

Kuwait University

Physics Department

Physics 105

Static Equilibrium, Force Decomposition, and Frictional Forces

Introduction

In this experiment, you will study a special case of motion, when the resultant force of all external forces acting on an object or a system of objects is equal to zero. This is called "**Static equilibrium of forces**". We will study this concept and develop the conditions required to maintain equilibrium for the object on which the forces are acting. Furthermore, our analysis focuses on three study cases. In the first study case, we compute the elasticity constant of a spring. The second case uses vector decomposition in studying Newton's 3rd Law. Finally, in the last study case we use the concept to resolve the forces acting on an object on an inclined plane and find the coefficient of static friction.

Objectives

- To study the Static equilibrium of forces and develop the conditions required to maintain equilibrium for a system of objects. (Newton's 2nd and 3rd Laws)
- To be able to calibrate a spring as a dynamometer and use it for the measurement of forces.
- To understand how to decompose forces into perpendicular components.
- To grasp the concept of retarding forces (Frictional forces).

Equipment to be used:

- Experimental board with mass hangers (mass 5 g), mass pieces (10 g, 20 g, 50 g, 100 g), pulleys (for changing the direction of forces), force ring, inclined plane.

- Measuring devices:

Dynamometer (spring balance with three different scales),

Degree plate (for measuring angles),

plumb (plumb-rule) for finding the vertical,

Ruler (for measuring displacements).

Theory

In this experiment the method of decomposing a force into components and expressing a force as a Cartesian vector will be used to solve problems involving equilibrium of particles. A particle of mass m under the influence of n number of forces is said to be in equilibrium provided it is at rest if originally at rest, or has a constant velocity if originally in motion. In both cases, the acceleration of the particle equals zero. Therefore, to maintain the state of equilibrium:

$$\sum_{i=1}^n F = m a = 0 \quad (1)$$

Since forces are vectors, and since we will mainly be dealing with forces that act in the xy plane, then Equation 1 states that the components of the net force must each be zero. Thus, the condition for equilibrium is written as:

$$\sum_{i=1}^n F_x = 0, \quad \sum_{i=1}^n F_y = 0. \quad (2)$$

Calibration of a spring as dynamometer

Suppose you are holding a simple coil spring of natural (or unstretched) length x . When you apply a force F_x to stretch the spring and cause a Δx amount of extension beyond its natural length, the spring exerts a restoring force $-F_s$ proportional to the extension Δx .

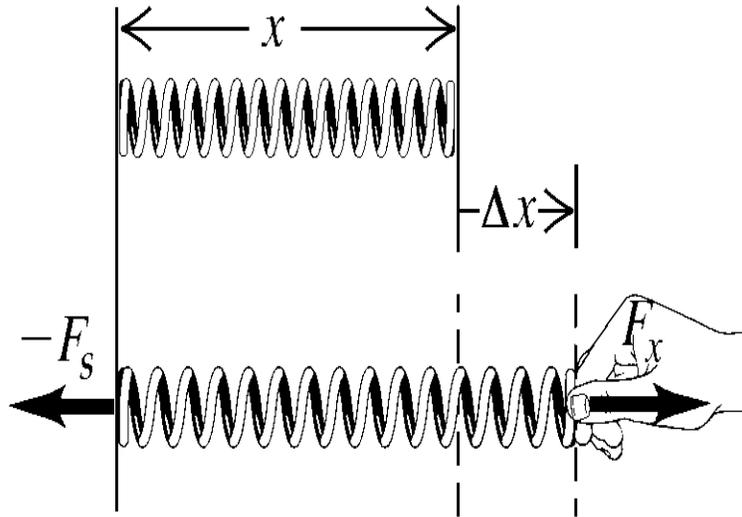


Figure 1. A spring of natural length x with force F_x applied to stretch it by Δx .

