Kuwait University Physics Department
Physics 105

Kinematics & Newton's 2^{nd} law

Introduction

The objective of this experiment is to study kinematics and Newton's 2^{nd} law for a uniformly accelerated one-dimensional motion. The experiment is carried out using an air track equipped with two photo-gate timers and a glider. In part one of the experiment, the displacement versus time is investigated. Part two studies the relation between the instantaneous and the average velocity. Part three relates the velocity to time. Finally, in part four Newton's 2^{nd} law is applied for a two bodies system.

Objectives

- To study displacement versus time for a uniformly accelerated motion.
- To be able to diffrenciat between average and instantaneous velocities.
- To acquire knowledge of using velocity-time graphs.
- To understand and be able to apply Newton's 2^{nd} Law.

  for a one-dimensional motion.

Equipment

- 2 m air track with glider, pulley, hanger, mass pieces and rubber bands.
- Photo-gate timer with two photo-gates.
- Laboratory Triple Balance

Theory

Displacement versus time:

If an object is set to move under the action of gravity on an inclined frictionless track, it will move with a constant acceleration

$$a \;=\; g\,sin\,\theta, \tag{1}$$

were g is the acceleration due to gravity and θ is the angle of inclination of the track. The displacement Δx at a given time t will be given as

$$\Delta x \;=\; v_o\,t \;+\; \frac{1}{2}\,a\,t^2, \tag{2}$$

were v_o is the velocity at t = 0. If the motion starts from rest then Equation 2 reduces to

$$\Delta x \;=\; \frac{1}{2}\,a\,t^2. \tag{3}$$

A plot of Δx versus t represents a parabola (a curve). See Figure 1.

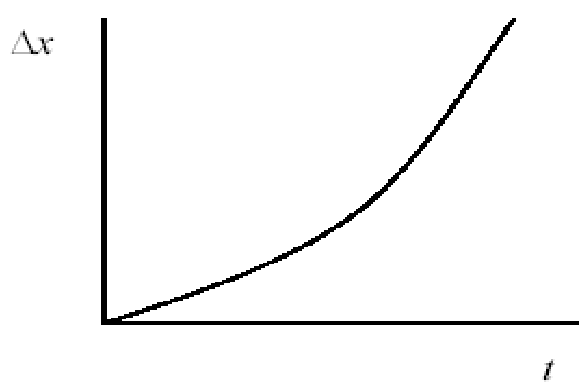

Figure 1

Instantaneous versus average velocity:

For a given interval of time Δt the object would have a displacement Δx, and the average velocity $\bar{v}$ for that interval is defined as

$$\bar{v} \; = \; \frac{\Delta x}{\Delta t}. \tag{4}$$

The importance of the average velocity lies in the fact that if an object moves with a constant velocity equals the average velocity it would cover the same displacement in the same interval of time. The instantaneous velocity v at a given time is defined as

$$v \; = \; \lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t} \right) = \frac{dx}{dt}. \tag{5}$$

Equation 5 indicates that the instantaneous velocity equals the slope of the tangent to the curve of Δx versus t at a given point. See Figure 2.

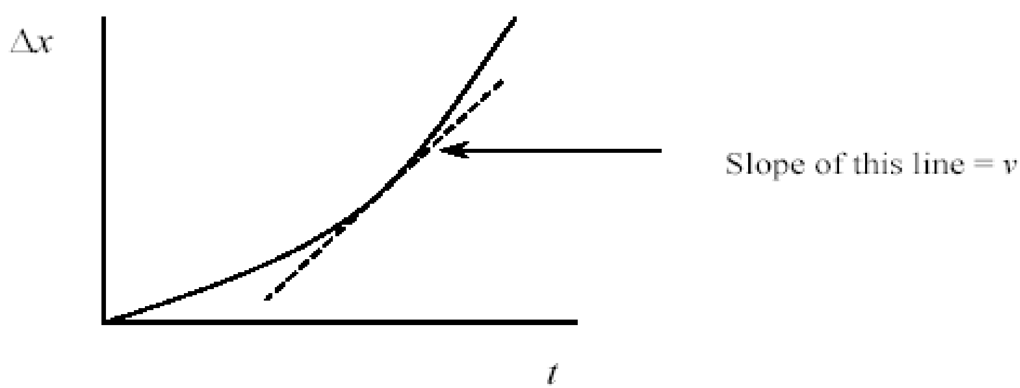

Figure 2

By decreasing the displacement Δx about a specific position x' and measuring the corresponding time interval Δt, if $\bar{v}$ is plotted against Δx, a curve would result, that saturates towards a limiting value as $\Delta x \to 0$. If the curve is extrapolated, it will intercept the $\bar{v}$ axis at a value equal to the instantaneous velocity v. See Figure 3.

