

PART I. Solve the following problems. Show your solutions in detail.

1. Three point charges $q_1 = q_2 = 2 \mu\text{C}$ and $q_3 = -2 \mu\text{C}$ are located on the vertices of an equilateral triangle with sides $a = 0.30 \text{ cm}$. What is the magnitude of the net force acting on the charge q_2 ? **[4 points]**

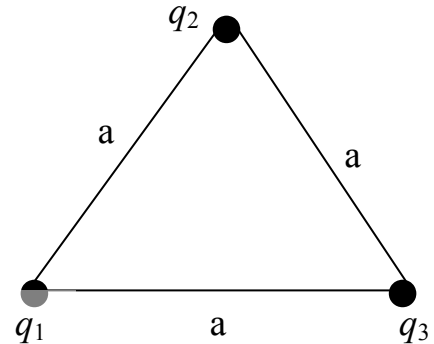
$$F_{21} = F_{23} = \frac{k|q_1||q_2|}{a^2} = 4 \times 10^3 \text{ N} \quad [1]$$

$$F_y = F_{21} \sin 60^\circ - F_{23} \sin 60^\circ = 0 \quad [1]$$

$$F_x = F_{21} \cos 60^\circ + F_{23} \cos 60^\circ$$

$$= 2F_{21} \cos 60^\circ = 4 \times 10^3 \text{ N} \quad [1]$$

$$F = \sqrt{F_x^2 + F_y^2} = 4 \times 10^3 \text{ N} \quad [1]$$

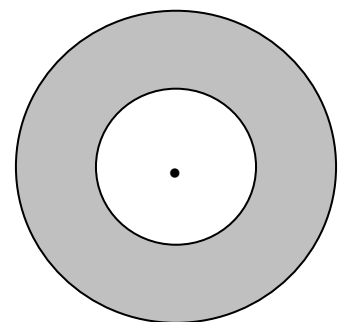


2. A spherical shell with inner radius $a = 1.0 \text{ m}$ and outer radius $b = 2.0 \text{ m}$ has a uniform volume charge density $\rho = 3 \times 10^{-6} \text{ C/m}^3$. What is magnitude of the electric field at 1.5 m from the center of the sphere? **[3 points]**

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\epsilon_0} = \frac{\rho V}{\epsilon_0} \quad [1]$$

$$E \cdot 4\pi r^2 = \frac{\rho \cdot (4\pi/3)(r^3 - a^3)}{\epsilon_0} \quad [1]$$

$$E = \frac{\rho(r^3 - a^3)}{3\epsilon_0 r^2} = 1.2 \times 10^5 \text{ N/C} \quad [1]$$



3. A charged ring of radius $R = 0.3 \text{ m}$ with the centre located at the origin has $Q = 6 \mu\text{C}$ distributed uniformly. A charged particle $q = 2 \mu\text{C}$ ($m = 3.6 \times 10^{-6} \text{ kg}$) is released at $x = 0.5 \text{ m}$. What is the speed of the particle when it reaches the point $x = 1.0 \text{ m}$? [3 points]

