



**PART I. Solve the following problems. Show your solutions in detail.**

1.  $C_1 = 3 \mu\text{F}$ ,  $C_2 = 6 \mu\text{F}$  and  $C_3 = 4 \mu\text{F}$  are connected to a battery as shown. If the charge on  $C_1$  is  $30 \mu\text{C}$ , what is the energy stored on  $C_3$ . Note:  $\epsilon$  is not given. [4 points]

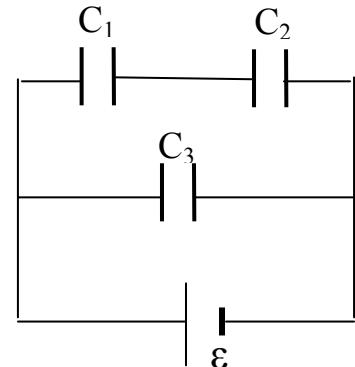
$$Q_{C1} = Q_{C2} = Q = 30 \mu\text{C}$$

$$V_1 = \frac{Q}{C_1} = 10 \text{ V} \quad [1]$$

$$V_2 = \frac{Q}{C_2} = 5 \text{ V} \quad [1]$$

$$\epsilon = V_1 + V_2 = 15 \text{ V} \quad [1]$$

$$U_3 = \frac{1}{2} C_3 \epsilon^2 = 450 \mu\text{J} \quad [1]$$



2. A parallel plate capacitor with the plate separation  $d = 3 \text{ mm}$  and the plate area  $A = 8 \text{ cm}^2$  has a charge  $Q = 120 \text{ pC}$  (the battery is disconnected). If a dielectric slab ( $K = 3$ ) of thickness  $1 \text{ mm}$  is placed between the capacitor plates as shown, what is the final potential difference across the capacitor? [4 points]

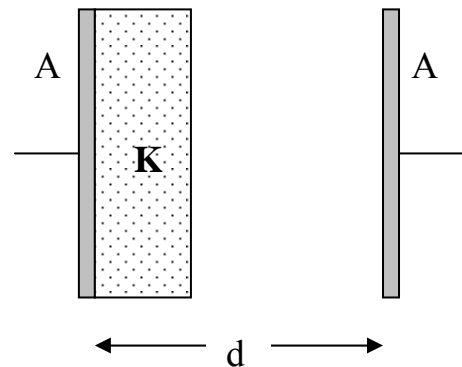
$$E_1 = \frac{\sigma}{K\epsilon_0} = \frac{Q}{K\epsilon_0 A}$$

$$E_2 = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \quad [1]$$

$$V_1 = E_1 \cdot d/3 = \frac{Qd}{3K\epsilon_0 A} = 5.6 \text{ V} \quad [1]$$

$$V_2 = E_2 \cdot 2d/3 = \frac{2Qd}{3\epsilon_0 A} = 33.9 \text{ V} \quad [1]$$

$$V = V_1 + V_2 = 5.6 + 33.9 = 39.5 \text{ V} \quad [1]$$



Alternately,

$$C_1 = \frac{K\epsilon_0 A}{d/3} = 21.2 \text{ pF} \quad [1]$$

$$C_2 = \frac{\epsilon_0 A}{2d/3} = 3.5 \text{ pF} \quad [1]$$

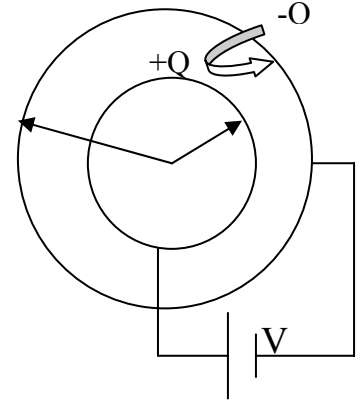
$$C = \frac{C_1 C_2}{C_1 + C_2} = 3.0 \text{ pF} \quad [1]$$

$$V = \frac{Q}{C} = 39.5 \text{ V} \quad [1]$$

3. A spherical capacitor consisting of two concentric conducting spheres of radii  $a$  and  $b$ , as shown in the figure, is connected to the potential difference  $V$ . The charge collected on the spheres is  $+Q$  and  $-Q$ , respectively.

