Kuwait University Physics Department

Physics 105

Linear Momentum

Intoduction

The objective of this experiment is to study the conservation of the total linear momentum and the total kinetic energy for a system of two bodies that encounter elastic and inelastic one dimentional head on collisions. The experiment is done using an air track with two gliders (representing the two bodies to be collided with each other) besides some other needed accessories. By measuring the velocities of the gliders before and after collision, the total linear momentum and kinetic energy before and after collision are compared.

Objectives

- To study conservation of linear momentum,
- To study conservation of kinetic energy,

  For elastic and inelastic head on collisions.

Equipment

- $2\,m$ long air track with 2 gliders, 2 ($1\,cm$ width) flags, other type flags, mass pieces, and rubber bands.
- Photo-gate timer with accessory photo-gate.
- Laboratory Triple Balance.

Theory

The linear momentum $\vec{p}$ of an object of mass m moving with velocity $\vec{v}$ is defined as

$$\vec{p} = m\vec{v}, \tag{1}$$

When two bodies with masses m_1 and m_2 moving along a straight line with velocities $\vec{v}_{1i}$ and $\vec{v}_{2i}$ encounter a head-on collision, their initial momenta $\vec{p}_{1i}$ and $\vec{p}_{2i}$ would change to final momenta $\vec{p}_{1f}$ and $\vec{p}_{2f}$ due to impulse; only when no external forces act on the system, the total linear momentum will be conserved. i.e.:

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}, \tag{2}$$

or

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}. \tag{3}$$

For the case of elastic collision, the total kinetic energy of the system will be conserved as well. i.e.:

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2. \tag{4}$$

Solving Equation 3 and Equation 4 for v_{1f} and v_{2f} we get

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}, \tag{5}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}, \tag{6}$$

Equations 5 & 6 predict the values of the final velocities for all combinations of masses and initial velocities of the colliding objects under the conditions given above. Considering the case when $v_{2i} = 0$, Equations 5 & 6 will be reduced to

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i}. \tag{7}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i}. \tag{8}$$

Now we study the cases when:

i. $\boldsymbol{m}_1 = \boldsymbol{m}_2$ then we would have:

$$v_{1f} = 0 \quad \text{and,} \quad v_{2f} = v_{1i}.$$

ii. $\boldsymbol{m}_1 > \boldsymbol{m}_2$:

$$v_{1i} > v_{1f} > 0 \quad \text{and,} \quad v_{2f} > v_{1i}.$$

iii. $\boldsymbol{m}_1 < \boldsymbol{m}_2$:

$$0 > v_{1f} > (-v_{1i}) \quad \text{and,} \quad v_{1i} > v_{2f} > 0.$$

iv **Completely inelastic collision**: (i.e. when the two bodies stick together) and, for $\boldsymbol{m}_1 = \boldsymbol{m}_2$, if $v_{2i} = 0$,

Equation 3 yields

$$v_{T_f} = 0.5\, v_{1i},$$

Equation 4 yields

$$KE_{T_f} = 0.5\, KE_{1i}.$$

Procedure

The photo-gate timer can be used to measure the time that a moving objects takes in crossing a certain position, so that to calculate its velocity. The photo-gate timer works in different modes; in this experiment it is used only in the gate-mode; therefore, it is important to know how does it function in this mode.

The photo-gate timer function in the Gate mode:

In this mode, the timer records the time duration for blocking any gate (whether the main or the accessory photo-gate). For example, if an object with width Δx crosses any of the two gates, the timer would measures the time t that the object takes to cross that gate, and thereby, it would be possible to calculate the speed v with which the object has passed the gate as

$$v = \frac{\Delta x}{t}. \tag{9}$$

The photo-gate timer function in the Gate mode with memory "ON":

With the memory switch set to "ON", the photo-gate timer is capable of measuring and recording successive time durations for blocking any gate (regardless of their order), and adding the measured values to its built in memory. For example, if an object with width Δx crosses the two gates successively, the timer would measure, record, and displays the time from the first gate t_1, then adds the time measured by the second gate t_2 to t_1, and save in the memory. Therefore, the steps for retrieving t_1 & t_2 can be abbreviated as:

- the displayed value represents t_1.
- set the memory switch to **Read** position
- t_2 = (the displayed value) $- t_1$.

Measurements one: (Elastic collision, $m_1 = m_2$, and $v_{2i} = 0$)

1) Level the air-track (**Figure 7**) by adjusting the two screws beneath the legs at one side of the air-track until a glider placed at the center of track becomes stationary (with the air pump is "ON").

2) Use two gliders with the $1\,cm$ flags mounted on top of each; fix 4 rubber band holders; one at each side of the gliders.

3) Put the main photo-gate timer at $x = 70\,cm$, the accessory photo-gate timer at $x = 110\,cm$ then, adjust the height of each gate such that the flag of each glider blocks the photo beam when it passes through.

4) Set the timer mode to **gate**, the range to $0.1\,ms$, and the memory switch to "ON".