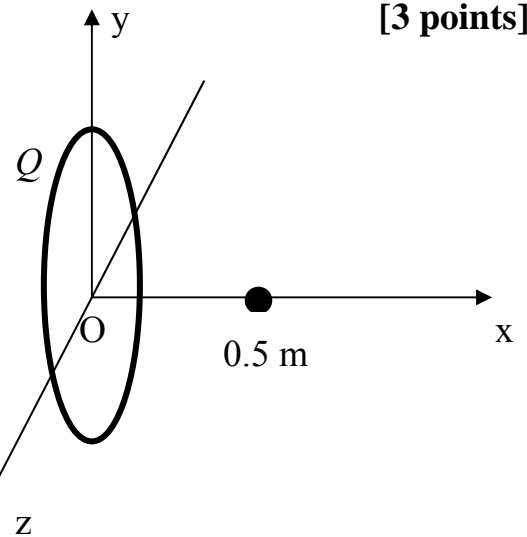
$$\Delta K = -\Delta U = q\Delta V$$

$$\frac{1}{2}mv^2 - 0 = -q(V_f - V_i) \quad [1]$$

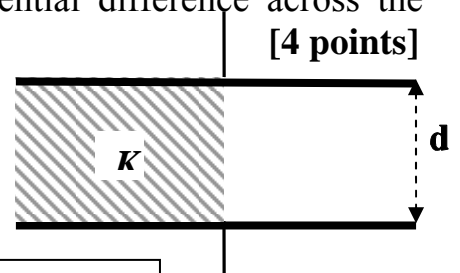
$$= -q \left(k \frac{Q}{\sqrt{R^2 + x_f^2}} - k \frac{Q}{\sqrt{R^2 + x_i^2}} \right)$$

$$= 8.2 \times 10^{-2} \text{ J} \quad [1]$$

$$v = \sqrt{\frac{2 \times 8.2 \times 10^{-2}}{3.6 \times 10^{-6}}} = 2.1 \times 10^2 \text{ m/s} \quad [1]$$



4. An air-filled parallel-plate capacitor $C = 16 \mu\text{F}$ is charged to $V = 16 \text{ V}$ and the battery is disconnected. The half of the space between the plates is now filled with a dielectric material $K = 5$, as shown. What is the potential difference across the capacitor plates now? [4 points]



Considering the two compartments of the capacitor as 1 and 2:

with air: $V_1 = V_2 = V \Rightarrow \sigma_1 = \sigma_2$ and $Q = (\sigma_1 + \sigma_2)A/2$

With the dielectric: $V_1^K = V_2^K \Rightarrow \frac{\sigma_1^K}{K} = \frac{\sigma_2^K}{1} \Rightarrow \sigma_1^K = 5\sigma_2^K$

And $Q = (\sigma_1^K + \sigma_2^K)A/2 = 6\sigma_2^K \times A/2 = 3\sigma_2^K A$

$$V_2^K = \frac{\sigma_2^K d}{\epsilon_0} = \frac{Qd}{3A\epsilon_0} = \frac{1}{3} \frac{Q}{C_0} = \frac{1}{3} V = \frac{16}{3} = 5.3 \text{ V}$$

An easier solution is: Consider the two compartments as two capacitors C_1 and C_2 in parallel.

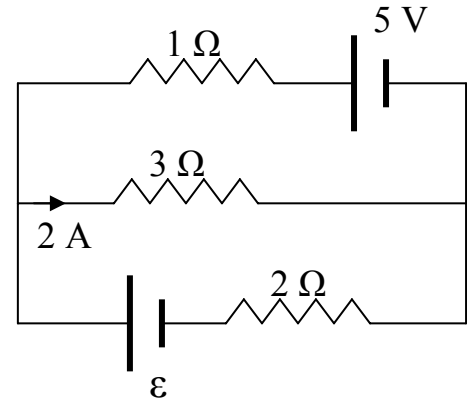
$$C_1 = \frac{K\epsilon_0 A/2}{d} = \frac{5}{2} C = 40 \mu\text{F} \text{ and } C_2 = \frac{\epsilon_0 A/2}{d} = \frac{C}{2} = 8 \mu\text{F}$$

$$C' = C_1 + C_2 = 40 + 8 = 48 \mu\text{F}$$

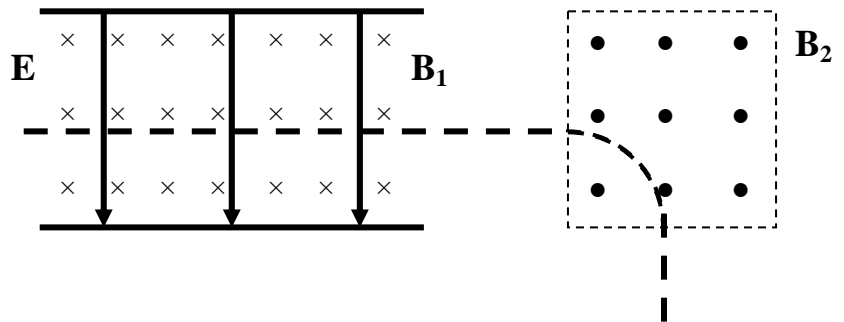
$$\text{New potential difference } V' = \frac{Q}{C'} = \frac{CV}{C'} = \frac{16}{48} \times V = \frac{V}{3} = 5.3 \text{ V}$$