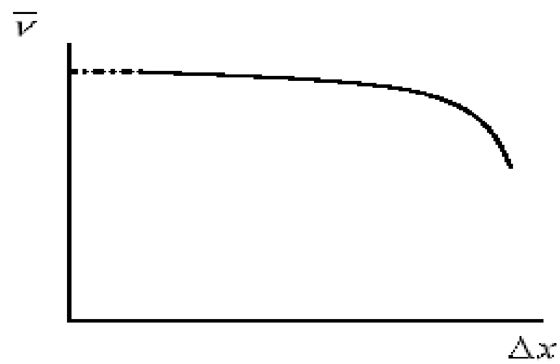

Figure 3

Velocity versus time:

The velocity of the object moving under a constant acceleration a at a given time t is given as

$$v \; = \; v_o \; + \; a\,t, \tag{6}$$

also, the velocity at the end of a displacement Δx is given as

$$v^2 \; = \; v_o^2 \; + \; 2\,a\,\Delta x. \tag{7}$$

Therefore a plot of v versus t results in a straight line were the slope represents the acceleration a. See Figure 4. Also, if we plot v^2 versus Δx we would get a straight line were the slope equals $2\,a$. See Figure 5. Furthermore, Equation 3 can be used to plot Δx vs t^2, which also represents a straight line were the slope represents $\frac{a}{2}$.

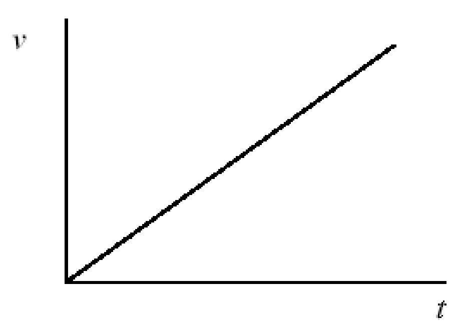

Figure 4

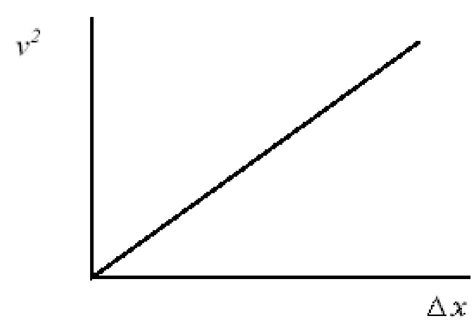

Figure 5

Newton's 2nd law:

If an object with mass M, which is free to move on a frictionless horizontal track, is connected by a string parallel to the track to a hanging mass m after passing over a frictionless and massless pulley, It will move with a uniformly accelerated motion such that

$$F = (M+m)\,a, \tag{8}$$

were $F = mg$, and a is the acceleration that the two mass system will move with. Now, if mass is transferred from M to m such that the total mass $(M+m)$ remains constant, F would increase, and consequently, the acceleration a, since F is directly proportional to a, therefore a plot of F versus a results in a straight line were the slope equals the total mass $(M+m)$. See Figure 6.

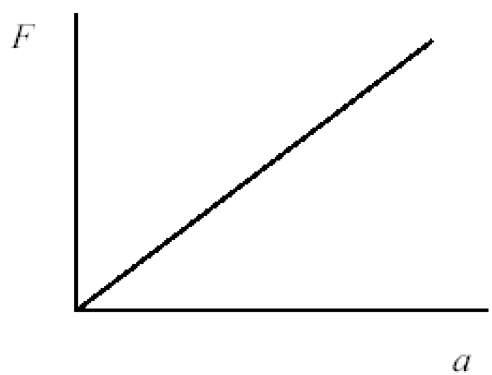

Figure 6

Procedure

Part I: Displacement versus time:

1) Let the air-track (see Fig. 7) be inclined by putting 50 g mass units beneath the legs of one side (in this case the inclination angle $\theta \cong 0.8^o$ and, the acceleration $a = (g sin\,\theta) \cong 0.14$ m/s^2).