Therefore, the force F_x can be written as

$$F_x = k (\Delta x).$$

Where k is a constant, known as the *Spring constant* or *Force constant*. It represents a measure of how elastic or how stiff this particular spring is. The extension in length (Δx), on the other hand, should be small because the elasticity feature of the spring may be damaged if you extend the spring beyond its elastic limit. The spring itself is found to exert a force F_s to the opposite direction of the applied force F_x . This force is known as restoring force because it tends to restore the spring to its natural length, and is given by **Hook's Law**:

$$F_s = -k (\Delta x). \quad (3)$$

We may utilize this property to calibrate a spring and use it as a measuring device for forces. By doing so, the spring is called **Dynamometer**. If we use a spring as a measuring device for masses, it is then called a **spring balance**.

In order to calculate the spring constant k of the Dynamometer in the lab, we apply a force of a known magnitude F_w , (that is equal to the weight of a known mass m). See Fig

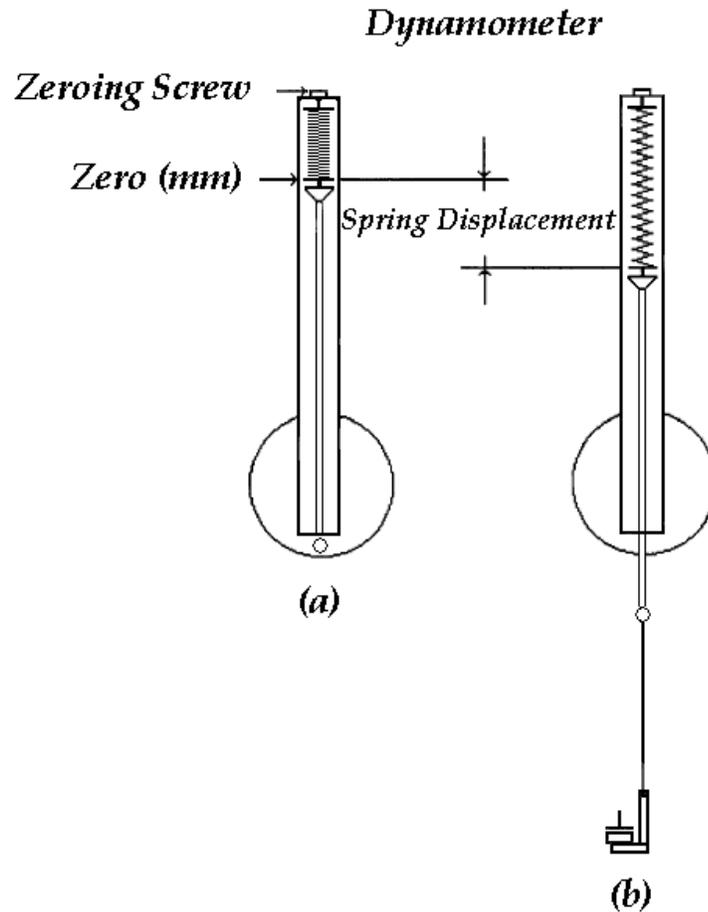


Figure 2. (a) The Dynamometer with its natural length
(b) The Dynamometer with applied force F_w .

Therefore, we can write

$$F_w + F_s = 0 \implies mg - k(\Delta y) = 0 \implies k = \left(\frac{mg}{\Delta y} \right) \quad (4)$$

Notes that the extension of spring length is denoted by Δy .

Decomposition of Forces

In this part of the experiment we test the validity of the conditions necessary for static equilibrium of forces, which are stated previously in Equation 2. Suppose we

have three forces acting on a circular object that is either at rest or moving with constant velocity. See Figure 3.

