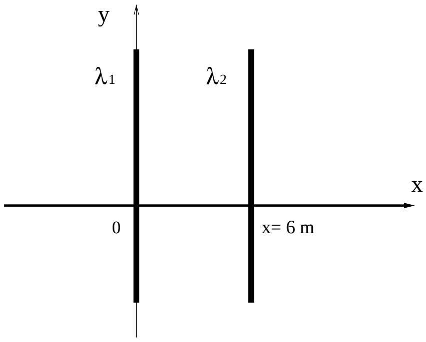
Problems

1. Infinite lines 1 and 2 of charge below have uniform linear charge densities $\lambda_1 = +2.0\,\text{nC/m}$ and $\lambda_2 = -1.5\,\text{nC/m}$, respectively. Find the magnitude of the net electric field at $x = 8.0\,\text{m}$. *(3 points)*

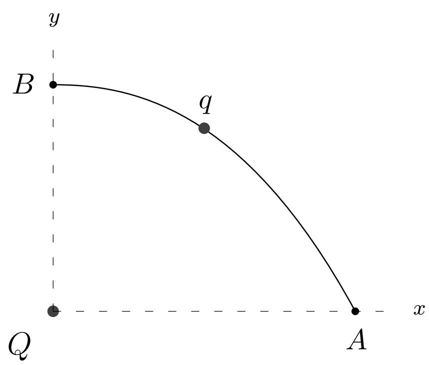


$$E_1 = \frac{2k\lambda_1}{(8.0\,\text{m})}, \quad E_2 = \frac{2k|\lambda_2|}{(2.0\,\text{m})}$$

$$E = E_2 - E_1$$

$$= +9.0\,\text{N/C}$$

2. Below, $Q = +4.0\,\mu\text{C}$ is a point charge fixed at the origin and $q = -3.5\,\text{pC}$ is another point charge. How much work is done by the electric force on q when it moves from A to B if $x_A = 4.0\,\mu\text{m}$ and $y_B = 3.0\,\mu\text{m}$? *(4 points)*

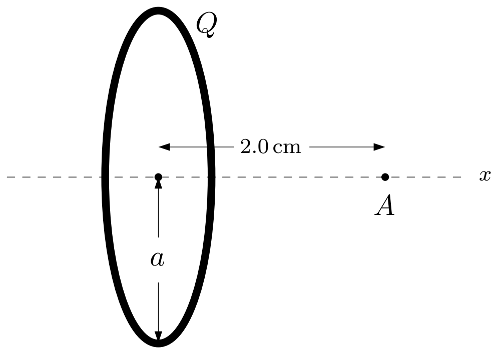


$$W_{\vec{E}} = -q\Delta V$$

$$= -q\left(\frac{kQ}{r_B} - \frac{kQ}{r_A}\right)$$

$$= 10.5\,\text{mJ}$$

3. In the figure below, the point A is on the axis of a uniform ring of charge $Q = 0.125\,\text{nC}$. Find the radius a if the electric field at point A has a magnitude of $1.44 \times 10^3\,\text{N/C}$. *(3 points)*



$$E = kQ\frac{x}{(x^2 + a^2)^{\frac{3}{2}}}$$

$$a^2 = \left(\frac{kQ}{E}\,x\right)^{\frac{2}{3}} - x^2$$

$$a = 1.5\,\text{cm}$$

4. Find the x-component of the net force on the $+2\,\mu$C charge on the x-axis below. All the charges in the figure are point charges.

(5 points)

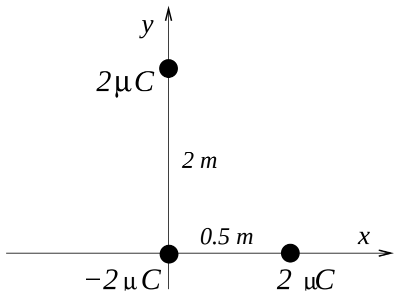
$$(E_{-2\,\mu\text{C}})_x = -E_{-2\,\mu\text{C}}$$

$$(E_{+2\,\mu\text{C}})_x = +E_{+2\,\mu\text{C}}\frac{0.5\,\text{m}}{2\,\text{m}}$$

$$F_x = (2\,\mu\text{C})(0.25E_{+2\,\mu\text{C}} - E_{-2\,\mu\text{C}})$$

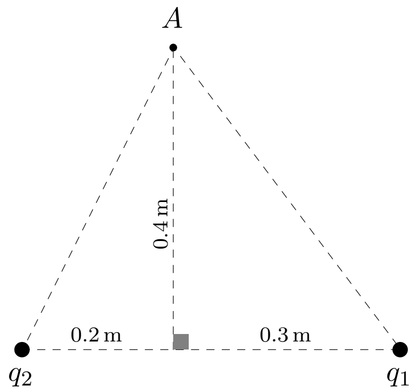
$$E_{-2\,\mu\text{C}} = 7.2\times10^4\,\text{N/C}$$
$$E_{+2\,\mu\text{C}} = 4.24\times10^3\,\text{N/C}$$

$$F_x = -0.14\,\text{N}$$



5. Find the net potential at point A below of point charges $q_1 = -6.20\,\text{nC}$ and $q_2 = +4.20\,\text{nC}$. Take potentials of point charges to be zero at infinity.