2) Put the main photo-gate timer at $x_1 = 10\,$cm and the accessory photo-gate timer at $x_2 = 20\,$cm, set the mode of the timer to **pulse** and the range to 1 ms.

3) Using a glider with the flag of 1 cm width mounted on top, adjust the height of each timer so that the flag of the glider blocks the photo beam when it passes through.

4) Hold the glider at the beginning of the track (such that the flag is just before the main photo-gate), reset the timer and, turn on the air pump then, release the glider. Record the time in **Table I**.

5) Repeat the step 4, two more times.

6) Change the position of the accessory photo-gate timer according to the table then, repeat steps 4 & 5.

Table I.

x_1 (cm)	x_2 (cm)	Δx (cm)	t_1 (s)	t_2 (s)	t_3 (s)	$\bar{t}$ (s)
10	20	10				
10	40	30				
10	60	50				
10	80	70				
10	100	90				
10	120	110				
10	140	130				
10	160	150				
10	180	170				

7) Plot Δx versus $\bar{t}$, then determine the slope of the tangent to the curve at $\Delta x = 90\,\text{cm}$ (Note that $\Delta x = 90\,\text{cm}$ corresponds to $x_2 = 100\,\text{cm}$). What does the slope represents?

....

8) Estimate the instantaneous velocity v at $x_2 = 100\,\text{cm}$ using Equation 7.

....

9) Determine the percentage error in the instantaneous velocity v:

$$\left(\frac{v_{estimated} - v_{measured}}{v_{estimated}}\right) \times 100 \; = \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Part II: Instantaneous versus average velocity::

1) Let the air-track be inclined as in part I.

2) Put the main photo-gate timer at $x_1 = 30\,\text{cm}$ and the accessory photo-gate timer at $x_2 = 170\,\text{cm}$, set the mode of the timer to **pulse** and the range to $1\,ms$.

3) Using a glider with the flag of 1 cm width mounted on top, adjust the height of each timer so that the flag of the glider blocks the photo beam when it passes through.

4) Hold the glider at the beginning of the track, reset the timer and turn on the air pump then, release the glider. Record the time in **Table II**.

5) Repeat the step 4, two more times.

6) Set the position of each timer according to **Table II** and repeat steps 4 & 5.

Table II.

x_1 (cm)	x_2 (cm)	Δx (cm)	Δt_1 (s)	Δt_2 (s)	Δt_3 (s)	$\bar{\Delta t}$ (s)	$\bar{v}$ (m/s)
30	170	140					
40	160	120					
60	140	80					
70	130	60					
80	120	40					
90	110	20					

7) Change the timer mode to **gate** and the range to $0.1\,ms$.

8) Replace the 1 cm flag with the 10 cm (black) flag.

9) Put the main photo-gate timer at $x = 95\,$cm, and remove the accessory photo-gate timer. Repeat steps 4 & 5, but record the time in **Table III**.

10) Replace the 10 cm flag with the 1 cm one, put the main photo-gate timer at $x = 99.5\,$cm, repeat steps 4 & 5 but, record the time in **Table III**.

Table III.

Flag width	Timer Position x	Δx (cm)	Δt_1 (s)	Δt_2 (s)	Δt_3 (s)	$\bar{\Delta t}$ (s)	$\bar{v}$ (m/s)
10 cm	95 cm	10					
1 cm	99.5 cm	1					

11) Plot $\bar{v}$ versus Δx, using the data from both the **Tables II** & **III** then, estimate the value of the instantaneous velocity at $x = 100\,\text{cm}$. Does it equal the value of v_{inst} that was estimated in part I?

....

12) Determine the percentage error in the instantaneous velocity v:

$$\left(\frac{v_{estimated} - v_{measured}}{v_{estimated}}\right) \times 100 \ = \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Part III: Velocity versus time:

1) Let the air-track be inclined as in part I.

2) Put the accessory photo-gate timer at $x_1 = 10\,\text{cm}$ and the main photo-gate timer at $x_2 = 60\,\text{cm}$.

3) Using a glider with the flag of 1 cm width mounted on top, adjust the height of each timer so that the flag of the glider blocks the photo beam when it passes through.

