**Kuwait University** **Department of Physics**

# Physics 102

## First Midterm Examination
## Spring Semester 2012-2013

**March 16, 2013**
**Time: 3:30 PM to 5:00 PM**

**Instructors: Drs. AbdelKarim, Afrousheh, Davis, Kokaj, Lajko, Marafi, Pichler, Rakhshani, Rao, Razee & Sharma**

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**Name:** .......................................................... **Student No.:** ....................................

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### Fundamental Constants

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|---|---|
| $k = \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$ | Coulomb's constant |
| $\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{ N}\cdot\text{m}^2)$ | Permitivity of free space |
| $\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$ | Permeability of free space |
| $e = 1.6 \times 10^{-19} \text{ C}$ | Elementary unit of charge |
| $m_e = 9.11 \times 10^{-31} \text{ kg}$ | Mass of an electron |
| $m_p = 1.67 \times 10^{-27} \text{ kg}$ | Mass of a proton |
| $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ | Conversion from eV to J |
| $N_A = 6.022 \times 10^{23}/\text{mol}$ | Avogadro's number |
| $g = 9.8 \text{ m/s}^2$ | Acceleration due to gravity |

### Prefixes of units

| | | | |
|---|---|---|---|
| $\text{m} = 10^{-3}$ | $\mu = 10^{-6}$ | $\text{n} = 10^{-9}$ | $\text{p} = 10^{-12}$ |
| $\text{k} = 10^{3}$ | $\text{M} = 10^{6}$ | $\text{G} = 10^{9}$ | $\text{T} = 10^{12}$ |

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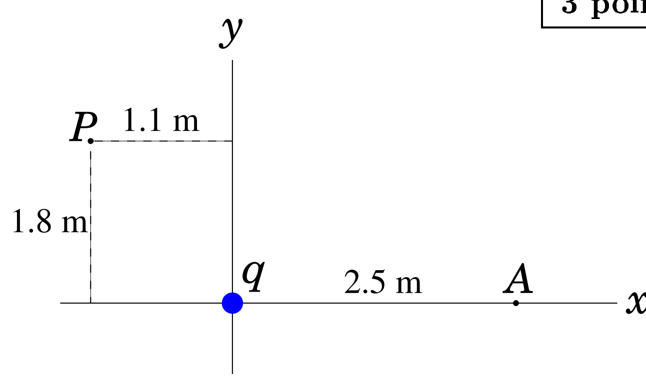
**For use by instructors only**

| Problems | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Questions | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| Marks | | | | | | | | | | |

## Part I: Solve all the problems. Show all your workings.

1. A point charge $q$ placed at the origin produces an electric field $\vec{E} = (5.3\,\hat{\imath} - 8.7\,\hat{\jmath})$ N/C at the point $P$ in the figure below. Find $\vec{E}$ at the point $A$ on the $x$-axis with $x$-coordinate $x_A = 2.5$ m.

**3 points**




**Solution:** Clearly, the charge $\boxed{q < 0}$. For the point $P$, $r = \sqrt{1.1^2 + 1.8^2} = 2.1$ m. Then considering the $x$-component of $\vec{E}$,

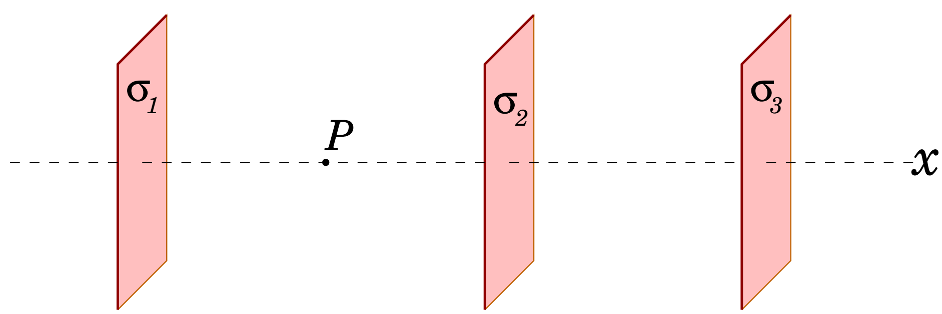
$$E_x = \frac{k|q|}{2.1^2}\,\frac{1.1}{2.1} = 5.3 \implies \boxed{|q| = 5.0 \times 10^{-9}\text{ C}}$$