(4 points)



$$r_1 = 0.5\,\text{m}, r_2 = \sqrt{0.2^2+0.4^2} = 0.45\,\text{m}$$

$$V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2}$$
$$= -27\,\text{V}$$

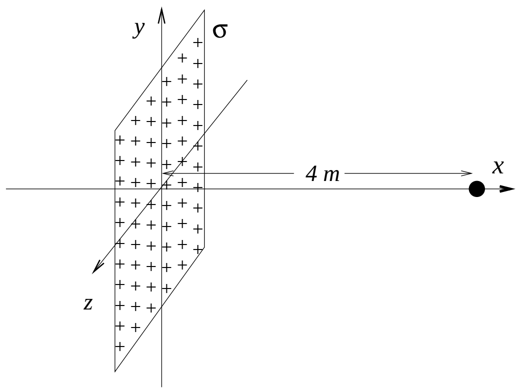
6. The yz-plane below is covered by charge of unknown *surface* density σ. An electron, released from rest at $x = 4\,\text{m}$, has a speed of $4\times10^5\,\text{m/s}$ after moving 2 m. Find σ.

(5 points)

$$a = \frac{v^2}{2|\Delta x|}$$

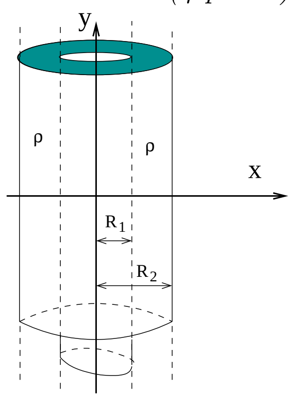
$$|\vec{E}| = \frac{ma}{e}$$
$$= 5.69\times10^{-12}a$$

$$\sigma = 2\epsilon_0|\vec{E}|$$
$$= 4.0\times10^{-12}\,\text{C/m}^2$$



7. An infinitely long *cylindrical* shell of charge (inner radius $R_1 = 0.2\,\text{m}$, outer radius $R_2 = 0.4\,\text{m}$) has uniform *volume* density $\rho = 2\,\mu\text{C/m}^3$. Find the electric field's magnitude at a distance of $2\,\text{m}$ from the axis of the shell (the y-axis).

(4 points)



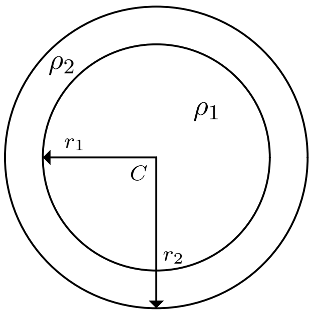
$$\Phi = E_\perp(2\pi r L)$$

$$\epsilon_0 \Phi = q_{\text{inside}} = \rho[\pi(R_2^2 - R_1^2)L]$$

$$E = |E_\perp| = \frac{\rho}{2\epsilon_0}(R_2^2 - R_1^2)\frac{1}{r}$$

$$= 6.8 \times 10^3\,\text{N/C}$$

8. Let r be the distance from the center C of the spherically symmetric charge shown in cross-section below. For $r < r_1$, the volume charge density is ρ_1; for $r_1 < r < r_2$, the volume charge density is ρ_2. (Both ρ_1 and ρ_2 are *positive* constants.)



