Kuwait University — Department of Physics

Physics 102

Final Examination
Summer Semester 2011-2012

July 30, 2012

Time: 2:00 PM to 4:00 PM

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Name: .... **Student No.:** ....

Fundamental Constants

$k = \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \ \mathrm{N \cdot m^2/C^2}$	Coulomb's constant
$\varepsilon_0 = 8.85 \times 10^{-12} \ \mathrm{C^2/(N \cdot m^2)}$	Permitivity of free space
$\mu_0 = 4\pi \times 10^{-7} \ \mathrm{T \cdot m/A}$	Permeability of free space
$e = 1.6 \times 10^{-19} \ \mathrm{C}$	Elementary unit of charge
$m_e = 9.11 \times 10^{-31} \ \mathrm{kg}$	Mass of an electron
$m_p = 1.67 \times 10^{-27} \ \mathrm{kg}$	Mass of a proton
$1 \ \mathrm{eV} = 1.6 \times 10^{-19} \ \mathrm{J}$	Conversion from eV to J
$g = 9.8 \ \mathrm{m/s^2}$	Acceleration due to gravity

Prefixes of units

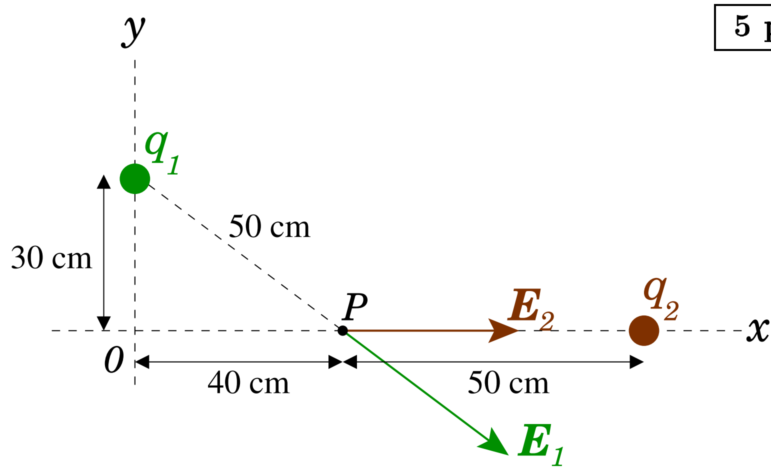
$\mathrm{m} = 10^{-3}$	$\mu = 10^{-6}$	$\mathrm{n} = 10^{-9}$	$\mathrm{p} = 10^{-12}$
$\mathrm{k} = 10^{3}$	$\mathrm{M} = 10^{6}$	$\mathrm{G} = 10^{9}$	$\mathrm{T} = 10^{12}$

For use by instructors only

Problems	1	2	3	4	5	6	7	8	9	10	Questions	Total
Marks												

Part I: Solve the following problems. Show all your workings

1. The point charges $q_1 = +9.0$ nC and $q_2 = -4.0$ nC are placed in the xy-plane as shown below. Find the electric field $\vec{E}$ at the point P in the figure. Express your answer using the unit vectors $\hat{i}$ and $\hat{j}$. **5 points**



Solution: The distance $r_1 = \sqrt{0.3^2 + 0.4^2} = 0.5$ m. The electric fields $\vec{E}_1$ and $\vec{E}_2$ at the point P are as shown in the figure. Then

$$\vec{E}_1 = \frac{k|q_1|}{0.5^2}\left(+\frac{0.4}{0.5}\,\hat{i} - \frac{0.3}{0.5}\,\hat{j}\right) = \left(+259.0\,\hat{i} - 194.0\,\hat{j}\right)\ \text{N/C}$$

$$\vec{E}_2 = \frac{k|q_2|}{0.5^2}\,\hat{i} = +144.0\,\hat{i}\ \text{N/C}$$

So

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \left(+403.0\,\hat{i} - 194.0\,\hat{j}\right)\ \text{N/C}$$

