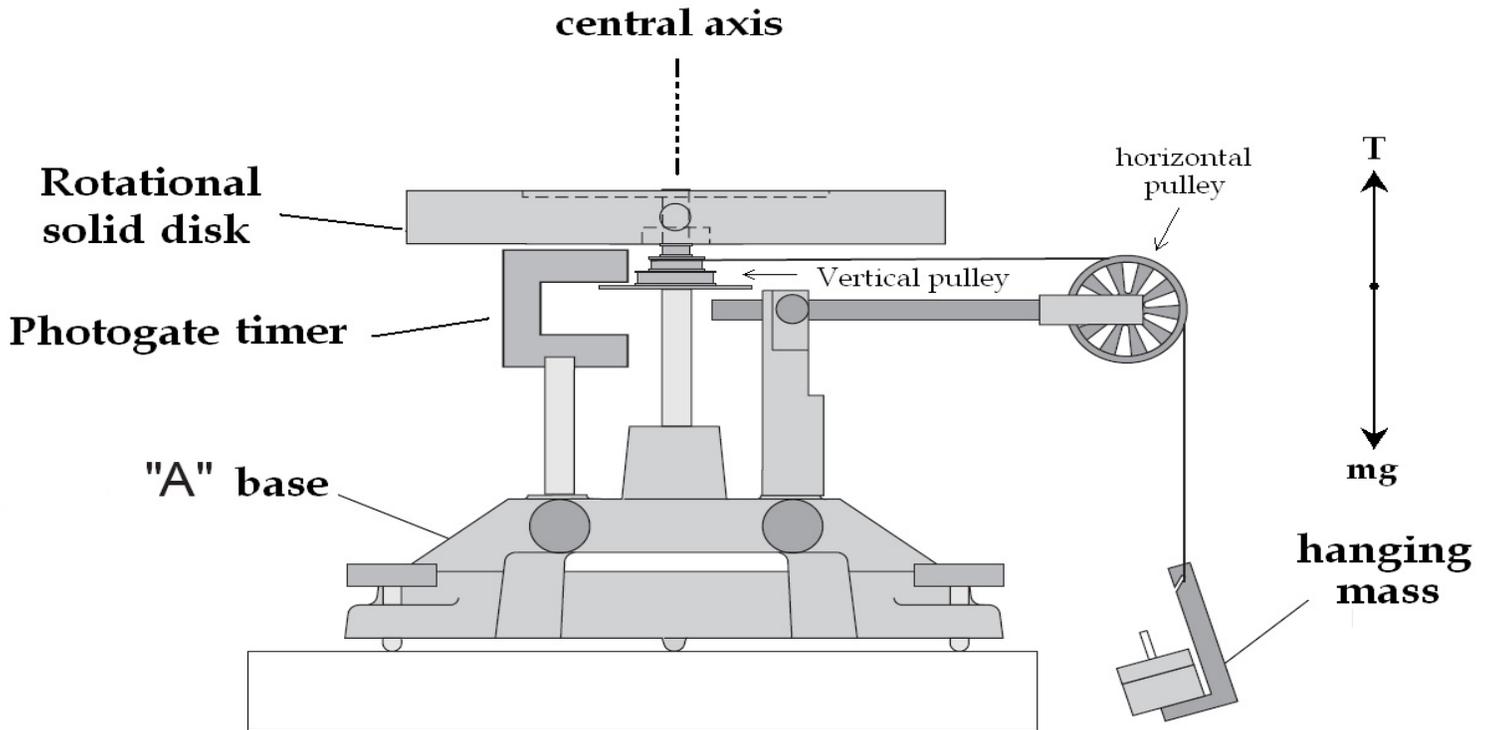

Figure 1. Rotational Apparatus and Free-Body Diagram

If the acceleration of the falling mass connected to the string is small compared to the acceleration due to gravity then the value of the torque is given by:

$$\tau = m_e g \, r_{vp} \tag{4}$$

where $m_e g$ is the weight of the effective mass m_e. Because the theory used to find the rotational inertia experimentally does not include friction then you should find out how much mass over the horizontal pulley is needed to overcome the kinetic friction and allow the hanging mass to drop at a constant speed. We refer to this mass as "friction mass" m_f and it will be subtracted from the hanging mass m_h used to accelerate the apparatus. therefore, $(m_e = m_h - m_f)$.