- (a) Show the distribution of the charges on the spheres. [1 point]
- (b) Using the above charges, write down the electric field  $\vec{E}$  between the two spheres. [1 point]

$$\vec{E} = k \frac{Q}{r^2} \hat{r}$$



- (c) Using the electric field obtained above, write down the expression for the potential difference between the spheres. [1 point]

$$V_b - V_a = - \int_a^b \vec{E} \cdot d\vec{r} = k \frac{Q}{r_b} - k \frac{Q}{r_a}$$

$$V = kQ \left( \frac{1}{r_a} - \frac{1}{r_b} \right) = \frac{kQ(r_b - r_a)}{r_b r_a}$$

- (d) Making use of the above potential difference, obtain the expression for the capacitance of the capacitor. [1 point]

$$V = \frac{kQ(r_b - r_a)}{r_b r_a} \Rightarrow$$

$$C = \frac{Q}{V} = \frac{r_b r_a}{k(r_b - r_a)} = \frac{4\pi\epsilon_0 r_b r_a}{r_b - r_a}$$

4. An electric field  $E = 800$  V/m is applied across a conducting wire of radius  $0.5$  mm. If the resistivity of the conductor is  $\rho = 1.7 \times 10^{-8}$   $\Omega \cdot m$ , how much charge passes through the wire in 5 minutes? [3 points]

$$J = \frac{E}{\rho} = 4.7 \times 10^{10} \text{ A/m}^2 \quad [1]$$

$$I = J \cdot \pi r^2 = 3.7 \times 10^4 \text{ A} \quad [1]$$

$$q = I \cdot t = 1.1 \times 10^7 \text{ C} \quad [1]$$

5. A copper wire carries a current of  $12$  A at  $20^\circ\text{C}$ . What is the current in the wire when its temperature rises to  $100^\circ\text{C}$ , assuming that the voltage applied to the wire does not change? The temperature coefficient of resistivity for copper is  $4 \times 10^{-3} \text{ }^\circ\text{C}^{-1}$ . [2 points]

$$R = R_0(1 + \alpha\Delta T)$$

$$I_0 = I(1 + \alpha\Delta T) \quad [1]$$

$$I = 9.1 \text{ A} \quad [1]$$

6. In the circuit shown,  $\varepsilon = 18 \text{ V}$ ,  $R_1 = 8 \Omega$ ,  $R_2 = 6 \Omega$ , and  $R_3 = 12 \Omega$ . What is the power dissipated in the resistor  $R_3$ ? [4 points]

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3} = 4 \Omega$$

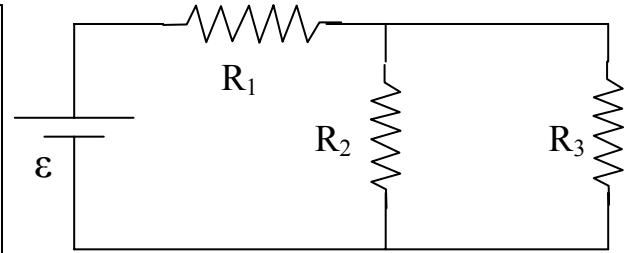
$$R_{eq} = R_1 + R_{23} = 12 \Omega \quad [1]$$

$$I_1 = \frac{\varepsilon}{R_{eq}} = 1.5 \text{ A} \quad [1]$$

Using the loop - rule for the outer loop:

$$+ \varepsilon - I_1 R_1 - I_3 R_3 = 0 \Rightarrow I_3 = 0.5 \text{ A} \quad [1]$$

$$P_{R_3} = I_3^2 R_3 = 3 \text{ W} \quad [1]$$



7. In the circuit shown, what is the e.m.f.  $\varepsilon_2$  of the second battery, if  $\varepsilon_1 = 6 \text{ V}$  and  $I_3 = 2.0 \text{ A}$ . [3 points]

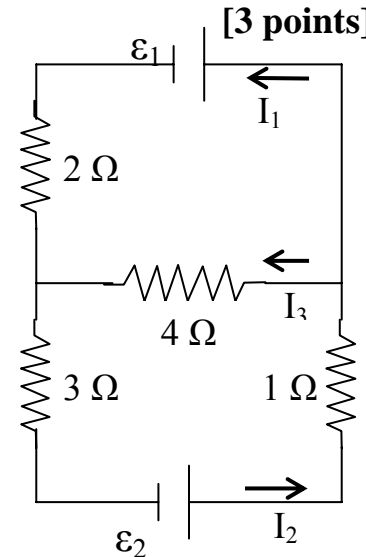
Upper loop: clockwise

$$+ I_1 \cdot 2 + \varepsilon_1 - I_3 \cdot 4 = 0 \Rightarrow I_1 = 1.0 \text{ A} \quad [1]$$

$$I_2 = I_1 + I_3 = 3.0 \text{ A} \quad [1]$$

Lower loop: clockwise

$$I_2 \cdot 3 + I_3 \cdot 4 + I_2 \cdot 1 - \varepsilon_2 = 0 \Rightarrow \varepsilon_2 = 20 \text{ V} \quad [1]$$



