

Kuwait University
Physics 105

Physics Department

Error Analysis, Processing of Lab Results, and Measurements of Density

Introduction

This experiment is an important experiment in which two essential concepts are being introduced. The first part of this experiment is designed to serve the goal of explaining the types of experimental errors a student will encounter in all other experiments further to come, and how to numerically analyze and calculate the error in the data. The second part of this experiment has the objective of illustrating to the students the most frequently used measuring devices and how to use them properly to get accurate data required. The importance of these two concepts is based on the fact that the student's entire work throughout the course will depend upon their understanding of these concepts.

Part A. Error Analysis, and processing of Lab Results

Objectives

- Measuring the length of a simple pendulum, using a 1m ruler.
- Measuring the oscillation period of the same pendulum, by a stop watch.
- Observing systematic and random errors.
- Calculating the average and the standard error of measured data using two different methods (i.e, the **tabulation** and the **calculator** methods).

Theory

1. Errors

Error is known to be the difference between a calculated or observed value and the true value. All experimental uncertainties are due to the presence of two types of experimental errors: **systematic errors** and **random errors**. The difference between random errors and systematic errors can be shown by repeating the measurement of a physical quantity several times under the same conditions. Random errors are statistical fluctuations or variations in the measured data produced by the experimenter's inability to take the same measurement in exactly the same way to get exactly the same reading. Therefore, the readings will be spread about the true value. See Figure 1 (a). Systematic errors, on the other hand, are reproducible inaccuracies that causes the measurements to constantly be in the same direction (either too high or too low). They are mostly due to defects in the measuring devices which make them continually present throughout the entire experiment. Therefore, the readings will always be displaced far from the true value see Figure 1 (b). For that reason, systematic errors are difficult to detect and cannot be analyzed statistically.

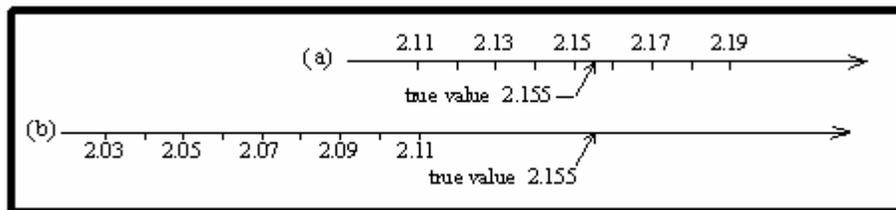


Figure 1: (a) A set of measurements taken with random errors only.
 (b) shows a set of measurements with both systematic and random errors.

- The **Accuracy** of an experiment is a measure of how close the result obtained for a given experiment is compared to the true value.
- The **Precision** of an experiment is a measure of how exactly the result is determined.

2. Statistical analysis of random errors

- **Recording** the data

Assume x represents a physical quantity, such as the period of a simple pendulum. In measuring x six times, the following data were recorded: $x_1 = 2.13$, $x_2 = 2.11$, $x_3 = 2.17$, $x_4 = 2.15$, $x_5 = 2.19$, and $x_6 = 2.18$.

- **Calculating the Mean Value**

For this set of measurements, to compute the mean value of your result (also known as the **Average value**) you should use the following equation :

$$\bar{x} = \sum_{i=1}^{i=n} \frac{x_i}{n} = \frac{x_1 + x_2 + \dots + x_n}{n} \quad (1)$$

where x represents the physical parameter measured, n equals the number of measurements, and x_i is the i^{th} measured value. Using Equation (1) for number of measurements is $n = 6$, the mean value of your results is $\bar{x} = 2.155$. see Figure 1(a).

- **Determining the Standard Error**

To compute the standard error of the measurements, we have to introduce the **standard deviation** (σ_x). The term refers to a statistical quantity which tells you how tightly your measurements are clustered around the mean value of your set of data. In other words, standard deviation is a good way to measure the spread of your data around the mean value. If the standard deviation is small then the spread is also small, which indicates that your data hold great deal of accuracy.

The following law is used to calculate the standard deviation:

$$\sigma_x = \sqrt{\frac{1}{n-1} \sum_{i=1}^{i=n} (x_i - \bar{x})^2} \quad (2)$$

$$= \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n - 1}}$$

Finally, the standard deviation is used to calculate the **standard error** ($\sigma_{\bar{x}}$), as follows: The uncertainty or the standard error in the mean value is defined as:

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} \quad (3)$$

The result to be reported from any measurement is

$$\bar{x} \pm \sigma_{\bar{x}} \quad (4)$$

3. Using the Pocket Calculator

The same analysis used to compute the mean value, standard deviation, and the standard error can also be carried out using the pocket calculator. To do so, change the mode in your calculator to the statistical mode, known as the “**SD**” mode, which will provide you with subroutines already programmed in your calculator to compute the mean value and the standard deviation required. A simple procedure is discussed to illustrate such a use of the pocket calculator:

- **Convert** the calculator to the “*SD*” mode.
- **Reset** the memory cells of the “*SD*” mode.
- **Enter** all the data (i.e, Enter each data and press the “*data*” or “*DT*” button after each one.
- To get the average **press the button** \bar{x} .
- To get the standard deviation **press the button** σ_{n-1} .