OR

Solution: The distance $r_1 = \sqrt{0.3^2 + 0.4^2} = 0.5$ m. The electric fields $\vec{E}_1$ and $\vec{E}_2$ at the point P are as shown in the figure. Then

$$E_1 = \frac{k|q_1|}{0.5^2} = 324.0\ \text{N/C}$$

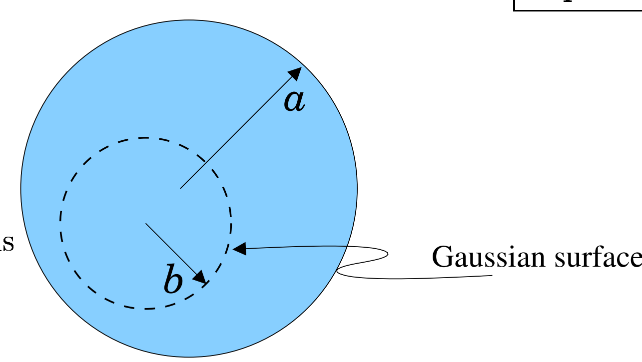
$$E_2 = \frac{k|q_2|}{0.5^2} = 144.0\ \text{N/C}$$

$$E_x = E_1 \times \frac{0.4}{0.5} + E_2 = 403.0\ \text{N/C}$$

$$E_y = -E_1 \times \frac{0.3}{0.5} = -194.0\ \text{N/C}$$

$$\vec{E} = \left(+403.0\,\hat{i} - 194.0\,\hat{j}\right)\ \text{N/C}$$

2. Charge $Q = +60.0\ \mu$C is distributed uniformly over the volume of a sphere of radius $a = 9.0$ cm. Find the electric flux through a spherical Gaussian surface of radius $b = 5.0$ cm (shown as dotted line) that is totally contained inside the charged sphere. **3 points**



Solution: The volume charge density is

$$\rho = \frac{Q}{4\pi a^3/3} = 1.96 \times 10^{-2}\ \text{C/m}^3$$

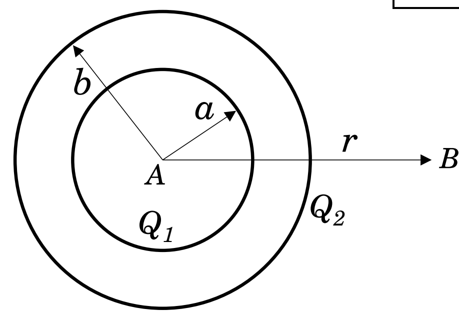
The charge enclosed by the Gaussian surface is

$$Q' = \frac{4\pi b^3}{3}\rho = 1.03 \times 10^{-5}\ \text{C}$$

Then the electric flux is

$$\Phi = \frac{Q'}{\varepsilon_0} = 1.2 \times 10^6\ \text{N} \cdot \text{m}^2/\text{C}$$

3. For the two concentric thin conducting spherical shells shown in the figure, the inner shell of radius $a = 8.0$ cm has a charge $Q_1 = +2.0\ \mu$C and the outer shell of radius $b = 15.0$ cm has a charge $Q_2 = -4.0\ \mu$C. Point A is at the centre and point B is at $r = 30.0$ cm from the centre. Find the potential difference, $(V_B - V_A)$. **3 points**



Solution: By the superposition principle, the electric potentials at A and B are (taking $V = 0$ at $r \to \infty$)

$$V_A = \frac{kQ_1}{a} + \frac{kQ_2}{b} = -1.5 \times 10^4 \text{ V}$$

$$V_B = \frac{kQ_1}{r} + \frac{kQ_2}{r} = -6.0 \times 10^4 \text{ V}$$

Then

$$(V_B - V_A) = -4.5 \times 10^4 \text{ V}$$

OR

Solution: The potential difference $(V_B - V_A)$ can be obtained by the intergration,

$$\begin{aligned}(V_B - V_A) &= \int_B^A \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{\ell}} \\ &= \int_r^b \frac{k(Q_1 + Q_2)}{r^2}\, dr + \int_b^a \frac{kQ_1}{r^2}\, dr \\ &= \frac{k(Q_1 + Q_2)}{r} - \frac{kQ_1}{a} - \frac{kQ_2}{b} \\ &= -6.0 \times 10^4 - 2.25 \times 10^5 \\ &\quad +2.40 \times 10^5 \\ &= -4.5 \times 10^4 \text{ V}\end{aligned}$$