Then $\vec{E}$ at $A$ is

$$\vec{E} = \frac{k|q|}{2.5^2}(-\hat{\imath}) \implies \boxed{\vec{E} = -7.2\,\hat{\imath}\text{ N/C}}$$

2. Three infinitely large sheets with uniform surface charge densities $\sigma_1 = +6.23$ nC/m$^2$, $\sigma_2 = -8.54$ nC/m$^2$ and $\sigma_3 = +4.44$ nC/m$^2$ are perpendicular to the $x$-axis as shown below. Find the electric field $\vec{E}$ at the point $P$ in the figure.

**3 points**



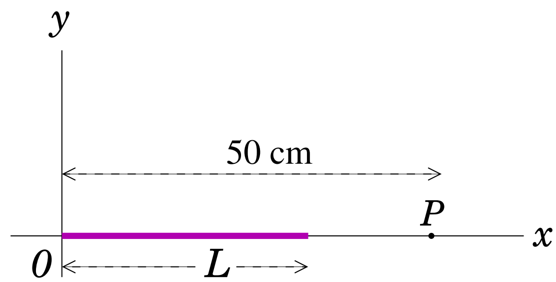

**Solution:** At $P$, the net electric field is

$$\vec{E} = \frac{|\sigma_1|}{2\varepsilon_0}\,\hat{\imath} + \frac{|\sigma_2|}{2\varepsilon_0}\,\hat{\imath} + \frac{|\sigma_3|}{2\varepsilon_0}\left(-\hat{\imath}\right)$$

$$\implies \vec{E} = 352\,\hat{\imath} + 482\,\hat{\imath} - 251\,\hat{\imath}$$

$$\implies \boxed{\vec{E} = 583\,\hat{\imath}\text{ N/C}}$$

3. In the figure below, the thin straight wire of length $L$ has a uniformly distributed charge of $Q = +3.0$ nC. Its electric field at point $P$ has magnitude $E = 300$ N/C. Find $L$. **5 points**




**Solution:** Choose a charge element $dq = \lambda dx$ at a distance $x$ from the origin. At $P$, its electric field $d\vec{E}$ is of magnitude

$$dE = \frac{k\,dq}{(0.5-x)^2} = \frac{k\lambda\,dx}{(0.5-x)^2}$$

Total magnitude ($\because$ all $d\vec{\boldsymbol{E}}$'s are parallel)

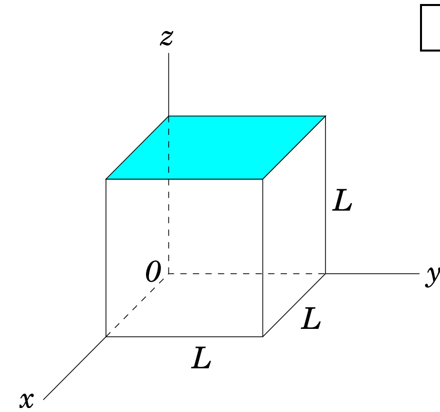
$$E = \int_0^L \frac{k\lambda\,dx}{(0.5-x)^2} = k\lambda\left(\frac{1}{0.5-x}\right)\Bigg|_0^L = k\lambda\,\frac{L}{0.5(0.5-L)}$$

$$\xrightarrow{Q=\lambda L} \boxed{E = \frac{kQ}{0.5(0.5-L)}}$$

So

$$\frac{kQ}{0.5(0.5-L)} = 300 \implies \boxed{L = 0.32\text{ m}}$$

4. The cubical surface of side length $L = 12.0$ cm shown is in the electric field $\vec{\boldsymbol{E}} = \left(950\,y\,\hat{\boldsymbol{i}} + 650\,z\,\hat{\boldsymbol{k}}\right)$ V/m. Find the electric flux through the top face of the cube. **3 points**




