Kuwait University **Department of Physics**

Physics 102

Second Midterm Examination (Fall Semester, 2010-2011)

December 18, 2010

Time: 3:00 PM to 5:00 PM

Instructors: Drs. AbdelKarim, Davis, Farhan, Lajko, Rakhshani, Razee & Sharma

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Name: .... **Student No.:** ....

Fundamental Constants

$k = \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \mathrm{N \cdot m^2/C^2}$	Coulomb's constant
$\varepsilon_0 = 8.85 \times 10^{-12} \mathrm{C^2/(\ N \cdot m^2)}$	Permitivity of free space
$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$	Permeability of free space
$\lvert e \rvert = 1.6 \times 10^{-19}\ \mathrm{C}$	Elementary unit of charge
$m_e = 9.11 \times 10^{-31}\ \mathrm{kg}$	Mass of an electron
$m_p = 1.67 \times 10^{-27}\ \mathrm{kg}$	Mass of a proton
$1\ \mathrm{eV} = 1.6 \times 10^{-19}\ \mathrm{J}$	Conversion from eV to J
$N_A = 6.022 \times 10^{23}/\mathrm{mol}$	Avogadro's number
$g = 9.8\ \mathrm{m/s^2}$	Acceleration due to gravity

Prefixes of units

$\mathrm{m} = 10^{-3}$	$\mu = 10^{-6}$	$\mathrm{n} = 10^{-9}$	$\mathrm{p} = 10^{-12}$
$\mathrm{k} = 10^{3}$	$\mathrm{M} = 10^{6}$	$\mathrm{G} = 10^{9}$	$\mathrm{T} = 10^{12}$

For use by instructors only

Problems	1	2	3	4	5	6	7	8	Questions	Total
Marks										

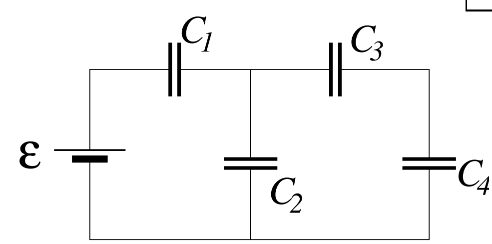
Part I: Solve the following problems. Show all your working

1. The magnitude of the electric field inside a paraffin-filled ($K = 2.2$) parallel-plate capacitor must not exceed the value $E = 9.4 \times 10^6$ V/m. If the capacitor needs to hold a plate-charge of at least $Q = 30.0$ nC, what must be the minimum plate-area? **2 points**

Solution: The electric field magnitude between the plates is

$$E = \frac{\sigma}{K\varepsilon_0} = \frac{Q}{KA\varepsilon_0} \implies A = \frac{Q}{K\varepsilon_0 E} = 1.6 \times 10^{-4}\ \text{m}^2$$

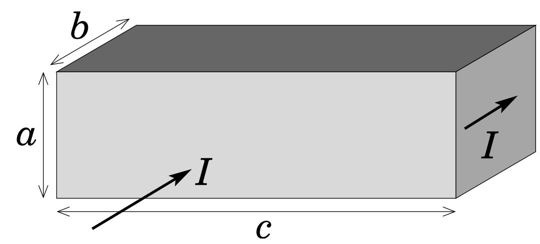
2. In the circuit below, $C_1 = C_2 = 18.0\ \mu$F and $C_3 = C_4 = 36.0\ \mu$F. The potential difference across the capacitor C_1 is $V_1 = 6.0$ V. How much electrostatic energy is stored in the capacitor C_2? **4 points**



Solution: See the circuit.

$$\begin{aligned}
C_3 \text{ and } C_4 \text{ are in series} &\implies C_{34} = \frac{C_3C_4}{C_3 + C_4} = 18.0\ \mu\text{F} \\
C_2 \text{ and } C_{34} \text{ are parallel} &\implies C_{234} = C_2 + C_{34} = 36.0\ \mu\text{F} \\
C_1 \text{ and } C_{234} \text{ are in series} &\implies Q_1 = Q_{234} \\
&\implies (18.0\ \mu\text{F}) \times 6.0\ \text{V} = (36.0\ \mu\text{F})\, V_2 \\
&\implies V_2 = 3.0\ \text{V} \\
&\implies U_2 = \frac{1}{2} C_2 V_2^2 = 8.1 \times 10^{-5}\ \text{J}
\end{aligned}$$

3. For the rectangular aluminum slab shown below $a = 4.0$ cm, $b = 2.0$ cm and $c = 8.0$ cm. What is the minimum resistance of this slab to the current flow? Indicate the direction of current (by an arrow) for which the resistance is minimum. The resistivity of aluminum is $\rho = 2.75 \times 10^{-8}$ Ω·m. **3 points**



Solution: The resistance is minimum when the length is minimum and the area of cross-section is maximum. In this case we should choose