5) Turn on the air pump then, put one of the gliders m_2 at $x = 100\,cm$; reset the timer then, push the other glider m_1 back against the rubber band (at the left side of the air track) for a fixed distance i,e, $\Delta x = 3\,cm$ and, release it.

6) After the collision takes place, stop both the gliders after each glider has passed one gate so that not to block the gates twice. Turn off the air pump then:
   - the displayed value represents t_{1_i}.
   - set the memory switch to **Read** position.
   - t_{2_f} = (the displayed value) $- t_{1_i}$.

7) Repeat steps 5 & 6 two more times.

Table I. (Elastic collision, $m_1 = m_2$, and $v_{2_i} = 0$)

Initial						Final					
t_{1_i}	t_{1_i}	t_{1_i}	$\bar{t_{1_i}}$	v_{1_i}	v_{2_i}	t_{2_f}	t_{2_f}	t_{2_f}	$\bar{t_{2_f}}$	v_{1_f}	v_{2_f}
					0					?	

Measurements:

$m_1 =$ .... , $m_2 =$ ....

Check for the conservation of momentum:

$P_{T_i} =$ .... , $P_{T_f} =$ ....

Is momentum conserved? ....

Determine the percentage error in the final total momentum:

$$\left(\frac{P_{T_i} - P_{T_f}}{P_{T_i}}\right) \times 100 \; = \; ....$$

Check for the conservation of kinetic energy:

$KE_{T_i} =$ .... , $KE_{T_f} =$ ....

Is kinetic energy conserved? ....

Determine the percentage error in the final kinetic energy:

$$\left(\frac{KE_{T_i} - KE_{T_f}}{KE_{T_i}}\right) \times 100 \; = \; ....$$

Measurements two: (Elastic collision, $m_1 > m_2$, and $v_{2_i} = 0$)

1) Add $100\,g$ to m_1 ($50\,g$ on each side of the glider).

2) Use only the main photo-gate timer to be at $x = 80\,cm$.

3) Set the timer mode to **gate**, and the range to $0.1\,ms$.

4) Turn on the air pump then, reset the timer then, push glider 1 (m_1) back against the rubber band (at the left side of the air track) for a fixed distance i,e, $\Delta x = 3\,cm$ and, release it.

5) After the collision takes place, stop both the gliders so that not to interrupt the gate any more. Turn off the air pump then, record t_{1_i} in **Table II**.

6) Repeat steps 4 & 5 two more times.

7) Now displace the main photo-gate timer to be at $x = 110\,cm$.

8) Set the memory switch to "ON".

9) Turn on the air pump then, put glider 2 (m_2) at $x = 100\,cm$; reset the timer then, push glider 1 (m_1) back against the rubber band (at the left side of the air track) for a fixed distance i.e. $\Delta x = 3\,cm$ and, release it.

10) After the collision takes place, stop both the gliders so that not to interrupt the gate more than once each. Turn off the air pump then:
    - the displayed value represents t_{2_f}.
    - set the memory switch to **Read** position.
    - $t_{1_f} =$ (the displayed value) $- t_{2_f}$.

11) Record t_{1_f} and t_{2_f} in **Table III**.

12) Repeat steps 9-11 two more times.

Table II. (Elastic collision, $m_1 > m_2$, and $v_{2_i} = 0$)

Initial					
t_{1_i}	t_{1_i}	t_{1_i}	$\bar{t_{1_i}}$	v_{1_i}	v_{2_i}
					0

Table III. (Elastic collision, $m_1 > m_2$, and $v_{2_i} = 0$)

Final									
t_{1_f}	t_{1_f}	t_{1_f}	$\bar{t_{1_f}}$	t_{2_f}	t_{2_f}	t_{2_f}	$\bar{t_{2_f}}$	v_{1_f}	v_{2_f}

Measurements:

$m_1 =$ .... , $m_2 =$ ....

Check for the conservation of momentum:

$P_{T_i} =$ .... , $P_{T_f} =$ ....

Is momentum conserved? ....

Determine the percentage error in the final total momentum:

$$\left(\frac{P_{T_i} - P_{T_f}}{P_{T_i}}\right) \times 100 \ = \ \text{...}$$

Check for the conservation of kinetic energy:

$KE_{T_i} =$ .... , $KE_{T_f} =$ ....

Is kinetic energy conserved? ....

Determine the percentage error in the final kinetic energy:

$$\left(\frac{KE_{T_i} - KE_{T_f}}{KE_{T_i}}\right) \times 100 \ = \ \text{...}$$

Measurements three: (Elastic collision, $m_1 < m_2$, and $v_{2_i} = 0$)

1) Displace the $100\,g$ from m_1 to m_2 ($50\,g$ on each side of the glider) or you can just exchange the two gliders! such that $m_1 < m_2$.

2) Use only the main photo-gate timer to be at $x = 80\,cm$.

3) Set the timer mode to **gate**, and the range to $0.1\,ms$.