Procedure

A.1 The length of the simple pendulum

- 1) **Measure** the length of the simple pendulum from the suspended point to the center of the sphere using a 1m ruler.
- 2) **Repeat** the measurement 6 times, **record** them in Table (1).
- 3) **Calculate** the mean length \bar{L} according to equation 1.
- 4) Use the calculator to **compute** the mean length \bar{L} directly.
- 5) **Complete** the data analysis of Table (2).
- 6) **Calculate** the standard deviation for the length σ_L according to equation 2.
- 7) Use the calculator to **calculate** the standard deviation σ_L .
- 8) **Calculate** the standard error $\sigma_{\bar{L}}$ using equation 3.

A.1 The length of the simple pendulum

Table 1.

quantity	unit	l_1	l_2	l_3	l_4	l_5	l_6
l_i							

\bar{l} (using equation 1) =.....

\bar{l} (using calculator) =.....

Table 2.

quantity	l_1	l_2	l_3	l_4	l_5	l_6
l_i						
$(l_i - \bar{l})$						
$(l_i - \bar{l})^2$						

σ_l (using equation 2) =.....

σ_l (by calculator) =.....

$\sigma_{\bar{l}}$ (using equation 3) =.....

A.2 The oscillation period of the simple pendulum

- 1) **Measure** the time (t) for n oscillations.
- 2) **Calculate** the period T ($T = t/n$) of one oscillation, and **record** your measurement in Table (3).
- 3) **Repeat** this process 6 times to get 6 values for the period T .
- 4) **Calculate** the mean period \bar{T} according to equation 1 applying the tabulation method.
- 5) Use the calculator to **get** the mean period \bar{T} .
- 6) **Complete** the data analysis of Table (4) then **calculate** the standard deviation for the period σ_T according to equation 2.
- 7) Use the calculator to **get** the standard deviation σ_T .
- 8) **Calculate** the standard error $\sigma_{\bar{T}}$ using equation 3.

A.2 The period of the simple pendulum

Table 3.

quantity	unit	T_1	T_2	T_3	T_4	T_5	T_6
T_i							

\bar{T} (using equation 1) =.....

\bar{T} (using calculator) =.....

Table 4.

quantity	T_1	T_2	T_3	T_4	T_5	T_6
T_i						
$(T_i - \bar{T})$						
$(T_i - \bar{T})^2$						

σ_T (using equation 2) =.....

σ_T (by calculator) =.....

$\sigma_{\bar{T}}$ (using equation 3) =.....

Part B. Measurements of Density

Objectives

- Determining the length, diameter, and mass of cylindrical objects.
- Calculating of the density of the cylindrical objects and comparing with the accepted values of the density of the metals.

Equipment list

- Four cylindrical objects of different sizes made from the same material (aluminum, steel, or brass).
- Measuring devices:

the *Micrometer Screw Gauge* (Figure 2).

the *Vernier Caliper* (Figure 3, 4).

the *Laboratory Triple Balance*.

Theory

Density ρ of any substance is defined as the mass m of a unit volume of that substance. The obvious way of finding it is by determining the mass of a known volume of the substance and dividing this mass by the volume

$$\rho = \frac{m}{V}, \quad (5)$$

The volume of any solid which has a simple geometric form may be determined from its dimensions; which, if the body is small, are most conveniently measured by a **Vernier Caliper** and a **Micrometer Screw Gauge**. The mass; however, is found by the use of the laboratory triple balance.

The **micrometer screw gauge**, (Figure 2), is most convenient for the accurate measurement of short lengths. The object is placed between the end of the screw and

the anvil. The distance through which the screw travels is measured by two scales:

- **Main Scale** which is divided into millimeters on the lower part and half millimeters on the upper part.
- **Rotating Scale** which is divided to (50) divisions each of which represents (0.01 mm).

The two scales are related to each other such that one complete revolution of the rotating scale equals half a millimeters on the main scale. The **Least Count** of the micrometer is equal to the ratio of the smallest division on the main scale to number of divisions on the rotating scale.