4) Set the mode of the timer to **pulse** and the range to $1\,m$s.

5) Hold the glider at the beginning of the track, reset the timer and, turn on the air pump then, release the glider. Record the time in **Table IV**.

6) Repeat the step 5, two more times.

7) Change the mode to **gate** and the range to $0.1\,m$s.

8) Remove accessory photo-gate timer then, repeat steps 5 & 6, but record the measured time under Δt.

9) Return the accessory photo-gate timer to its previous position (i.e. at $x_1 =$ 10 cm then, change the position of the main photo-gate timer according to **Table IV** and, repeat steps 4-8.

10) For each Δx in the table, calculate:

$$v = \frac{0.01}{\bar{\Delta t}} \qquad \text{and} \qquad a = \frac{v}{\bar{t}}$$

Table IV: $x_1 = 10\,\text{cm}$

		Pulse Mode, $1\,ms$				Gate Mode, $0.1\,ms$					
x_2 (cm)	Δx (cm)	t_1 (s)	t_2 (s)	t_3 (s)	$\bar{t}$ (s)	Δt_1 (s)	Δt_2 (s)	Δt_3 (s)	$\bar{\Delta t}$ (s)	v (m/s)	a (m/s^2)
60											
80											
100											
120											
140											

$\bar{a}$ = ...

11) Plot a graph for v versus $\bar{t}$, from which determine the slope. What does the slope represents?

....

12) Determine the percentage error in the acceleration a:

$$\left(\frac{a_{estimated} - a_{measured}}{a_{estimated}}\right) \times 100 \ = \ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

Part IV: Newton's 2^{nd} Law:

1) Level the air track such that it is completely horizontal, by balancing a glider in the middle of the air-track while the air pump is on.

2) Put the accessory photo-gate timer at $x_1 = 70\,\text{cm}$ and the main photo-gate timer at $x_2 = 130\,\text{cm}$. See Figure 8.

3) Using a glider with the flag of $1\,cm$ width mounted on top, adjust the height of each timer so that the flag of the glider blocks the photo beam when it passes through.

4) Put $40\,g$ mass of $10\,g$ units on each side of the glider (i.e. net mass of $80\,g$).

5) Use a suitable length string, connect the glider to a mass hanger through the pulley such that the flag of the glider is to be located at $x = 68\,\text{cm}$ while the mass hanger is at maximum height from the ground.

6) Set the mode of the timer to **pulse** and the range to $1\,m$s.

7) Hold the glider such that its flag is at $x = 68\,\text{cm}$, reset the timer and, turn on the air pump then, release the glider. Record the time in **Table V**.

8) Repeat the step 7, two more times.

9) Change the mode to **gate** and the range to $0.1\,m$s.

10) Remove accessory photo-gate timer then, repeat steps 7 & 8, but record the measured time under Δt.

11) Return the accessory photo-gate timer to its previous position (i.e. at $x_1 = 70\,\text{cm}$) and, move $20\,g$ from the glider to the mass hanger (take $10\,g$ from each side), then repeat steps 6-10.

12) For each combination of M & m determine:

$$v = \frac{0.01}{\bar{\Delta t}} \qquad \text{and} \qquad a = \frac{v}{\bar{t}}$$

13) Plot a graph of F versus a, determine the slope. What does the slope represents?

....

Table V: $x_1 = 70\,\text{cm}$, $x_2 = 130\,\text{cm}$, $x_{flag} = 68\,\text{cm}$

			Pulse Mode, $1\,ms$				Gate Mode, $0.1\,ms$					
M (kg)	m (kg)	F (N)	t_1 (s)	t_2 (s)	t_3 (s)	$\bar{t}$ (s)	Δt_1 (s)	Δt_2 (s)	Δt_3 (s)	$\bar{\Delta t}$ (s)	v (m/s)	a (m/s^2)

$\bar{a}$ = ...