4. A copper cube 5.0 cm on a side is formed into a cylindrical wire of length 8.0 m. The resistivity of copper is $\rho = 1.68 \times 10^{-8}\ \Omega$·m. Find the resistance of this wire. **2 points**

Solution: The area of crossection is

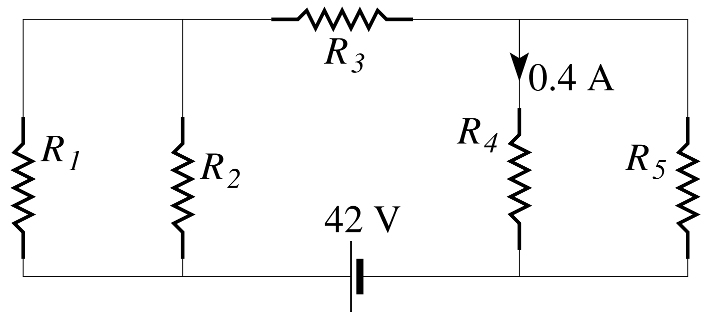
$$A = \frac{(0.05)^3}{L} = 1.56 \times 10^{-5} \text{ m}^2$$

Then , the resistance is

$$R = \rho \frac{L}{A} = 8.6 \times 10^{-3}\ \Omega$$

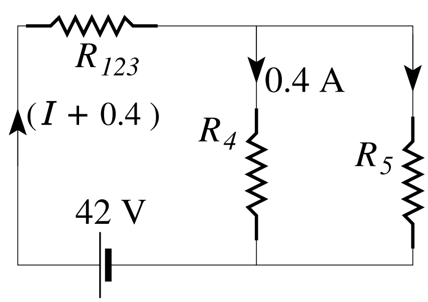
5. In the circuit shown, $R_1 = 6.0\ \Omega$, $R_2 = 12.0\ \Omega$, $R_3 = 11.0\ \Omega$ and $R_5 = 30.0\ \Omega$. R_4 is unknown. The current in R_4 is 0.4 A as shown. Find R_4.

5 points



Solution: The equivalent circuit is as shown with

$$R_{123} = \frac{R_1 R_2}{R_1 + R_2} + R_3 = 15.0\ \Omega$$



Let the current in R_5 be I so the current in R_{123} is $(I + 0.4)$. Then the loop rule for the big loop gives

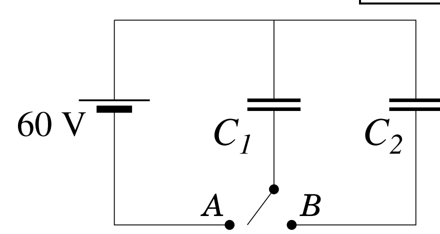
$$-15.0\ (I + 0.4) - 30.0\ I + 42.0 = 0 \implies I = 0.8\ \text{A}$$

Then the loop rule for the right loop gives

$$0.4 \times R_4 - 0.8 \times 30 = 0 \implies R_4 = 60.0\ \Omega$$

6. In the circuit shown, $C_1 = 4.0\ \mu\text{F}$ and $C_2 = 6.0\ \mu\text{F}$. Capacitor C_2 is initially uncharged and the switch is set to position A. Then the switch is moved to position B. Find the final plate-charge on C_2.