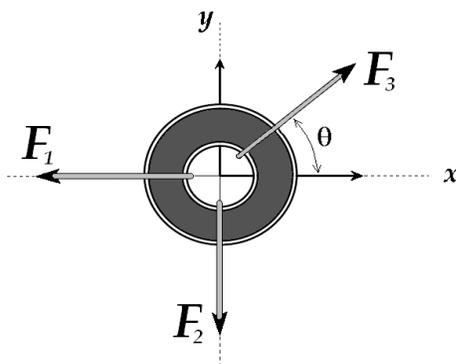


Figure 3. Free-Body Diagram

If the object is at rest, then:

- along the x -direction, we should have:

$$F_1 = F_{3_x} \implies F_1 = F_3 \cos \theta$$

- along the y -direction we should have:

$$F_2 = F_{3_y} \implies F_2 = F_3 \sin \theta$$

Refer to Figure 4. The exact same analysis is carried out to show that the force provided by the dynamometer F_d , and applied to the force ring, is in fact equal in magnitude to the x components of the force F applied to the same force ring by the hanging mass m . Similarly, along the y direction, we should show that the magnitude of the force F' applied to the force ring by the hanging mass m' is equal to the magnitude of the y component of the force F . See Figure 4.

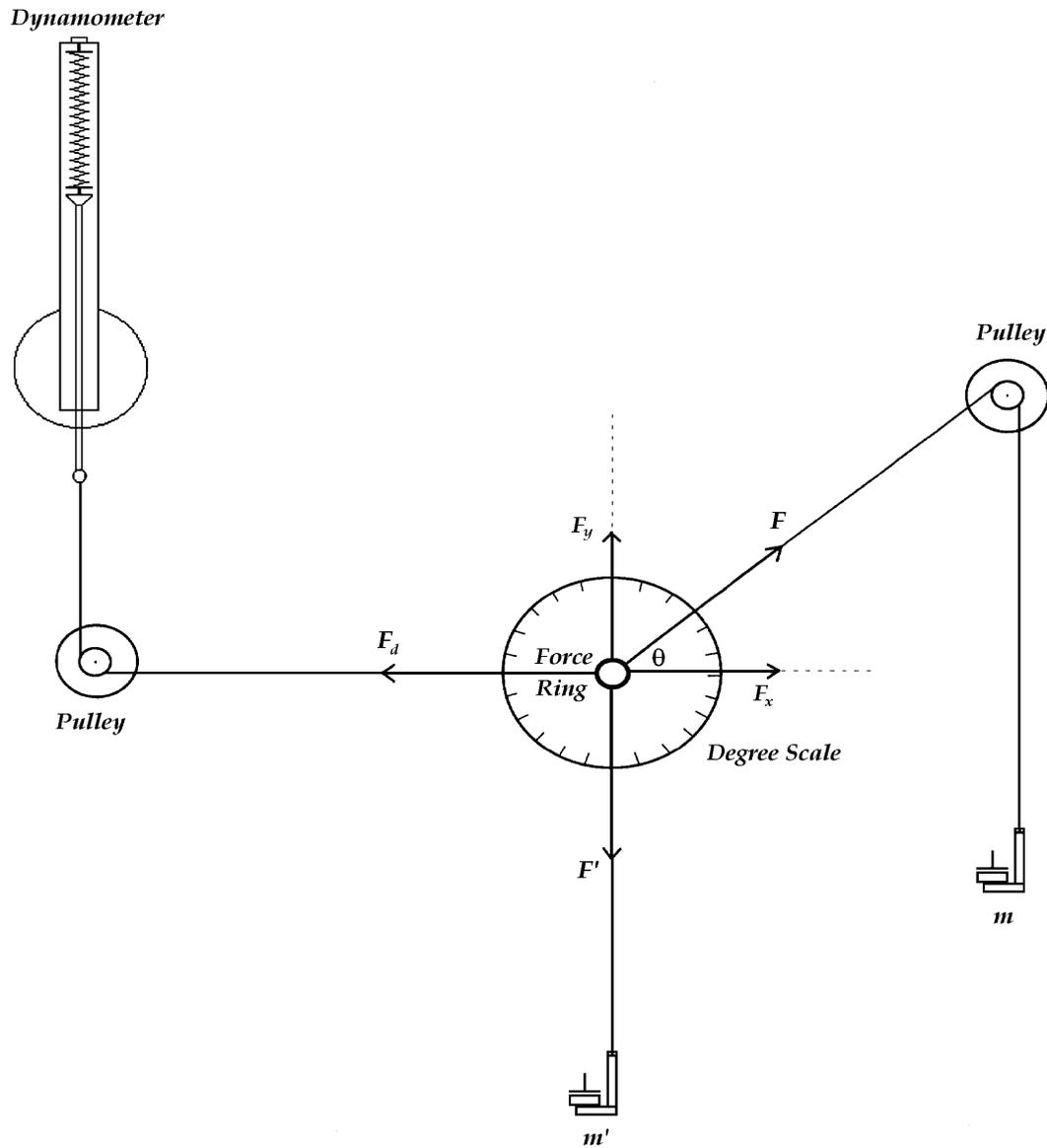


Figure 4. Experimental board with three forces applied to the force ring.

The degree scale is used to measure the angle between the applied force F and the positive direction of the x axis.

Forces along the Inclined Plane