8. A capacitor  $C = 10 \mu\text{F}$  is connected to a circuit as shown, where the switch  $S$  has been open for a long time. What is the change in the charge on the capacitor long time after  $S$  is closed? [4 points]

Initial charge:  $Q_i = C\varepsilon = 120 \mu\text{C} \quad [1]$

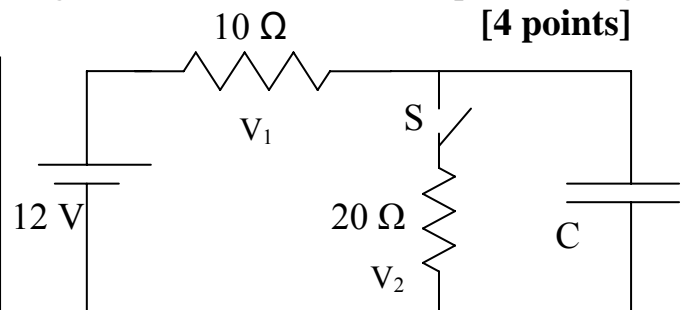
Long time after closing the switch:

$$I = \frac{12}{(10+20)} = 0.4 \text{ A} \quad [1]$$

$$V_2 = I \cdot 20 = 8 \text{ V}$$

$$Q_f = CV_2 = 80 \mu\text{C} \quad [1]$$

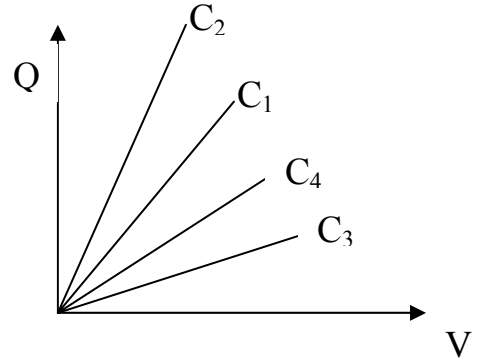
$$\Delta Q = Q_f - Q_i = -40 \mu\text{C} \quad [1]$$



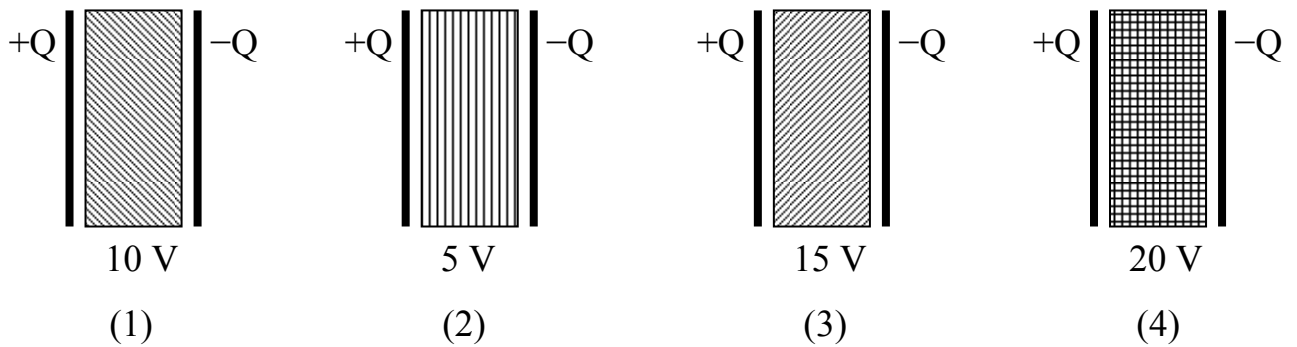
**PART II: Conceptual Questions (each carries 1 point). Tick the best answer:**

1. Charge  $Q$  is shown for four different capacitors as a function of the voltage applied. Which of the following is true?