4) Turn on the air pump then, reset the timer then, push glider 1 (m_1) back against the rubber band (at the left side of the air track) for a fixed distance i,e, $\Delta x = 3\,cm$ and, release it.

5) After the collision takes place, stop both the gliders so that not to interrupt the gate any more. Turn off the air pump then, record t_{1_i} in **Table IV**.

6) Repeat steps 4 & 5 two more times.

7) Put the main photo-gate timer at $x = 70\,cm$, the accessory photo-gate timer at $x = 110\,cm$, and set the memory switch to "ON".

8) Turn on the air pump then, put glider 2 (m_2) at $x = 100\,cm$; push the glider 1 (m_1) back against the rubber band (at the left side of the air track) for a fixed distance i.e. $\Delta x = 3\,cm$ and, release it.

9) Immediately after the collision takes place, reset the timer!

10) After the gliders cross both the gates (glider 2 will pass first!), stop them so that not to interrupt the gates any more. Turn off the air pump then:
    - the displayed value represents t_{2_f}.
    - set the memory switch to **Read** position.
    - $t_{1_f} =$ (the displayed value) $- t_{2_f}$.

11) Record t_{1_f} and t_{2_f} in **Table V**. Repeat steps 9-12 two more times.

Table IV. (Elastic collision, $m_1 < m_2$, and $v_{2_i} = 0$)

Initial					
t_{1_i}	t_{1_i}	t_{1_i}	$\bar{t_{1_i}}$	v_{1_i}	v_{2_i}
					0

Table V. (Elastic collision, $m_1 < m_2$, and $v_{2_i} = 0$)

Final									
t_{1_f}	t_{1_f}	t_{1_f}	$\bar{t_{1_f}}$	t_{2_f}	t_{2_f}	t_{2_f}	$\bar{t_{2_f}}$	v_{1_f}	v_{2_f}

Measurements:

$m_1 =$ .... , $m_2 =$ ....

Check for the conservation of momentum:

$P_{T_i} =$ .... , $P_{T_f} =$ ....

Is momentum conserved? ....

Determine the percentage error in the final total momentum:

$$\left(\frac{P_{T_i} - P_{T_f}}{P_{T_i}}\right) \times 100 \ = \ \text{...}$$

Check for the conservation of kinetic energy:

$KE_{T_i} =$ .... , $KE_{T_f} =$ ....

Is kinetic energy conserved? ....

Determine the percentage error in the final kinetic energy:

$$\left(\frac{KE_{T_i} - KE_{T_f}}{KE_{T_i}}\right) \times 100 \ = \ \text{...}$$

Measurements four: (Elastic collision, $m_1 = m_2$, and $v_{2i} = 0$)

1) Remove the extra masses from m_2, such that $m_1 = m_2$ and remove the flag from glider 2.

2) Replace the rubber band holders that face each other in the two gliders, with two flags: one supplied with a nail and the other supplied with hard wax.

3) Put the main photo-gate timer at $x = 70\,cm$, the accessory photo-gate timer at $x = 110\,cm$ then, adjust the height of each gate such that the flag of glider 1 blocks the photo beam when it passes through.

4) Set the timer mode to **gate**, the range to $0.1\,ms$, and the memory switch to "ON".

5) Turn on the air pump then, put glider 2 (m_2) at $x = 100\,cm$; reset the timer then, push the other glider (m_1) back against the rubber band (at the left side of the air track) for a fixed distance i.e. $\Delta x = 3\,cm$ and, release it.

6) After the collision takes place (the two gliders are supposed to stick together!), stop the gliders after they have passed the second gate so that not to block the gates twice. Turn off the air pump then:
   - the displayed value represents t_{1_i}.
   - set the memory switch to **Read** position.
   - $t_{T_f} =$ (the displayed value) $-t_{1_i}$.

7) Record t_{1_i} and t_{T_f} in **Table VI**.

8) Repeat steps 5-7 two more times.

Table VI. (Completely inelastic collision, $m_1 = m_2$, and $v_{2_i} = 0$)

Initial						Final				
t_{1_i}	t_{1_i}	t_{1_i}	$\bar{t_{1_i}}$	v_{1_i}	v_{2_i}	t_{T_f}	t_{T_f}	t_{T_f}	$\bar{t_{T_f}}$	v_{T_f}
					0					

Measurements:

$m_1 =$ .... , $m_2 =$ ....

Check for the conservation of momentum:

$P_{T_i} =$ .... , $P_{T_f} =$ ....

Is momentum conserved? ....

Determine the percentage error in the final total momentum:

$$\left(\frac{P_{T_i} - P_{T_f}}{P_{T_i}}\right) \times 100 \;=\; \text{...}$$

Check for the conservation of kinetic energy:

$KE_{T_i} =$ .... , $KE_{T_f} =$ ....

Is kinetic energy conserved? ....

Determine the percentage error in the final kinetic energy:

$$\left(\frac{KE_{T_i} - KE_{T_f}}{KE_{T_i}}\right) \times 100 \;=\; \text{..}$$