5. In the circuit given, the current through the $3\ \Omega$ resistor is $2\ \text{A}$. What is the e.m.f. of the battery ε ? [3 points]

Upper loop : clockwise
$I_1 \cdot 1 - 5 + 2 \times 3 = 0 \Rightarrow I_1 = -1\ \text{A}$ [1]
Junction rule: $I_1 + I_2 = 2$
$\Rightarrow I_2 = 2 - I_1 = 2 - (-1) = 3\ \text{A}$ [1]
Lower loop : clockwise
$-2 \times 3 - 3 \times 2 + \varepsilon = 0$
$\Rightarrow \varepsilon = 12\ \text{V}$ [1]



6. A charged particle ($q = 8.0\ \mu\text{C}$ and $m = 1.2 \times 10^{-9}\ \text{kg}$) is traveling straight in crossed fields $E = 360\ \text{V/m}$ and $B_1 = 0.15\ \text{T}$. Then it enters perpendicular to a uniform magnetic field $B_2 = 1.8\ \text{T}$. It comes out of the field as shown. How much distance does it travel inside the field B_2 ? [4 points]



$qE = qvB_1$	[1]
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$v = \frac{E}{B_1} = 2400\ \text{m/s}$	[1]
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$r = \frac{mv}{qB_2} = 0.20\ \text{m}$	[1]
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$s = \frac{2\pi r}{4} = 0.31\ \text{m}$	[1]
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7. A conducting wire of length a carrying a current I lies along the y -axis as shown.

- (a) Write down the magnitude of the magnetic field dB at the point P due the infinitesimal element dy of the wire. [1 point]

$$dB = \frac{\mu_0 I}{4\pi} \frac{dy \sin \theta}{r^2}$$

- (b) Using the formula $\sin \theta = \sin (\pi-\theta)$, obtain the expression for dB in terms of x , y and dy . [1 point]

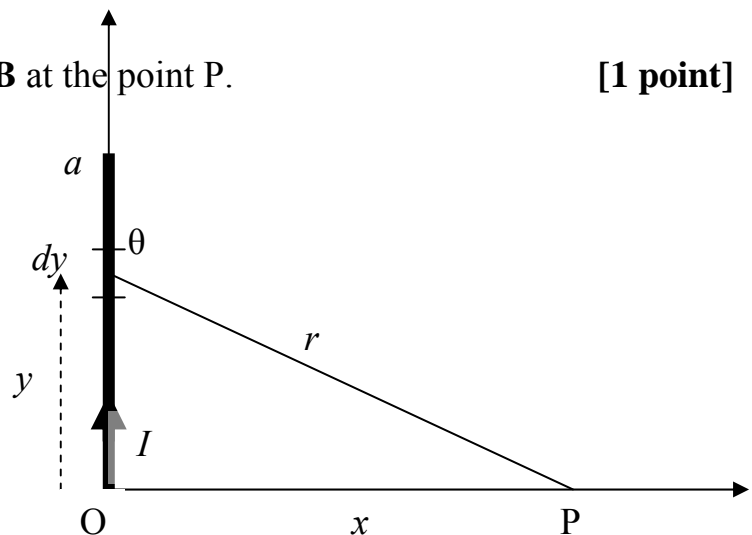
$$dB = \frac{\mu_0 I}{4\pi} \frac{xdy}{(x^2 + y^2)^{3/2}}$$

- (c) Write down the integral for the resultant magnetic field B produced at P due to the whole wire. [1 point]

$$B = \frac{\mu_0 I}{4\pi} \int_0^a \frac{xdy}{(x^2 + y^2)^{3/2}}$$

- (d) Draw the direction of the field \mathbf{B} at the point P. [1 point]