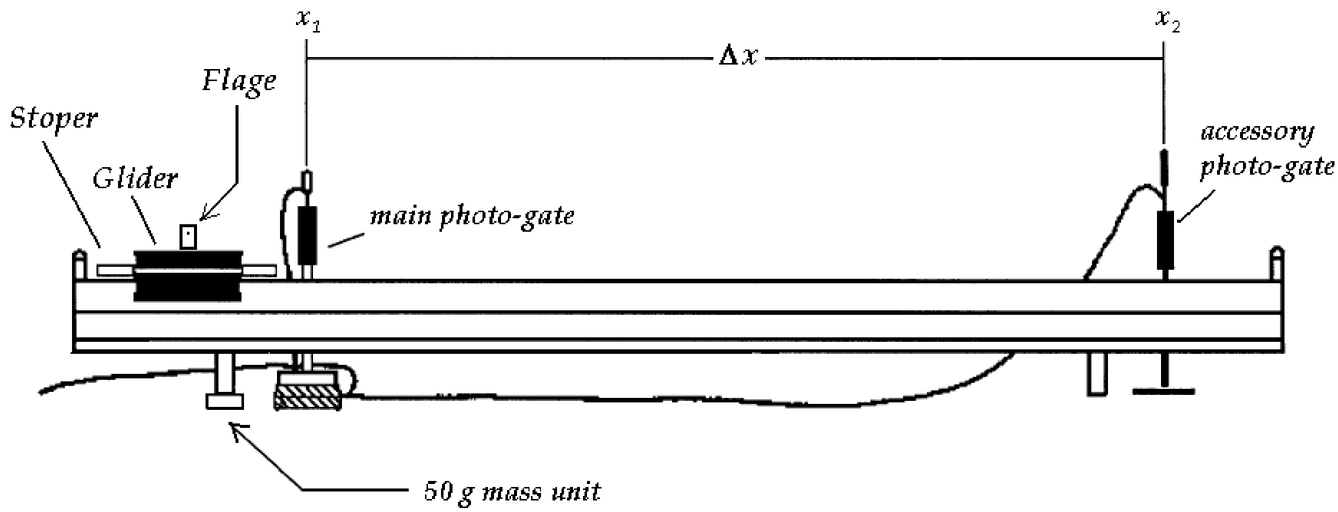

Figure 7. Air Track setup used in Part I: Displacement versus time

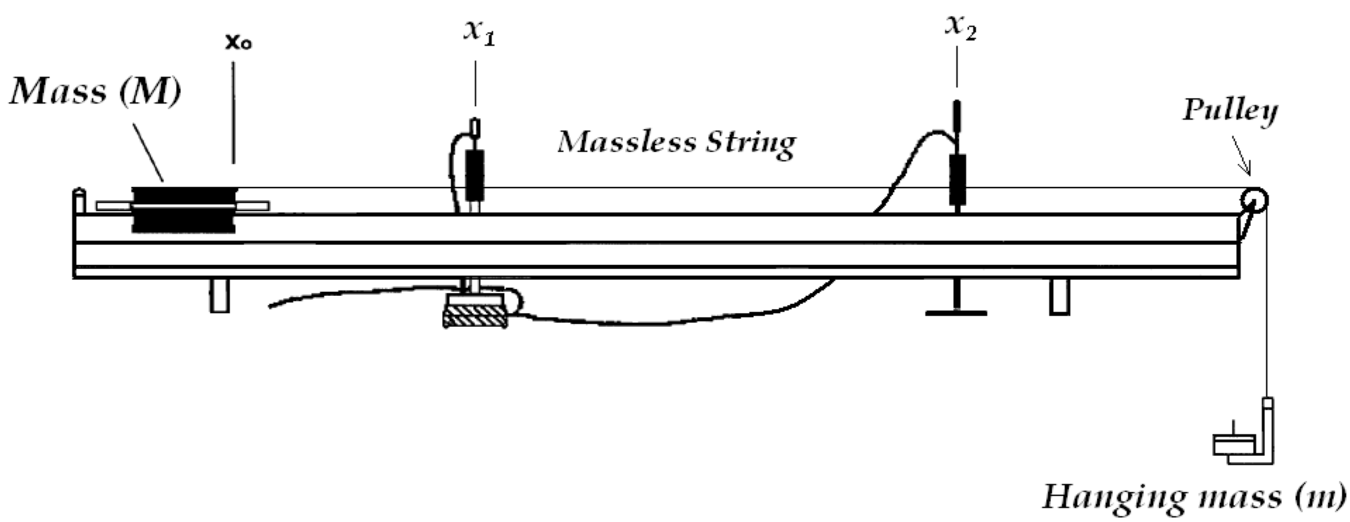

Figure 8. Air Track setup used in Part IV: Newton's 2^{nd} Law