4 points



Solution: With the switch at position A, the charge on C_3 is

$$Q_3 = C_3 \times 60 = 2.40 \times 10^{-4}\ \text{C}$$

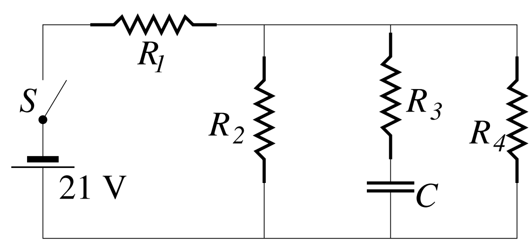
When the switch is moved to B, C_3 and C_4 share the charge Q_3.

$$Q_3' + Q_4' = Q_3 \implies (C_3 + C_4)\, V = Q_3 \implies V = 24.0\ \text{V}$$

Then

$$Q_4' = 24.0 \times C_4 = 1.44 \times 10^{-4}\ \text{C}$$

7. In the circuit shown, $R_1 = 12.0\ \Omega$, $R_2 = 8.0\ \Omega$, $R_3 = 10.0\ \Omega$, $R_4 = 12.0\ \Omega$ and $C = 3.0\ \mu$F. Determine the energy stored in the capacitor long time after the switch is closed. **5 points**



Solution: Long time after S is closed there is no current in R_3. Then the equivalent reistance is

$$R_{eq} = R_1 + \frac{R_2 R_4}{R_2 + R_4} = 16.8\ \Omega$$

Then

$$I_1 = I_{24} = \frac{21}{R_{eq}} = 1.25\ \text{A}$$

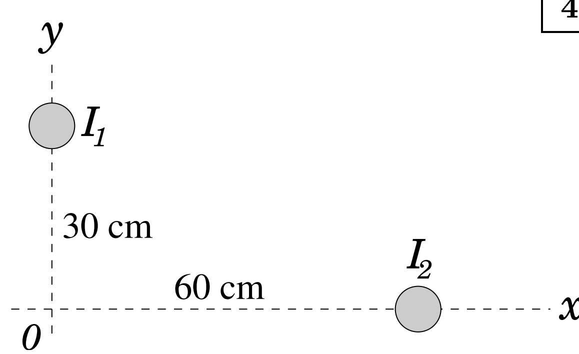
Then

$$V_C = V_{24} = I_{24} R_{24} = 6.0\ \text{V}$$

Then the energy in C is

$$U = \frac{1}{2} C V_C^2 = 5.4 \times 10^{-5}\ \text{J}$$

8. Two long parallel wires carrying currents I_1 and I_2 are perpendicular to the xy-plane as shown. The net magnetic field at the origin is $\vec{\boldsymbol{B}} = \left(1.2 \times 10^{-6}\ \hat{\boldsymbol{\imath}} + 4.8 \times 10^{-6}\ \hat{\boldsymbol{\jmath}}\right)$ T. Find I_1, I_2 and their directions. **4 points**



Solution: By right-hand-rule we can see that

$$B_x \text{ is due to } I_1 \implies I_1 \text{ is } \textbf{out-of-the-plane}$$

$$B_y \text{ is due to } I_2 \implies I_2 \text{ is } \textbf{into-the-plane}$$

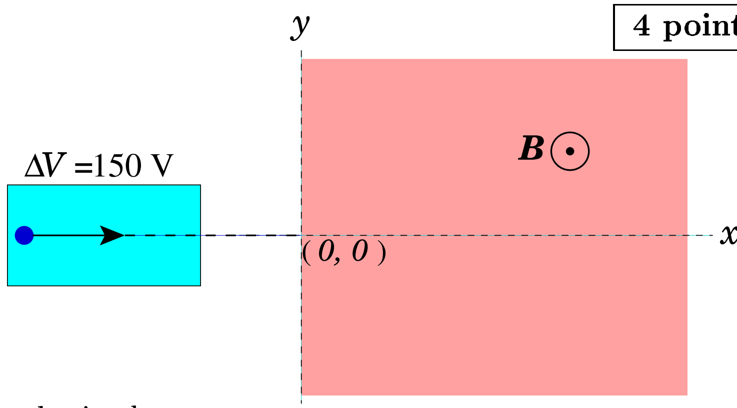
For the magnitudes

$$\frac{\mu_0 I_1}{2\pi(0.3)} = 1.2 \times 10^{-6} \implies I_1 = 1.8\ \text{A}$$

$$\frac{\mu_0 I_2}{2\pi(0.6)} = 4.8 \times 10^{-6} \implies I_1 = 14.4\ \text{A}$$