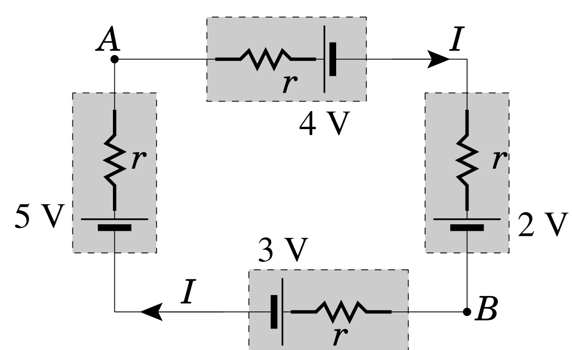
$$L = b = 2.0 \text{ cm} \qquad \text{and} \qquad A = ac = 32.0 \text{ cm}^2$$

Then

$$R = \rho \frac{L}{A} = 1.72 \times 10^{-7} \ \Omega$$

The current I is parallel to the side with length b (shown by arrows in the figure).

4. In the circuit below, the real batteries have equal internal resistances $r = 2.0$ Ω. Find the potential difference $(V_A - V_B)$ between points A and B. **3 points**



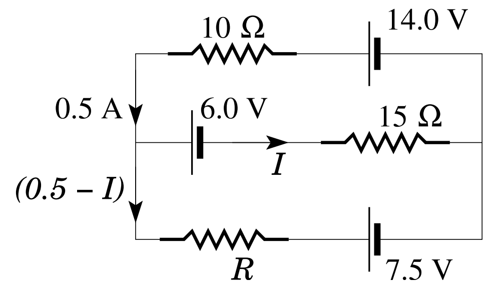
Solution: Let the current I be clockwise, as shown. The loop rule gives

$$5 - 2I - 2I - 4 - 2I - 2 - 2I - 3 = 0 \implies 8I = -4 \implies I = -0.5 \text{ A}$$

Then

$$V_A - 2I - 4 - 2I - 2 = V_B \implies V_A - V_B = 6 + 4I \implies V_A - V_B = 4.0 \text{ V}$$

5. In the circuit below, the current in the 10 Ω resistor is 0.5 A as shown. Find the unknown resistance R. **3 points**

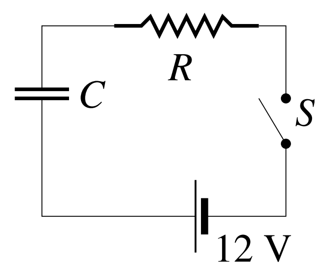


Solution: Let the current in the 15 Ω resistor be I to the right, then current in R is $(0.5 - I)$ to the right, as shown. Then

Upper loop clockwise: $15I + 6 + 10 \times 0.5 - 14 = 0 \implies I = 0.2$ A

Lower loop clockwise: $7.5 + (0.5 - I)\,R - 6 - 15I = 0 \implies R = 5\ \Omega$

6. In the circuit below, $C = 8.0\ \mu$F while the resistance of R is not known. The switch S is closed at $t = 0$ when the capacitor was uncharged. Find the time t at which the plate-charge on C is $Q = 40.0\ \mu$C and the power dissipated in R is $P = 8.2 \times 10^{-3}$ W. **4 points**



Solution: The loop rule with the current in the clockwise direction gives us

$$12 - \frac{Q}{C} - V_R = 0 \implies V_R = 12 - \frac{Q}{C} \implies V_R = 7\ \text{V}$$

Then

$$R = \frac{V_R^2}{P} = \frac{7^2}{8.2 \times 10^{-3}} = 6.0 \times 10^3\ \Omega$$

The charge $Q(t)$ is given by

$$Q = \mathcal{E}C\left(1 - e^{-t/RC}\right) \implies t = -RC\ \ln\left(1 - \frac{Q}{\mathcal{E}C}\right) = 2.6 \times 10^{-2}\ \text{s}$$

7. In a region, the electric field $\vec{\boldsymbol{E}} = 1.2 \times 10^3\ \hat{\boldsymbol{\imath}}$ V/m and the magnetic field $\vec{\boldsymbol{B}}$ is in the $+y$-direction. A point charge $q = +3.0$ mC with velocity $\vec{\boldsymbol{v}} = \left(-1.9 \times 10^5\ \hat{\boldsymbol{\jmath}} + 9.5 \times 10^5\ \hat{\boldsymbol{k}}\right)$ m/s feels a force $\vec{\boldsymbol{F}} = -1.5\ \hat{\boldsymbol{\imath}}$ N. Find the magnitude of $\vec{\boldsymbol{B}}$. **4 points**

Solution: Let the magnetic field be

$$\vec{\boldsymbol{B}} = B\ \hat{\boldsymbol{\jmath}}$$

The net force on the particle is

$$\vec{\boldsymbol{F}} = q\vec{\boldsymbol{E}} + q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} \implies -1.5\ \hat{\boldsymbol{\imath}} = 3.6\ \hat{\boldsymbol{\imath}} - \left(2.85 \times 10^3\right)\ B\ \hat{\boldsymbol{\imath}}$$

$$\implies -5.1 = -\left(2.85 \times 10^3\right)\ B \implies \boxed{B = 1.8 \times 10^{-3}\ \text{T}}$$