**Solution:** The area element in the given plane is

$$d\vec{\boldsymbol{A}} = dA\,\hat{\boldsymbol{k}}$$

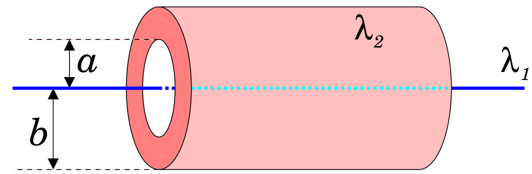
So the electric flux through this plane is

$$\Phi = \int_S \left(950\,y\,\hat{\boldsymbol{i}} + 650\,z\,\hat{\boldsymbol{k}}\right)\Big|_{z=0.12} \cdot dA\,\hat{\boldsymbol{k}} = 78\,A = 78 \times (0.12)^2$$

$$\implies \boxed{\Phi = 1.12\ \text{N}\cdot\text{m}^2/\text{C}}$$

5. A long wire with linear charge density $\lambda_1$ is on the axis of a **conducting** cylindrical shell (inner radius $a = 2.0$ cm, outer radius $b = 5.0$ cm). The shell has linear charge density $\lambda_2 = -8.5$ nC/m. The *net* electric field at $r = 8.0$ cm from the wire is **inward** with magnitude $E = 720$ N/C. Find the surface charge density $\sigma_{inner}$ on the inner surface of the shell.

**5 points**




**Solution:** We have cylindrical symmetry here, so we take a cylindrical Gaussian surface of length $L$ and radius $r = 8.0$ cm. Then

$$\Phi_E = -2\pi r L E \qquad \text{(Since the electric field is inward)}$$

$$Q_{\text{encl}} = \lambda_1 L + \lambda_2 L$$

So

$$-2\pi r L E \varepsilon_0 = \lambda_1 L + \lambda_2 L \implies \lambda_1 + \lambda_2 = -2\pi\varepsilon_0 r E = -3.2 \times 10^{-9} \text{ C/m}$$

$$\implies \boxed{\lambda_1 = +5.3 \times 10^{-9} \text{ C/m}}$$

For any Gaussian surface of radius $r$ such that $a < r < b$, the net charge inside the surface is zero. Then

$$\lambda_1 L + 2\pi a L\, \sigma_{inner} = 0 \implies \sigma_{inner} = \frac{-\lambda_1}{2\pi a} \implies \boxed{\sigma_{inner} = -4.2 \times 10^{-8} \text{ C/m}^2}$$

6. A uniformly charged thin ring has radius $a = 16.0$ cm and total charge $Q = +24.0\ \mu$C. Where must a point charge $q = -18.0\ \mu$C be placed on the axis of the ring for the electric potential at the centre of the ring to be zero? (Take $V \to 0$ as $r \to \infty$.)

**3 points**

**Solution:** Let $d$ be the distance of $q$ from the centre of the ring. Then

$$\frac{kQ}{a} + \frac{kq}{d} = 0 \implies d = \frac{-q\, a}{Q} \implies \boxed{d = 0.12 \text{ m}}$$

7. A proton moves directly towards a stationary nucleus of charge $Q$. The proton has a speed of $4.15 \times 10^6$ m/s and an acceleration of $4.31 \times 10^{26}$ m/s$^2$ when it is $8.00 \times 10^{-14}$ m from the centre of the nucleus. What is the distance of closest approach (to the centre of the nucleus)?

**4 points**

**Solution:** First we need to find $Q$. The acceleration of the proton is

$$a = \frac{kQe}{mr^2} = 4.31 \times 10^{26} \implies \boxed{Q = 3.2 \times 10^{-18}\ \text{C}}$$