9. A uniform magnetic field of magnitude $B = 0.2$ T directed out-of-the-page is present in the region defined by $x > 0$. An electron is accelerated from rest through a potential difference of $\Delta V = 150.0$ V. Then it enters the region of magnetic field along the x-axis at $x = 0$ as shown. Find the coordinate $(x,\ y)$ of the point at which the electron exits the magnetic field region.

4 points



Solution: The speed of the electron is obtained as

$$\frac{1}{2}mv^2 = e\,\Delta V \implies v = \sqrt{\frac{2e\,\Delta V}{m}} = 7.26 \times 10^6 \text{ m/s}$$

Then the radius of the path is

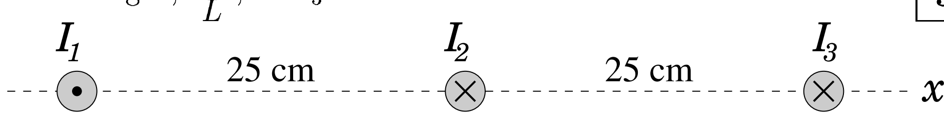
$$R = \frac{mv}{Be} = 2.06 \times 10^{-4} \text{ m}$$

By the right-hand-rule, the electron will move upward. So the coordinates of the exit point are

$$x = 0 \qquad y = +2R = 4.12 \times 10^{-4} \text{ m}$$

10. Three long parallel wires carrying currents are shown below. The dotted line is the x-axis. $I_1 = 8.0$ A out-of-the-plane, $I_2 = 5.0$ A into-the-plane and $I_3 = 15.0$ A into-the-plane. Find the net force per unit length, $\frac{\vec{F}_3}{L}$, on I_3.

3 points



Solution: The two forces on I_3 are

$$\frac{\vec{F}_{31}}{L} = \frac{\mu_0 I_1 I_3}{2\pi(0.50)}\,\hat{\imath} = 4.8 \times 10^{-5}\,\hat{\imath} \text{ N/m}$$

$$\frac{\vec{F}_{32}}{L} = \frac{\mu_0 I_2 I_3}{2\pi(0.25)}\left(-\,\hat{\imath}\right) = -6.0 \times 10^{-5}\,\hat{\imath} \text{ N/m}$$

Then

$$\frac{\vec{F}_3}{L} = \frac{\vec{F}_{31}}{L} + \frac{\vec{F}_{32}}{L} - 1.2 \times 10^{-5}\,\hat{\imath} \text{ N/m}$$

Part II: Conceptual Questions

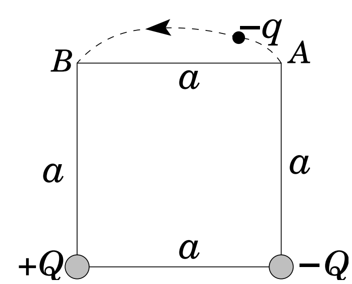
In the following, tick ($\surd$) the best answer. Each question carries 1 point.

1. An air-filled parallel-plate capacitor is connected to a battery. While the battery is still connected, a slab of material with dielectric constant K is inserted between the plates. Then which statement is true?

√(a) Both the plate-charge and the energy stored increase.
(b) The plate-charge remains the same, while the energy stored increases.
(c) The plate-charge decreases, while the energy increases.
(d) The plate-charge increases, while the energy remains the same.