8. A current-carrying wire has radius a and resistivity ρ. The electric field in the wire is uniform but varies in time according to the relationship

$$E = (E_0 t + 3)\ \text{V/m}$$

(a) Express the time-varying current density J in terms of E_0 and ρ. **1 point**

$$J = \frac{E}{\rho} = \frac{1}{\rho}(E_0 t + 3)$$

(b) Express the time-varying current in the wire in terms of E_0, ρ and a. **1 point**

$$I = JA = \pi a^2 J = \frac{\pi a^2}{\rho}(E_0 t + 3)$$

(c) Derive an expression for the total charge passing a cross section of the wire between $t = 2$ s and $t = 4$ s. Express your answer in terms of E_0, ρ and a. **2 points**

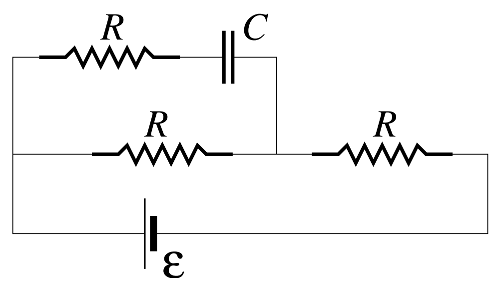
$$Q = \int_2^4 I\ dt = \frac{\pi a^2}{\rho}\int_2^4 (E_0 t + 3)\ dt = \frac{\pi a^2}{\rho}\left(E_0\frac{t^2}{2} + 3t\right)\bigg|_2^4 = \frac{\pi a^2}{\rho}(6E_0 + 6)$$

Part II: Conceptual Questions

In the following, tick (√) the best answer. Each question carries 1 point.

1. Two initially uncharged capacitors, with $C_1 > C_2$, are connected in series to a source of emf. Which of the following statements is true?
   (a) There is more charge stored in C_1.
   (b) There is more charge stored in C_2.
   (c) The potential difference across C_1 is equal to that across C_2.
   (d) $\boxed{\text{The charge in } C_1 \text{ is equal to the charge in } C_2}$.

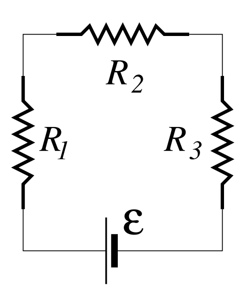
2. In the circuit shown, what is the final plate-charge on the capacitor?
   (a) $Q = \mathcal{E}C$.
   (b) $Q = \frac{2}{3}\mathcal{E}C$.
   (c) $\boxed{Q = \frac{1}{2}\mathcal{E}C}$.
   (d) $Q = \frac{1}{3}\mathcal{E}C$.



3. Which of the following statements is false?
   (a) The electrons in a wire carrying a current normally move very slowly.
   (b) $\boxed{\text{A battery supplies the same current to any device to which it is connected}}$.
   (c) The electric current leaving the positive terminal of the battery is the same as the current arriving at the negative terminal.
   (d) The conventional electric current flows in a direction opposite to the motion of electrons.

4. The equivalent resistance of a group of parallel resistors is
   (a) equal to the smallest resistance in the group.
   (b) equal to the average resistance in the group.
   (c) $\boxed{\text{less than the smallest resistance in the group}}$.
   (d) larger than the largest resistance in the group.

5. Three resistors, $R_1 > R_2 > R_3$ are connected to a source of emf as shown. If V_1, V_2 and V_3 are the potential differences across R_1, R_2 and R_3, then
   (a) $V_1 = V_2 = V_3$.
   (b) $\boxed{V_1 > V_2 > V_3}$.
   (c) $V_1 = V_2 > V_3$.
   (d) $V_1 < V_2 < V_3$.



6. A negatively charged particle is launched with velocity $\vec{v}$ in a region of uniform magnetic field $\vec{B}$. Then its speed will

   (a) increase if $\vec{v}$ and $\vec{B}$ are in same direction.

   (b) increase if $\vec{v}$ and $\vec{B}$ are in opposite directions.

   (c) increase if $\vec{v}$ and $\vec{B}$ are perpendicular to each other.

   (d) $\boxed{\text{remain constant throughout, however } \vec{v} \text{ may change}}$.

7. Two wires of same material, but different lengths L_1 and L_2, and different crossectional areas A_1 and A_2 are connected to same potential differences. Then the current densities will be related as

   (a) $\boxed{\dfrac{J_1}{J_2} = \dfrac{L_2}{L_1}}$.

   (b) $\dfrac{J_1}{J_2} = \dfrac{A_2}{A_1}$.

   (c) $\dfrac{J_1}{J_2} = \dfrac{A_1 L_2}{A_2 L_1}$.

   (d) $\dfrac{J_1}{J_2} = \dfrac{L_1}{L_2}$.

8. In the circuit shown, the three resistors dissipate equal power. Then

   (a) $R_1 = R_2 = R_3$.

   (b) $R_2 = R_3 = 2R_1$.

   (c) $\boxed{R_2 = R_3 = 4R_1}$.

   (d) $R_2 = R_3 = \dfrac{1}{2}R_1$.