The term r_{vp} is the moment arm, which is the radius of the vertical pulley fixed on the vertical shaft.

The experimental value of the moment of inertia I of the solid disk is compared to the Theoretical moment of inertia $I_{theoretical}$ of the disk about its central axis which is given by

$$I_{theocratical} = \frac{1}{2}MR^2 \tag{5}$$

where M is the mass of the solid disk and R is its radius.

Part two: Conservation of Mechanical Energy

In this part of the experiment we study how the potential energy from the hanging mass is transformed into rotational kinetic energy of the disk. In terms of moment of inertia the rotational kinetic energy, denoted by K_{rot}, of a rigid body is given by:

$$K_{rot} = \frac{1}{2}I\omega^2 \tag{6}$$

and the translational kinatic energy, K_{trn}, of the effective mass as it moves slowly downward toward the ground is given by

$$K_{trn} = \frac{1}{2}m_e v^2 \tag{7}$$

Then conservation of mechanical energy requires that

$$m_e g\, h = \frac{1}{2}I\omega_f^2 + \frac{1}{2}m_e v_f^2 \tag{8}$$

however, the translational kinetic energy K_{trn} of the falling mass is very small (almost negligible) then the chose to ignore it. That reduces Equation (8) to

$$m_e g\, h \simeq \frac{1}{2}I\omega_f^2 \tag{9}$$

Procedure:

Leveling the "A" Base

1) **Set up** the equipment according to Figure 2.

2) **Attach** the 300g square mass onto either end of the aluminum track and tighten the thumbscrew.

3) **Adjust** the leveling screw on the right leg of the base until the end of the track with the square mass is aligned over the leveling screw of the left leg of the base.

4) **Rotate** the track 90° so it becomes parallel to the right side of the "A" base and adjust the other leveling screw until the track remains at rest in this position.

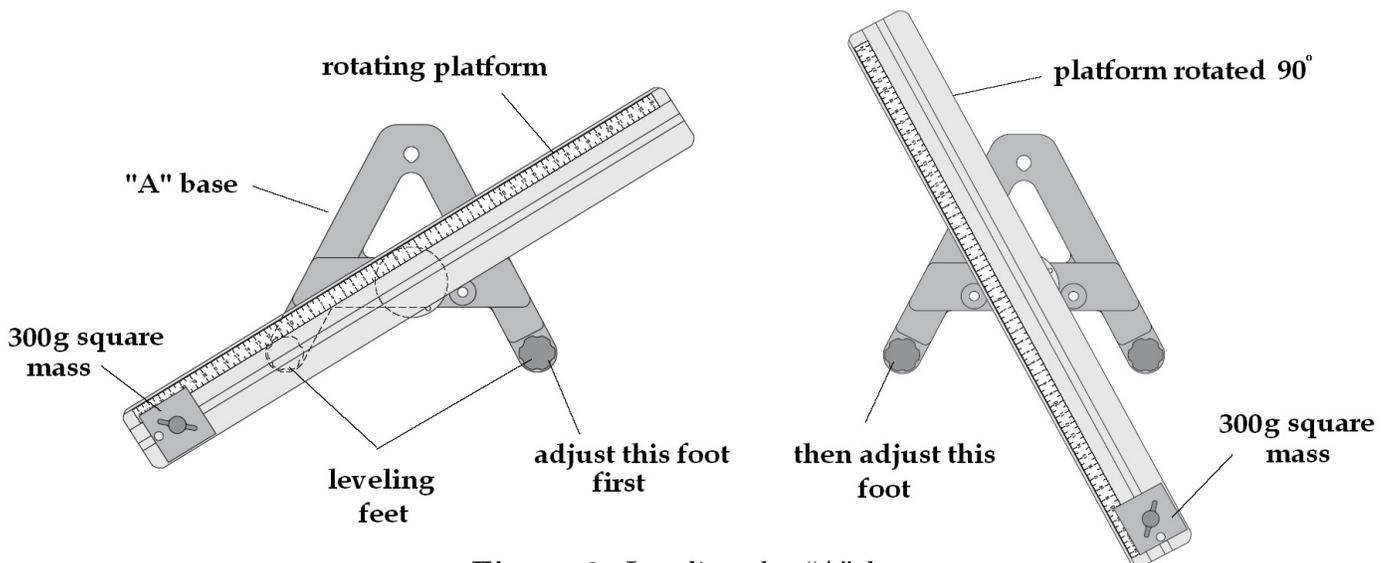5) The track should remain at rest regardless of its orientation otherwise **repeat** steps 3 and 4.

