Kuwait University — Department of Physics

Physics 102
First Midterm Examination (Summer Semester, 2009-2010)

July 12, 2010
Time: 6:00 PM to 8:00 PM

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Name: .... **Student No.:** ....

Fundamental Constants

$k = \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \mathrm{N \cdot m^2/C^2}$	Coulomb's constant
$\varepsilon_0 = 8.85 \times 10^{-12} \mathrm{C^2/(\ N \cdot m^2)}$	Permitivity of free space
$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$	Permeability of free space
$\lvert e \rvert = 1.6 \times 10^{-19}\ \mathrm{C}$	Elementary unit of charge
$m_e = 9.11 \times 10^{-31}\ \mathrm{kg}$	Mass of an electron
$m_p = 1.67 \times 10^{-27}\ \mathrm{kg}$	Mass of a proton
$1\ \mathrm{eV} = 1.6 \times 10^{-19}\ \mathrm{J}$	Conversion from eV to J
$N_A = 6.022 \times 10^{23}/\mathrm{mol}$	Avogadro's number
$g = 9.8\ \mathrm{m/s^2}$	Acceleration due to gravity

Prefixes of units

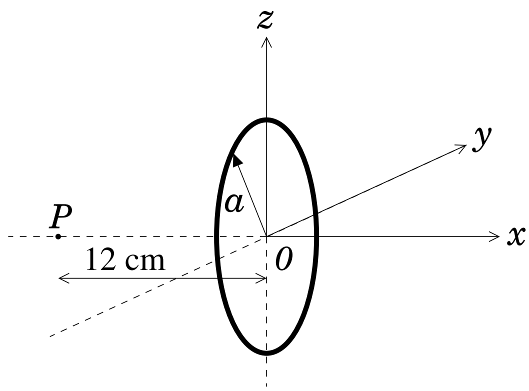
$\mathrm{m} = 10^{-3}$	$\mu = 10^{-6}$	$\mathrm{n} = 10^{-9}$	$\mathrm{p} = 10^{-12}$
$\mathrm{k} = 10^{3}$	$\mathrm{M} = 10^{6}$	$\mathrm{G} = 10^{9}$	$\mathrm{T} = 10^{12}$

For use by instructors only

Problems	1	2	3	4	5	6	7	8	Questions	Total
Marks										

Part I: Solve the following problems. Show all your workings

1. A thin ring of radius $a = 4.0$ cm contains a uniformly distributed charge Q. It is perpendicular to the x-axis with its centre at the origin, as shown. The electric field at $x = -12.0$ cm on the x-axis (point P in the figure) is $\vec{E} = 2.3 \times 10^3\ \hat{\imath}$ N/C. Find the total charge Q on the ring. **2 points**

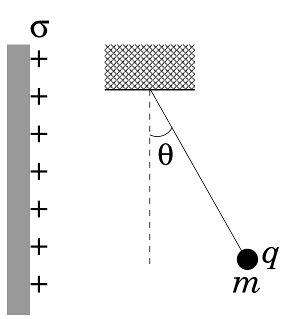


Solution: Since $\vec{E}$ is towards the charge Q must be negative, $Q < 0$. The magnitude of the electric field is

$$\frac{k\,|Qx|}{(x^2+a^2)^{3/2}} = 2.3 \times 10^3 \implies |Q| = \frac{(x^2+a^2)^{3/2}}{k|x|} \times 2.3 \times 10^3 = 4.3 \times 10^{-9} \text{ C}$$

$$\text{So} \qquad Q = -4.3 \times 10^{-9} \text{ C}$$

2. A small metallic sphere of mass $m = 0.02$ g and carrying a charge $q = +6.25 \times 10^{-8}$ C hangs from an insulating thread near a very large flat non-conducting vertical sheet with uniform charge density $\sigma = +8.63 \times 10^{-8}$ C/m^2 as shown. Find the angle θ the thread makes with the vertical. **2 points**

