Physics 102 - Formula Sheet

$$F_{12} = k \; \frac{|Q_1 Q_2|}{r^2} = \frac{1}{4\pi\varepsilon_0} \; \frac{|Q_1 Q_2|}{r^2}$$

Force on a charge q due to electric field:

$$\vec{F} = q\vec{E}$$

Electric field due to a charge distribution:

$$\vec{E} = k \int_{Q} \frac{dq}{r^{2}} \, \hat{r} = \frac{1}{4\pi\varepsilon_{0}} \, \int_{Q} \frac{dq}{r^{2}} \, \hat{r}$$

$$\Phi_E = \oint_S ec{m{E}} \cdot dec{m{A}} = rac{Q_{enc}}{arepsilon_0}$$

Electric field calculations:

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Charge distribution	Electric field E (magnitude)	
Point charge	$E = \frac{k Q }{r^2}$	
Charged ring of radius a (along its axis, which is taken to be the X -axis)	$E = \frac{k Qx }{(x^2 + a^2)^{3/2}}$	
Charged disc of radius a (along its axis, which is taken to be the X -axis)	$E = \frac{ \sigma }{2\varepsilon_0} \left[1 - \frac{ x }{\sqrt{x^2 + a^2}} \right]$	
Infinite non-conducting charged sheet perpendicular to the X -axis	$E = \frac{ \sigma }{2\varepsilon_0}$	
Infinite line with uniform charge distribution	$E = \frac{ \lambda }{2\pi\varepsilon_0 r} = \frac{2k \lambda }{r}$	
Sphere of radius a with uniform charge distribution	$E = \frac{k Q }{r^2} \qquad r \ge a$ $E = \frac{k Q r}{a^3} \qquad r \le a$	
	$E = \frac{\kappa Q r}{a^3} \qquad r \le a$	

Electric potential:

$$V = k \int_Q \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0} \int_Q \frac{dq}{r} \qquad \text{relative to } V = 0 \text{ at } r \to \infty$$

Relationship between \vec{E} and V: $V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$ $E_i = -\frac{\partial V}{\partial x_i}$

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Work done by the field in moving a charge q from a to b: $W_{ab} = U_a - U_b = q (V_a - V_b)$

Potential energy of a system of point charges: $U = \sum_{i \in I} \frac{kQ_iQ_j}{r_{ij}}$

Electric potential calculations (Relative to V=0 at ∞):

Charge distribution	Electric potential V
Point charge	$\frac{kQ}{r}$
Charged ring of radius a (along its axis, which is taken to be the X -axis)	$\frac{kQ}{\sqrt{x^2 + a^2}}$
Charged disc of radius a (along its axis, which is taken to be the X -axis)	$\frac{\sigma}{2\varepsilon_0} \left[\sqrt{x^2 + a^2} - x \right]$
Sphere of radius a with uniform charge distribution	$\frac{kQ}{r} \qquad \qquad r \ge a$
	$\frac{kQ}{2a} \left[3 - \frac{r^2}{a^2} \right] \qquad r \le a$

Capacitors: $C = \frac{Q}{V}$

Capacitance of different capacitors (for air or vacuum, K=1):

Capacitor	Capacitance
Parallel-plate capacitor with plate-area A and thickness d	$K\varepsilon_0 \frac{A}{d}$
Spherical capacitor of radii a and b	$K\varepsilon_0 \frac{4\pi ab}{b-a}$
Isolated sphere of radius a	$K\varepsilon_0 4\pi a$
Cylindrical capacitor of radii a and b , and length L	$K\varepsilon_0 \frac{2\pi L}{\ln(b/a)}$

Capacitor combinations:

Series connection:
$$\frac{1}{C_{eq}} = \sum_{i=1}^{N} \frac{1}{C_i}$$
 Parallel connection: $C_{eq} = \sum_{i=1}^{N} C_i$

Energy stored:
$$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$
 Energy density: $u = \frac{1}{2}K\varepsilon_0 E^2$

Electric current and resistance:
$$I = \frac{dQ}{dt} = \int \vec{J} \cdot d\vec{A}$$
 $\vec{J} = nq\vec{v}_d$ $Q = \int I \ dt$

$$I = \frac{dQ}{dt} = \int \vec{J} \cdot d\vec{A}$$

$$\vec{m{J}} = nq\vec{m{v}}_d$$

$$Q = \int I \, dt$$

$$V = IR$$

$$R = \rho \frac{L}{A}$$

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 $\rho_T = \rho_0 \left[1 + \alpha \left(T - T_0 \right) \right]$ $P = IV = I^2 R = \frac{V^2}{R}$

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Resistor combinations:

Series connection:
$$R_{eq} = \sum_{i=1}^{N} R_i$$
 Parallel connection: $\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}$

$$\frac{1}{R_{eg}} = \sum_{i=1}^{N} \frac{1}{R_i}$$

RC circuits: (Time Constant, T = RC)

Charging:
$$Q(t) = C\mathcal{E}\left(1 - e^{-t/\tau}\right)$$
 Discharging: $Q(t) = Q_0 e^{-t/\tau}$

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Charged particle in a uniform magnetic field: $\vec{F} = q\vec{v} \times \vec{B}$ $R = \frac{mv_{\perp}}{Ba}$ $T = \frac{2\pi m}{Ba}$

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$$R = \frac{mv_{\perp}}{Ba}$$

$$T = \frac{2\pi m}{Ra}$$

Force on a current carrying conductor in a uniform magnetic field:

$$\vec{m{F}} = I \vec{m{L}} imes \vec{m{B}}$$

Magnetic field of a moving point charge:

$$\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi} \, \frac{q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{r}}}{r^3}$$

Magnetic field of a current element (Biot and Savart law): $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \vec{r}}{r^3}$

$$d\vec{B} = \frac{\mu_0}{4\pi} \, \frac{I \, d\vec{l} \times \vec{r}}{r^3}$$

Amperes's law:

$$\oint ec{m{B}} \!\cdot\! dec{m{l}} = \mu_0 I_{enc}$$

Magnetic field calculations:

Shape of the conductor carrying current	Magnetic field B (magnitude)
Long straight wire	$B = \frac{\mu_0 I}{2\pi r}$
Circular loop of radius a (along its axis, which is taken to be the X -axis)	$B = \frac{\mu_0 I a^2}{2 \left(x^2 + a^2\right)^{3/2}}$
Long solenoid	$B = \mu_0 nI$

Force between two long parallel straight wires carrying current: $\frac{F}{I_c} = \frac{\mu_0 I_1 I_2}{2\pi d}$

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

<u>Faraday's law of induction:</u> $\mathcal{E} = -N \frac{d\Phi_B}{dt}$ $\Phi_B = \int \vec{B} \cdot d\vec{A}$

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$$\Phi_B = \int \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}}$$

$$F = \frac{B^2 L^2 r}{R}$$

Motional emf:
$$\mathcal{E} = BLv$$
 $F = \frac{B^2L^2v}{R}$ Induced electric field: $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$