In this part of the experiment we use the method of vector decomposition to resolve forces acting on an object on a rough inclined surface. Suppose you have an object of mass m tied by a massless string and allowed to slide down on a rough inclined plane until it come to rest. See Figure 5.

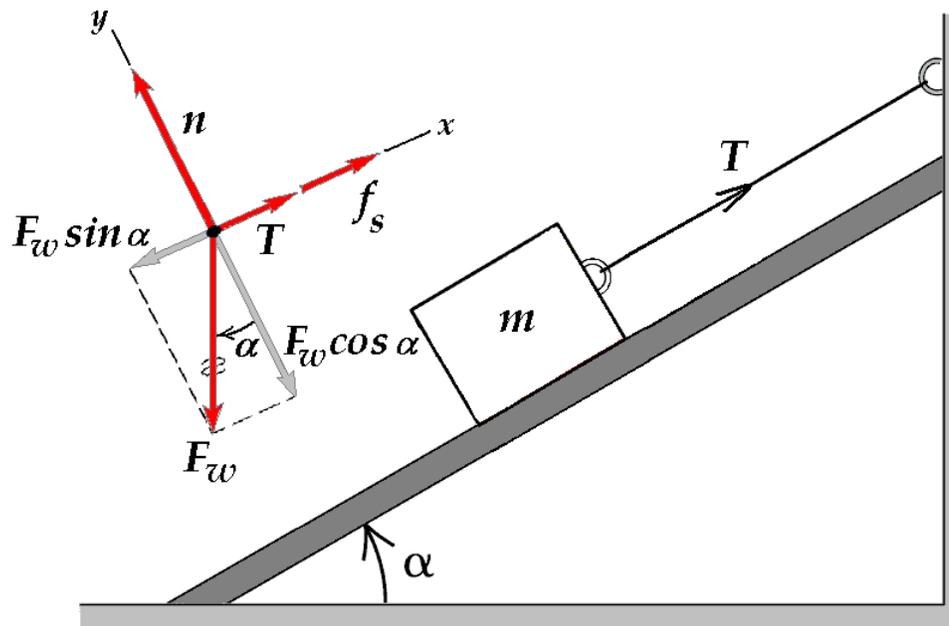


Figure 5. Object on rough inclined plane

As seen in the figures, the two forces that are acting parallel to the inclined plane and tend to prevent the block from sliding down are the tensional force T and the force of static friction f_s . Both forces are acting along the positive x -direction. The ramp exerts also a normal force N on the block that is perpendicular to the ramp. The force due to the weight of the block F_w makes an angle α with the ramp. Resolving this force into its xy -components gives:

$$F_{w_x} = F_w \sin \alpha = mg \sin \alpha \quad \text{and} \quad F_{w_y} = F_w \cos \alpha = mg \cos \alpha$$

Using the fact that the block is at equilibrium:

Along x -direction:

$$T + f_s = mg \sin \alpha \implies f_s = mg \sin \alpha - T \quad (a)$$

Similarly, along the y -direction:

$$N = mg \cos \alpha \quad (b)$$

The magnitude of the static friction force f_s usually increases when the normal force n increases. Therefore, the magnitude of the static friction can be written as

$$f_s = \mu_s N \quad (5)$$

Where μ_s is a constant called the **coefficient of static friction**. Using the results obtained earlier in (a) and (b), Equation (5) is rewritten:

$$mg \sin \alpha - T = \mu_s mg \cos \alpha \implies \mu_s = \frac{mg \sin \alpha - T}{mg \cos \alpha} \quad (6)$$

The objective of this part of the experiment is to calculate the value of the coefficient of static friction when a block of wood rests on ramp. The exact same analysis that was discussed above theoretically is followed. your experimental setup is illustrated in Figure 6 below. A minor change you should take care of is that in the experimental approach, the tensional force T is replaced by a force F_d from the dynamometer. Therefore,

$$\mu_s = \frac{mg \sin \theta - F_d}{mg \cos \theta} \quad (7)$$

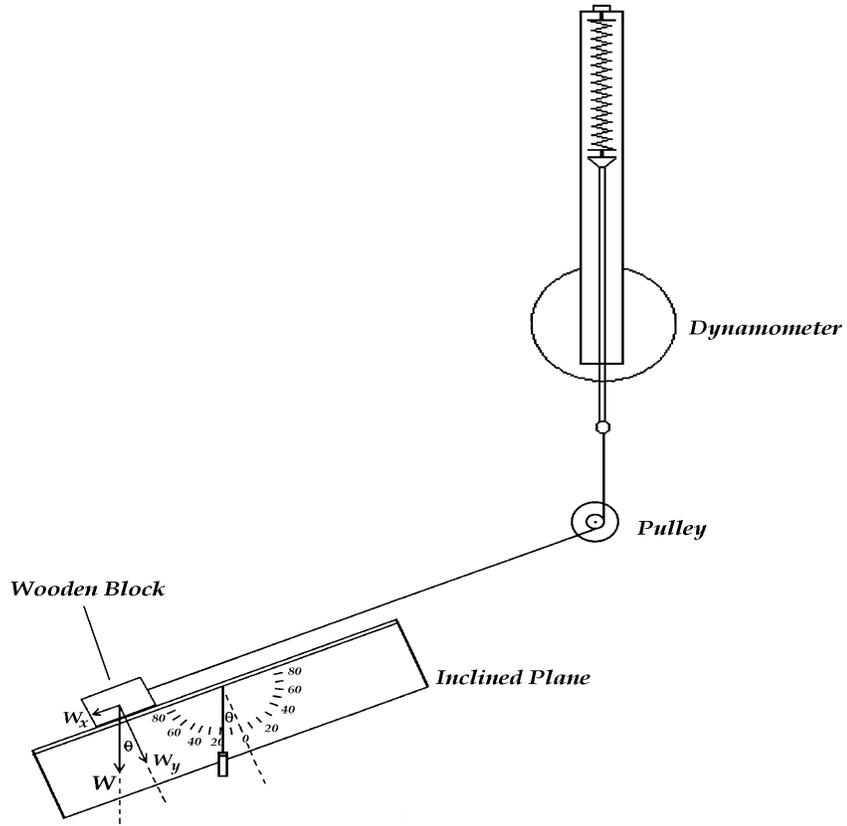


Figure 6. The experimental board with the inclined plane, block, and dynamometer

Procedure

Part I: Calibration of a spring as dynamometer

- 1) **Hang** the spring balance on the experimental board (Figure 2.). Be sure the spring hangs vertically in the plastic tube.
- 2) With no force applied to the spring balance, **adjust** the zeroing screw on the top of the spring balance until the indicator is aligned with the 0 mm mark on the millimeter scale of the spring balance as shown in Figure 2(a).
- 3) **Hang** the 5 gram mass hanger with a 50 gram mass from the spring balance.
- 4) **Measure** the spring displacement Δy on the millimeter scale as shown in Figure 2(b). **Record** the values of the total mass m_{total} and the spring displacement Δy in Table 5.1.
- 5) **Repeat** step 4 with other total masses according to Table 5.1.
- 6) **Calculate** the total weight in newtons for each value of the total mass m_{total} . **Record** your results in the table. Don't forget to convert the unit of mass to kilogram before multiplying by g .
- 7) **Calculate** the spring constant k using Equation (4).
- 8) **Plot** the applied force $F_w = mg$ versus spring displacement Δy . You will use this graph as calibration curve of the dynamometer whenever it is used as a device for measuring forces. The slope of the graph is the spring constant k for the spring used in the dynamometer.

Table 5.1: Calibration of a spring as dynamometer

m_{total} (g)	Δy (mm)	$F_w = mg$ (N)	$k = \frac{F_w}{\Delta y}$ (N/m)
75			
95			
115			
135			
175			
195			
225			

Spring constant ($k \pm \sigma_{\bar{k}}$) from the table =

Spring constant (k) from the graph =

Part II: Force Decomposition