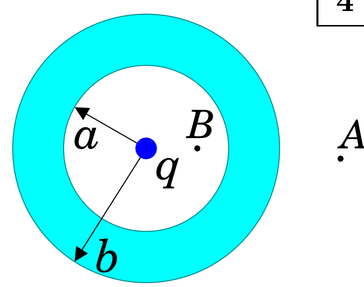
Let $d$ be the distance of closest approach. Then the work-energy principle implies

$$0 - \frac{1}{2}m(4.15 \times 10^6)^2 = \frac{kQe}{8.00 \times 10^{-14}} - \frac{kQe}{d} \implies \frac{kQe}{d} = 7.2 \times 10^{-14}$$

$$\implies \boxed{d = 6.4 \times 10^{-14}\ \text{m}}$$

8. A point charge $q$ is placed at the centre of an electrically neutral spherical conducting shell of inner radius $a = 3.0$ cm and outer radius $b = 5.0$ cm. The electric potential at the point $A$ in the figure is $V_A = -150$ ($r_A = 9.0$ cm from the centre). Find $V_B$ at $B$ if $r_B = 2.0$ cm. (Assume that $V \to 0$ as $r \to \infty$.)

**4 points**




**Solution:** Since the electric field in a conductor must be zero, to comply with the Gauss' law, the charge on the inner and outer surfaces of the shell are $q_{inner} = -q$ and $q_{outer} = +q$ respectively. Then, by the superposition principle,

<u>at $r_A = 9.0$ cm:</u>
$$V_A = \frac{kq}{r_A} + \frac{k(-q)}{r_A} + \frac{kq}{r_A}$$

$$\implies -150 = \frac{kq}{r_A} \implies \boxed{q = -1.5 \times 10^{-9}\ \text{C}}$$

Then

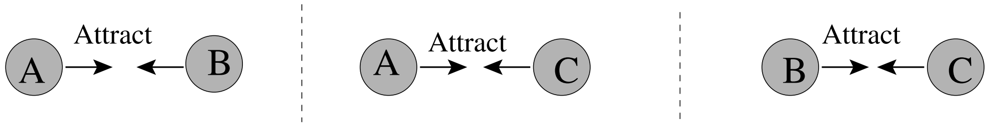
<u>at $r_B = 2.0$ cm:</u>
$$V_B = \frac{kq}{r_B} + \frac{k(-q)}{a} + \frac{kq}{b}$$

$$\implies V_B = kq\left(\frac{1}{r_B} - \frac{1}{a} + \frac{1}{b}\right) \implies \boxed{V_B = -495\ \text{V}}$$

# Part II: Conceptual Questions

Each question carries 1 point. Tick the **best** answer.

1. The forces between three small metallic spheres $A$, $B$ and $C$ are as shown when they are put together, two at a time. The sphere $A$ has **positive** charge.




One can conclude that:

(a) $B$ and $C$ are both negative.

(b) $B$ and $C$ are both neutral.

(c) either $B$ is negative and $C$ is positive, or $B$ is positive and $C$ is negative.

✓(d) either $B$ is neutral and $C$ is negative, or $C$ is neutral and $B$ is negative.

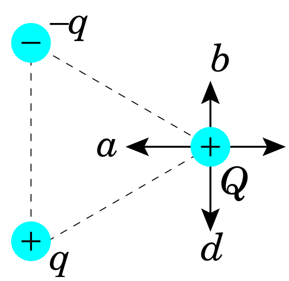
2. Point charges $q$, $-q$ and $Q$ (positive) are at the corners of an equilateral triangle. The net force on $Q$ is in the direction of:

(a) $a$

✓(b) $b$

(c) $c$

(d) $d$



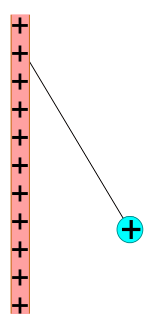

3. A charged ball is suspended (with an insulating thread) from a large uniformly charged vertical plate (see figure). If the charge on the ball is decreased, the tension in the thread:

(a) increases.

✓(b) decreases.

(c) remains the same.

(d) becomes zero.