Find the charge inside a Gaussian surface of radius r (and center C), if:

a) $r < r_1$, *(1 point)*

$$\boldsymbol{Q = \rho_1 \frac{4\pi}{3} r^3}$$

b) $r_1 < r < r_2$. *(1 point)*

$$\boldsymbol{Q = \rho_1 \frac{4\pi}{3} r_1^3 + \rho_2 \frac{4\pi}{3}(r^3 - r_1^3)}$$

Find the magnitude of the electric field at a distance r from C if:

c) $r < r_1$, *(1 point)*

$$\boldsymbol{E = \frac{\rho_1}{3\epsilon_0} r}$$

d) $r_1 < r < r_2$. *(1 point)*

$$\boldsymbol{E = \frac{\rho_2}{3\epsilon_0} r + \frac{(\rho_1 - \rho_2)}{3\epsilon_0}\frac{r_1^3}{r^2}}$$

Conceptual Questions

1. Consider the system of 3 point charges in the figure below. The work done by the electric field as the charges repel each other to infinity is:

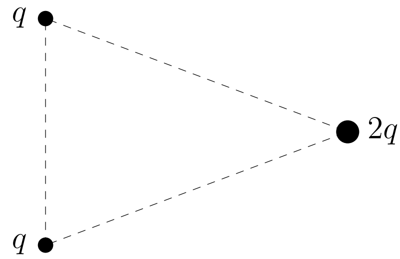
a) positive; ✓

b) negative;

c) zero;

d) positive if $q > 0$ and negative if $q < 0$;

e) none of the above.



2. The electron volt is a unit for:

a) electric potential;

b) kinetic energy of electrons;

c) energy; ✓

d) potential differences;

e) none of the above.

3. In the system of charges in the figure below, the net force on the positive point charge q due to the spherically symmetric charges Q (> 0) and $-Q$ (< 0) is in the direction of:

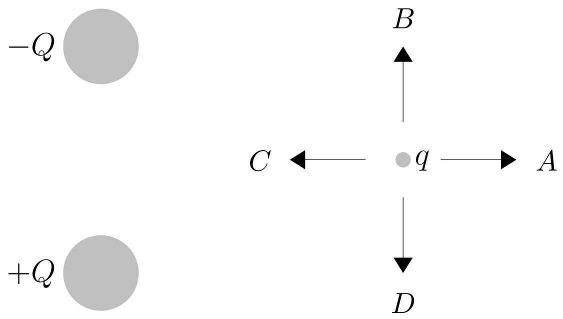
a) A;

b) B; ✓

c) C;

d) D;

e) none of the above.



4. Below, point charges Q_1 and Q_2 are such that $Q_2 = -2Q_1$. The electric potential is zero at:

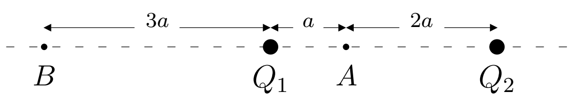
a) A only;

b) B only;

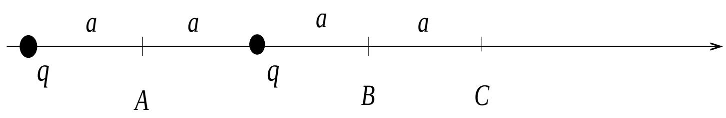
c) A and B; ✓

d) neither A nor B;

e) to the right of Q_2.



5. For the two identical point charges q below, the electric field is largest at:



a) A;

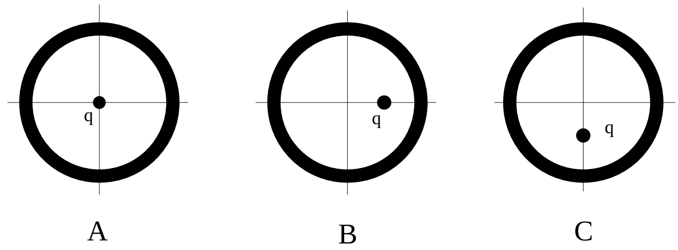
b) B; ✓

c) C;

d) A and B;

e) A and C.

6. Inside each of the three identical conducting spherical shells A, B and C below is a point charge q. The *inner* surface charge density is uniform for:



a) A; ✓

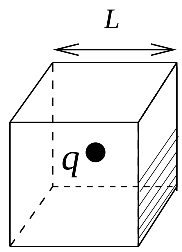
b) B;

c) C;

d) A, B and C;

e) none of the above.

7. A point charge q is at the center of the cube below. The electric flux through the shaded half of the right-hand face of the cube is:



a) q/ϵ_0;

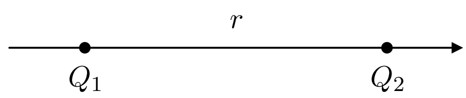
b) $q/(2\epsilon_0)$;

c) $q/(6\epsilon_0)$;

d) $q/(12\epsilon_0)$; ✓

e) none of the above.

8. The point charges Q_1 and Q_2 below have masses m_1 and m_2, respectively, and are free to move. The ratio a_1/a_2 of the magnitudes of their accelerations is:



a) 1;

b) m_1/m_2;

c) m_2/m_1; ✓

d) $(|Q_1|/m_1)(|Q_2|/m_2)^{-1}$;

e) none of the above.