- a.  $C_1 > C_2 > C_3 > C_4$
- b.  $C_2 > C_3 > C_4 > C_1$
- c.  $C_2 > C_1 > C_4 > C_3$  (Ans)
- d.  $C_4 > C_3 > C_1 > C_2$

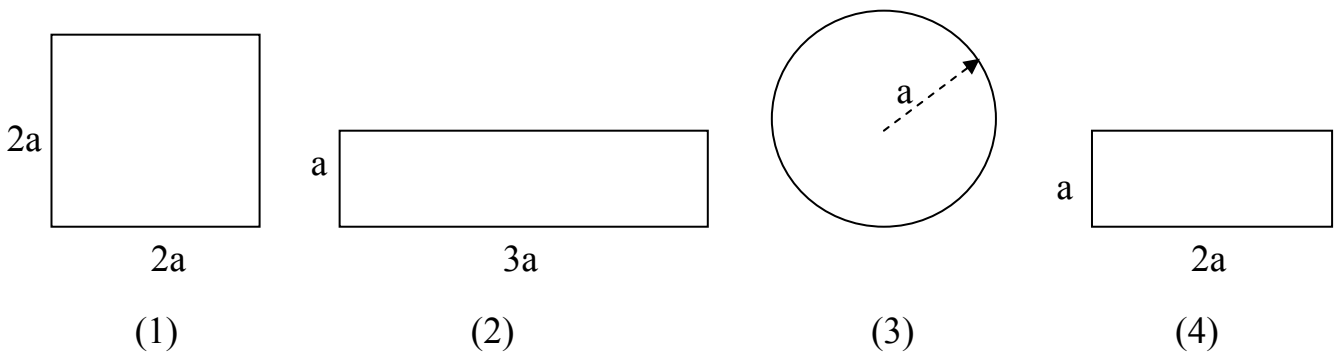


2. Four identical capacitors with the same charge  $Q$  are filled completely with different dielectric materials. The resulting potential difference across the capacitors is shown below. Which capacitor has the highest induced charge on the surface of the dielectric?



- a. 1
- b. 2 (Ans)
- c. 3
- d. 4

3. Cross-sectional area of four conductors of the same material is shown in the figure. If the same potential difference is applied across the same length of the conductors, which conductor gives the largest current?



- a. 1 (Ans)
- b. 2
- c. 3
- d. 4

4. Tick the correct answer.

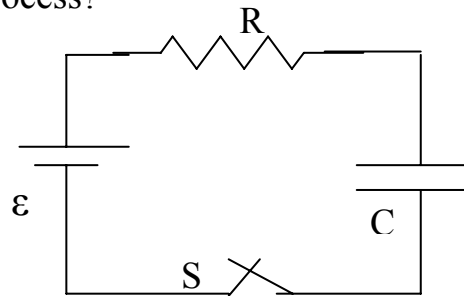
- a. Conductivity of a metallic conductor increases with an increase in temperature.
- b. Conductivity of a metallic conductor decreases with increase in temperature. (Ans)
- c. Resistivity of a metallic conductor decreases with an increase in temperature.
- d. Resistivity of a metallic conductor does not change with change in temperature.

5. An air-filled parallel-plate capacitor has a charge on its plates. The distance between the plates and the area of the plates are doubled, keeping the charge as constant. Then, the energy density stored on the capacitor

- a. does not change.
- b. becomes four times the initial value.
- c. becomes one quarter of the initial value. (Ans)
- d. becomes double of the initial value.

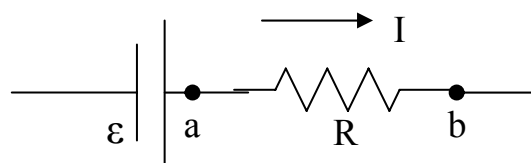
6. A capacitor is charged in an RC-circuit as shown. If the resistor (which is made of copper) is heated, what happens to the charging process?

- a. The capacitor will charge faster.
- b. The capacitor will charge slower. (Ans)
- c. The capacitor will charge as before.
- d. The capacitor will not charge.



7. For the current directed to the right, which statement is correct?

- a.  $V_a > V_b$  (Ans)
- b.  $V_a < V_b$
- c.  $V_a = V_b$
- d.  $V_a = V_b - \varepsilon$



8. Current in a conductor of length  $L$  is  $I_1$  when a voltage  $V$  is applied across it. The conductor is then cut into two halves. If the same voltage is applied across one half of the conductor (as shown on the right), then the current  $I_2$  is now:

- a.  $I_1$ .
- b.  $I_1/2$ .
- c.  $2I_1$ . (Ans)
- d.  $4I_1$ .