Direction of \mathbf{B} : Into the page



8. A conducting loop with the sides a and b is perpendicular to a uniform magnetic field $\mathbf{B} = 1.2 \text{ T}$ as shown. It is pulled completely out of the field in 0.23 s . If the loop has a resistance of 3.6Ω , what is the magnitude and direction of the average current induced during the pull out? Given, $a = 0.20 \text{ m}$, $b = 0.30 \text{ m}$. [3 points]

$$\Delta\Phi = \Phi_f - \Phi_i = 0 - Bab = -7.2 \times 10^{-2} \text{ Wb} \quad [1]$$

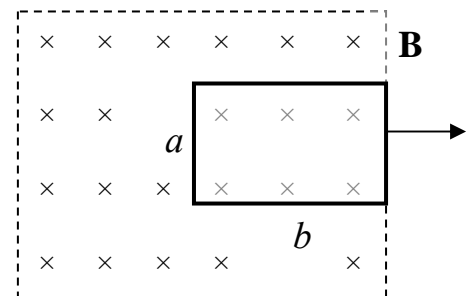
$$|\mathcal{E}| = \left| \frac{\Delta\Phi}{\Delta t} \right| = \frac{7.2 \times 10^{-2}}{0.23} = 0.31 \text{ V} \quad [1]$$

$$I = \frac{\mathcal{E}}{R} = 8.6 \times 10^{-2} \text{ A} = 86 \text{ mA} \quad [1]$$

OR:

$$\mathcal{E} = BLv = Ba \times \frac{b}{t} = 0.31 \text{ V} \quad [1+1]$$

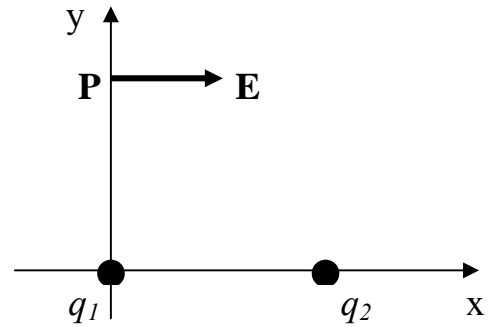
$$I = \frac{\mathcal{E}}{R} = 8.6 \times 10^{-2} \text{ A} = 86 \text{ mA} \quad [1]$$



PART II: Conceptual Questions (each carries 1 point). Tick the best answer:

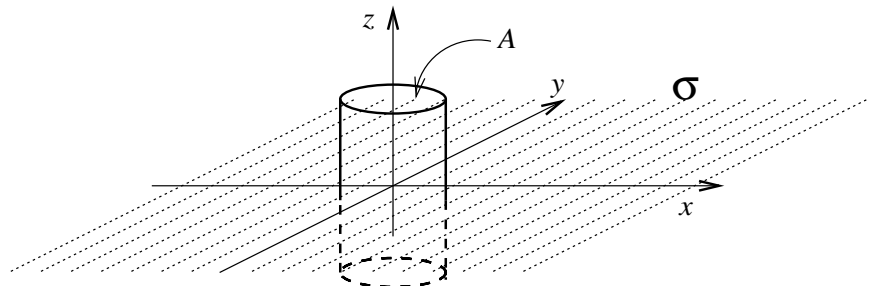
1. The net electric field at point P due to the charges q_1 and q_2 points along the +x-axis as shown. The signs of the charges are:

- a. $q_1 > 0$ and $q_2 > 0$.
- b. $q_1 < 0$ and $q_2 < 0$.
- c. $q_1 > 0$ and $q_2 < 0$. (Ans)
- d. $q_1 < 0$ and $q_2 > 0$.



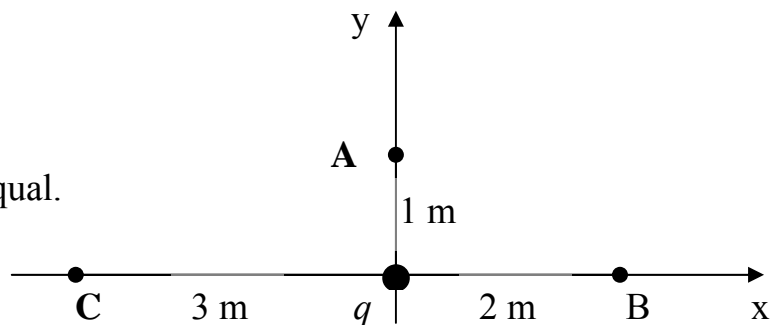
2. The x-y plane has a uniform surface charge density σ . A cylinder with cross-sectional area A lies along the z-axis. The electric flux through the top face of the cylinder is

- a. 0.
- b. $\frac{\sigma A}{4\epsilon_0}$.
- c. $\frac{\sigma A}{2\epsilon_0}$. (Ans)
- d. $\frac{\sigma A}{\epsilon_0}$.