- 1) **Set up** the equipment with mass hanger, masses, pulleys, force ring and string as shown in Figure 4.
- 2) Set up the spring balance and a pulley so the string from the balance runs horizontally from the bottom of the pulley to the force ring.
- 3) **Hang** the first mass hanger directly from the force ring, with the string running diagonally on the experimental board, and through the right pulley.
- 4) **Attach** masses to the first mass hanger. this mass is denoted by (m).
- 5) **Hang** a second mass hanger vertically and directly from the force ring.
- 6) **Attach** masses to the second mass hanger. this mass is denoted by (m').
- 7) Pull the spring balance toward or away from the pulley to adjust the horizontal x component of the force.
- 8) Adjust the position of the right pulley until the force ring is centered at the circle drawn on the degree scale. If we reach that, then equilibrium is achieved.
- 9) **Record** the masses used:

$$m = \dots\dots\dots \quad m' = \dots\dots\dots$$

- 10) **Calculate** the forces:

$$F = mg = \dots\dots\dots \quad F' = m'g = \dots\dots\dots$$

- 11) **Measure** the spring displacement, and use the previously calculated value of k to **calculate** the spring force F_d .

$$\Delta y = \dots\dots\dots \quad F_d = k(\Delta y) = \dots\dots\dots$$

- 12) **Record** the direction (angle θ) the force F makes with the horizontal, and **resolve** this force into its xy components.

$$\theta = \dots\dots\dots \quad F_x = \dots\dots\dots \quad F_y = \dots\dots\dots$$

- 13) Compare the force F' with the calculated value of F_y and comment on your result.
- 14) Compare the force F_d with the calculated value of F_x and comment on your result.

Part III: Forces along an inclined plane

- 1) Use the laboratory balance to measure the mass m of the wooden block.
- 2) **Setup the equipment** with inclined plane, angular plate, pulley, wooden block and spring balance as shown in Figure 6. Note that in this Figure the x axis is selected along the incline, therefore you have to make sure that the string is parallel to the inclined plane.
- 3) **Adjust the angle of inclination** to different values shown in Table 5.2. For each angle θ , **calculate** F_{w_x} and **measure** the force of the spring balance F_d . Record the results in Table 5.2.
- 4) **Check** whether there is a static friction force by calculating the force difference according to $f_s = F_{w_x} - F_d$. **Record** the values in Table 5.2.
- 5) Consequently **calculate** μ_s according to the equation 7.
- 6) **Calculate** the average of μ_s and the standard error of your results.

Table 5.2: Forces along the Inclined Plane.

Angle of inclination (θ)	$F_d = k(\Delta y)$ (N)	$F_{w_x} = mg \sin \theta$ (N)	$f_s = F_{w_x} - F_d$ (N)	μ_s
30°				
40°				
50°				

the coefficient of static friction ($\bar{\mu}_s \pm \sigma_{\bar{\mu}_s}$) =