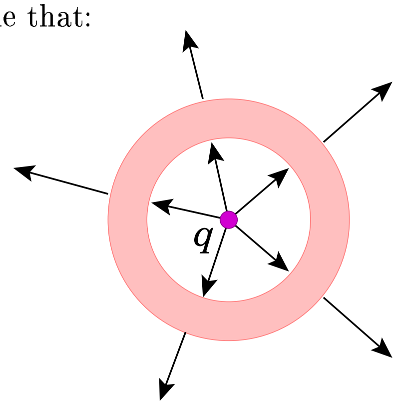
4. A charged metallic spherical shell with net charge $Q$ has a point charge $q$ at its centre. The electric field lines are as shown. Then, we conclude that:

(a) $Q$ must be positive.

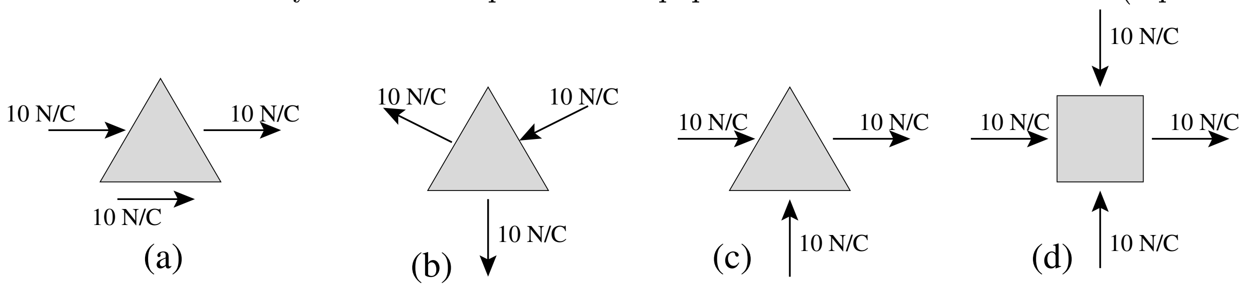
✓(b) $Q$ must be greater than $-q$.

(c) $Q$ must be greater than $+q$.

(d) $Q$ must be negative.



5. The figures shown below are cross sections of three-dimensional closed surfaces. The electric field is everywhere in the plane of the paper and uniform over each face (separately).




Which surfaces contain a net negative charge?

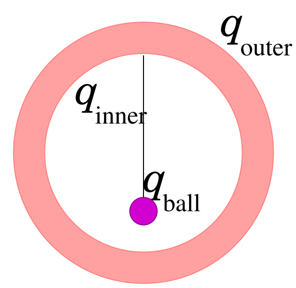
(a) $c$

(b) $d$

√(c) $c$ and $d$

(d) $a$, $b$ and $d$

6. A neutral small conducting ball hangs by an insulating thread inside a conducting spherical shell. A positive charge is placed on the outer surface of the shell. The charges on the conductors are such that:




(a) $q_{outer} = q_{ball} = -q_{inner}$

√(b) $q_{outer} > 0$, $q_{ball} = 0 = q_{inner}$

(c) $q_{outer} > q_{ball} = -q_{inner}$

(d) $q_{outer} > q_{ball} > |q_{inner}|$

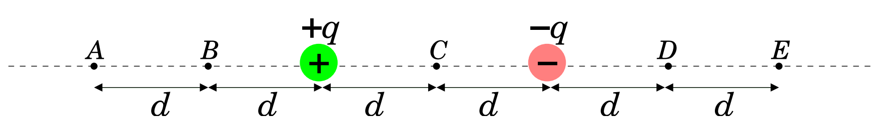
7. A positive point charge $+q$ and a negative point charge $-q$ are shown below with five points $A$, $B$, $C$, $D$ and $E$ around them. Then:

(a) $V_B$ is the most positive, $V_E$ is the most negative.

√(b) $V_B$ is the most positive, $V_D$ is the most negative.

(c) $V_B$ is the most positive, $V_C$ is the most negative.

(d) $V_A$ is the most positive, $V_C$ is the most negative.




8. The figure below shows a uniform electric field. The dotted lines 1 and 2 are two planar surfaces. The potential differences between the points marked in the figure are such that:

(a) $V_{ab} = V_{cd} = V_{ef}$

√(b) $V_{ab} < V_{cd} < V_{ef}$

(c) $V_{ab} > V_{cd} > V_{ef}$

(d) $V_{ac} = V_{bd}$