3. Three points A, B, and C are around a point charge $q (>0)$. The magnitude of the potential difference is the smallest between the points

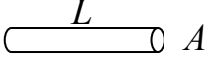
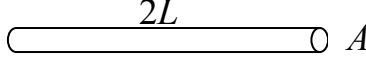
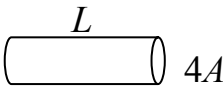
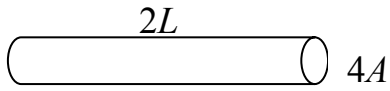
- a. A and B.
- b. B and C. (Ans)
- c. A and C.
- d. All potential differences are equal.



4. Two capacitors A and B are identical. Initially the capacitor A is charged so that it stores 8 J of energy, whereas the capacitor B is uncharged. The capacitors are then connected in parallel. The total energy on the capacitors is now:

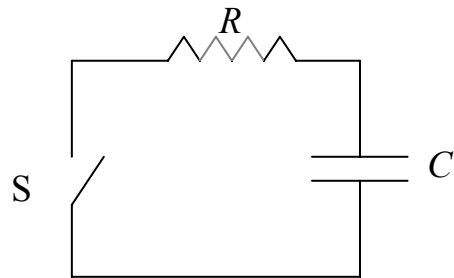
- a. 16 J
- b. 8 J
- c. 4 J (Ans)
- d. 2 J

5. Four cylindrical wires are made of the same material. The length and the cross-section of the wires are shown in the figures. Rank the wires according to their resistance from the lowest to the greatest.

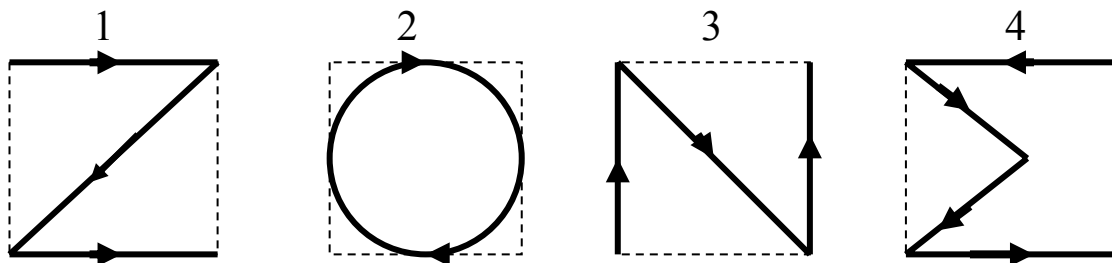
- a. $R_3 < R_4 < R_1 < R_2$ (Ans) 1  2 
- b. $R_3 < R_2 = R_4 < R_1$
- c. $R_2 < R_1 < R_4 < R_3$
- d. $R_4 < R_2 < R_3 < R_1$ 3  4 

6. In the figure below, initially the capacitor is charged to a potential difference V_0 and the switch S is open. Immediately after the switch is closed, the current in the circuit is

- a. 0.
 b. $V_0 C$.
 c. V_0/R . (Ans)
 d. $V_0/(2R)$.



7. Four elements of wires with the same current I are placed in a plane perpendicular to a uniform magnetic field B as shown in the figure. All the wires fit in a square of side L . Rank the wires according to the magnitude of the magnetic force.



- a. $1 < 2 < 3 < 4$
 b. $2 < 3 < 1 < 4$
 c. $2 < 4 < 1 = 3$ (Ans)
 d. $3 < 4 < 2 < 1$

8. Cross-sections of two long parallel wires with identical currents I but in opposite directions are shown. At which point is the magnetic field highest?

- a. A
 b. B (Ans)
 c. C
 d. D