Figure 2. Leveling the "A" base

Part one: Moment of Inertia of a Solid Disk

1) **Set up** the rotating apparatus as shown in Figure 1 with the disk placed onto the vertical shaft.

2) Using a hook, **fix** the string to the hole in the vertical pulley and **wind** the string around that pulley by rotating the disk. The total length of the string should deliver a suitable height h, for which the disk can make at least 8 revolutions before the hanging mass falls to the ground.

3) **Measure** and **record** the following parameters:

   * The radius of the rotating disk R_d = ...
   * The mass of the disk M_d = ...
   * The radius of the vertical pulley r_{vp} = ...
   * The friction mass m_f = ...
   * The hanging mass m_h = ...
   * The effective mass m_e = ...
   * The torque τ using equation (4)= ....
   * The moment of inertia of the disk using equation (5)= ....

4) Use the free spoke of the vertical pulley and the photogate timer to measure

   - The odd periods of revolutions of the disk in the first attempt

   - The even periods of revolutions of the disk in the second attempt

   For the purpose of measurement of the odd periods **set** the photogate timer to **pulse** mode and select the $1ms$ resolution. **Reset** the photogate timer and **release** the disk when the free space of the vertical pulley is shortly before the photogate head. Whenever a reading appears on the timer, immediately **record** it and **reset** the photogate timer quickly so it can measure the next odd period. **Record** your data in Table 2.

5) For measuring the even periods, we **reset** the photogate timer as soon as it starts timing the first period and then continue exactly as in the previous step.

6) **Calculate** the total time t elapsed after $n = 1, 2, 3, 4, ...$ revolutions in Table 2, and **plot** the graph of the angular position θ versus t^2. **Determine** the angular acceleration from this graph and then **calculate** the moment of inertia of the disk using Equation (1).

7) **Compare** the experimental value of the moment of inertia of the disk with the theoretical value calculated.

Table 2.

n	$\theta = 2\pi n$	t_{odd} (s)	t_{even} (s)	t (s)	t^2 (s^2)
1					
2					
3					
4					
5					
6					
7					
8					

Angular acceleration α (from the graph)= ....

Experimental value of I (from Equation 1) = ...

Theoratical value of I (from Equatios 5) = ...

Part two: Conservation of Mechanical Energy

1) **Set up** the rotational apparatus as shown in Figure 1.

2) **Measure** the falling distance h and **record** it in Table 3. **Adjust** the length of the string such that at the moment the falling mass hits the ground, the hook would be released automatically from the threaded hole.

3) **Set** the photogate timer to **pulse** mode and **measure** the period T_f in the phase of uniform motion just after the acceleration phase is over. **Record** it Table 3.

4) **Calculate** the angular speed $\omega_f = \frac{2\pi}{T_f}$ and **record** it in Table 3.

5) **Repeat** the previous steps for other hanging masses given in Table 3.

6) **Plot** a graph of the rotational kinetic energy versus initial potential energy of the effective mass. You should get a straight line through the origin with slope equal to 1.

Table 3. Falling Height $h =$ ....,   Moment of Inertia $I =$ ....

m_h (g)	$m_e gh$ (J)	T_f (s)	$\omega_f = \frac{2\pi}{T_f}$ (rad/s)	K_{rot} (J)
17				
20				
23				
26				
29				
55				