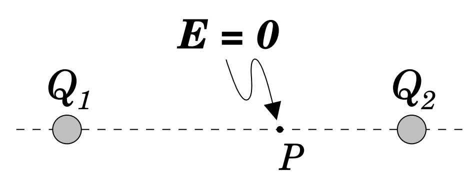
2. Two point charges $+Q$ and $-Q$ are placed on the corners of a square of side a as shown below. A negative charge $-q$ is moved by an external agent from the corner A to the corner B. The work done by the external agent is

(a) positive and the potential energy of the system increases.
(b) negative and the potential energy of the system increases.
√(c) negative and the potential energy of the system decreases.
(d) positive and the potential energy of the system decreases.



3. In the figure below, at the point P between the two point charges Q_1 and Q_2, the net electric field $\vec{E} = 0$. Then which statement is true about the potential energy U of these two charges (Taking $U = 0$ at $r \to \infty$).

√(a) U is definitely positive.
(b) U is definitely negitive.
(c) U is definitely zero.
(d) U may be positive, negative or zero depending upon the charges.

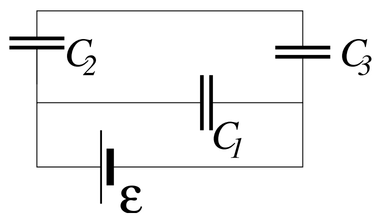


4. A charge Q is placed on an isolated solid metallic cylinder. How will this charge be distributed?

(a) The charge will be distributed uniformly over the volume of the cylinder.
√(b) The charge will be distributed over the surface of the cylinder.
(c) The charge will settle as a point charge at the centre of the cylinder.
(d) The charge will continuously move on the surface creating a current.

5. The three identical capacitors, $C_1 = C_2 = C_3$, are connected as shown to a battery. If Q_1, Q_2 and Q_3 are the plate-charges respectively, then

√(a) $Q_1 > Q_2 = Q_3$.
(b) $Q_2 = Q_3 > Q_1$.
(c) $Q_1 = Q_2 = Q_3$.
(d) $Q_2 > Q_1 > Q_3$.

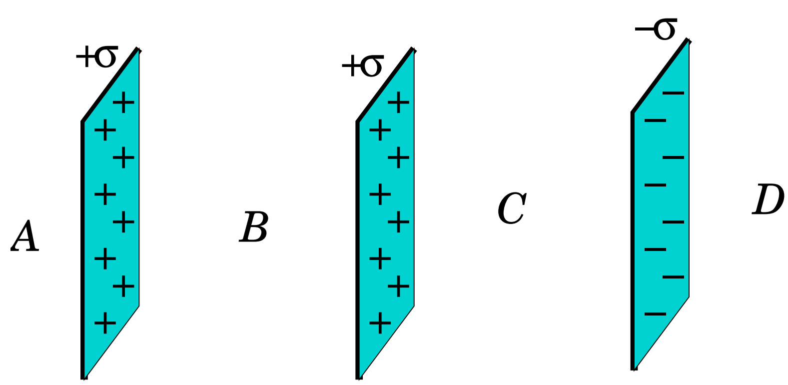


6. A charged particle is moving in a steady magnetic field. Its kinetic energy

(a) remains constant only if the path is circular.
(b) remains constant only if it is moving parallel to the magnetic field.
(c) remains constant only if the magnetic field is uniform.
√(d) remains constant whatever the nature of its path and the magnetic field.

7. Three large parallel sheets of charge are shown. The surface charge densities are $+\sigma$, $+\sigma$ and $-\sigma$ respectively as shown below. In which region, the magnitude of the electric field is largest?

(a) A
(b) B
√(c) C
(d) D



8. Two point charges, q_1 and q_2 are placed at the two vertices of a triangle. The direction of the net electric field $\vec{E}$ at the third vertex P is as shown in the figure. Then we conclude that

(a) $q_1 < 0$, $q_2 < 0$.
(b) $q_1 > 0$, $q_2 > 0$.
√(c) $q_1 < 0$, $q_2 > 0$.
(d) $q_1 > 0$, $q_2 < 0$.