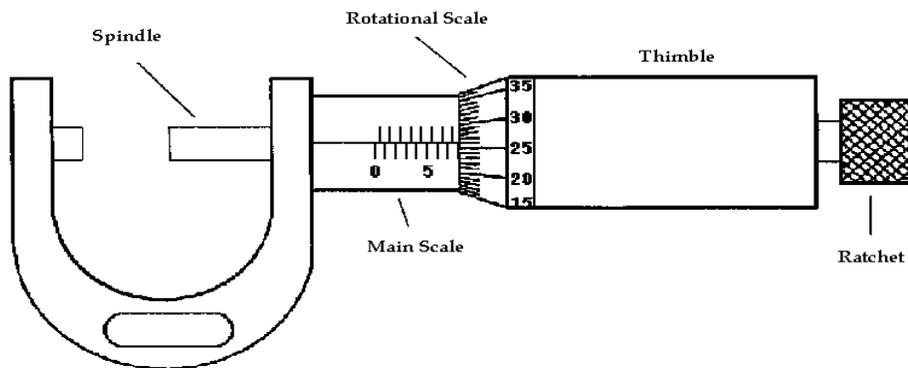


Figure 2. Micrometer Screw Gauge

The **Vernier Caliper**, (Figure 3), is considered also as a convenient device for accurate measurement of short lengths of objects. The object placed between the jaws. The distance through which the jaws travel is measured by two scales:

- **Main Scale** which is divided into inches on the upper part and centimeters on the lower part.

- **Vernier Scale** which is divided to (20) divisions each of which represents (0.005 cm).

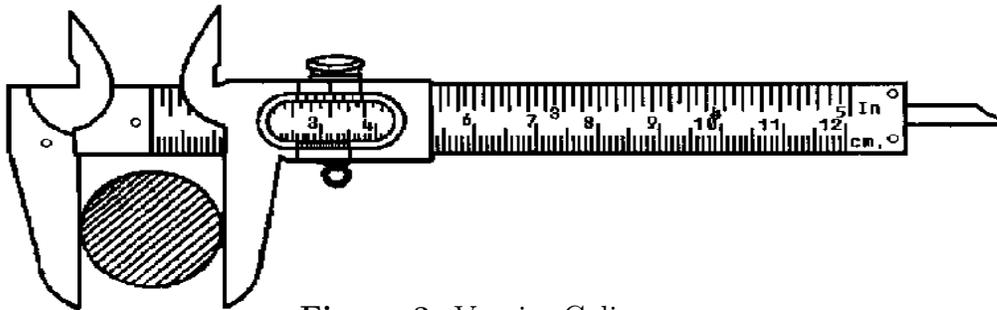


Figure 3. Vernier Caliper

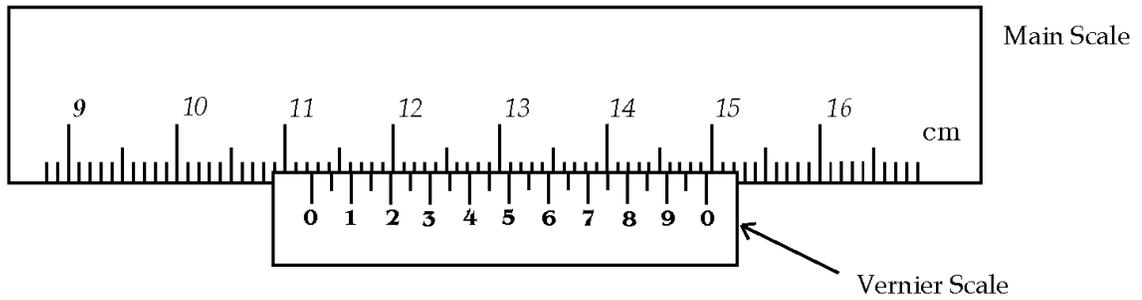


Figure 4. Magnified view of the Vernier Caliper

Procedure

B. Measurements of density

- 1) Use the micrometer screw gauge to **measure** the diameter (d) of the cylindrical objects. **Record** your data in Table (5).
- 2) Use the vernier caliper to **measure** the length (l) of the cylindrical objects. **Record** your data in Table (5).
- 3) Use the laboratory triple balance to **measure** the mass (m) of the cylindrical objects. **Record** your data in Table (5).
- 4) **Calculate** the volume for each cylindrical object using the measured values of diameter and length.
- 5) **Calculate** the density (ρ) for each piece.
- 6) **Compute** the average value of the density ($\bar{\rho}$).
- 7) **Calculate** the standard error ($\sigma_{\bar{\rho}}$) of your results.
- 8) Use your data on Table (5) to **plot** a graph of the mass m (y -axis) of each cylindrical object versus its volume V (x -axis). According to equation (5) the graph should be a straight line through the origin. **Get** the average density ($\bar{\rho}$) from the slope of this line.
- 9) **Compare** the measured density with the accepted values for the density of metals.

B. Measurements of Density

Table 5

Object	diameter d (cm)	length l (cm)	mass m (g)	volume V (cm ³)	density (ρ) (g/cm ³)
cylinder 1					
cylinder 2					
cylinder 3					
cylinder 4					

$\bar{\rho} \pm \sigma_{\bar{\rho}}$ (from Table 5) =.....

$\bar{\rho}$ (from the graph) =.....