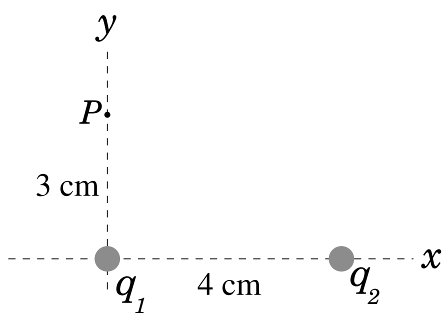


Solution: Let T be the tension in the thread. The free-body diagram gives us

$$T\ \cos\theta = mg \qquad \text{and} \qquad T\ \sin\theta = qE = \frac{q\sigma}{2\varepsilon_0}$$

$$\implies \theta = \tan^{-1}\left(\frac{q\sigma}{2\varepsilon_0 mg}\right) = 57.3^o$$

3. Two point charges, $q_1 = +2.0$ nC and $q_2 = -6.0$ nC, lie along the x-axis as shown. Find the net electric field $\vec{\boldsymbol{E}}$ at the point P on the y-axis. **4 points**



Solution: At the point P, we have

$$\vec{\boldsymbol{E}}_1 = \frac{k|q_1|}{0.03^2}\,\hat{\boldsymbol{j}} = 2.00 \times 10^4\,\hat{\boldsymbol{j}}\ \text{N/C}$$

$$\vec{\boldsymbol{E}}_2 = \frac{k|q_2|}{0.05^2}\left(\frac{0.04}{0.05}\,\hat{\boldsymbol{i}} - \frac{0.03}{0.05}\,\hat{\boldsymbol{j}}\right) = \left(1.73 \times 10^4\,\hat{\boldsymbol{i}} - 1.30 \times 10^4\,\hat{\boldsymbol{j}}\right)\ \text{N/C}$$

$$\vec{\boldsymbol{E}} = \vec{\boldsymbol{E}}_1 + \vec{\boldsymbol{E}}_2 = \left(1.73 \times 10^4\,\hat{\boldsymbol{i}} + 7.04 \times 10^3\,\hat{\boldsymbol{j}}\right)\ \text{N/C}$$

4. A sphere of radius $a = 4.0$ cm has a charge Q uniformly distributed over its volume. The magnitude of the electric field at a distance of $r = 3.0$ cm from the centre is $E = 6.33 \times 10^3$ N/C. Find the magnitude of the electric field at a distance of $r = 6.0$ cm from the centre. **2 points**

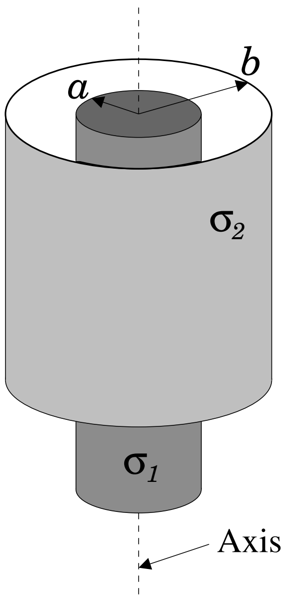
Solution: At $r = 3.0$ cm (so $r < a$),

$$E = \frac{kQr}{a^3} \implies Q = \frac{Ea^3}{kr} = 1.50 \times 10^{-9}\ \text{C}$$

At $r = 6.0$ cm ($r > a$),

$$E = \frac{kQ}{r^2} = 3.75 \times 10^3\ \text{N/C}$$

5. A long *conducting* cylinder of radius $a = 1.5$ cm and surface charge density σ_1 is coaxial with a long thin conducting hollow cylinder of radius $b = 3.0$ cm with surface charge density σ_2. The net electric field $E = 0$ at a distance of $r = 5.0$ cm from the axis. If $\sigma_2 = -3.12 \times 10^{-8}$ C/m^2, find the electric field (its magnitude and direction) at a distance of $r = 2.0$ cm from the axis. **5 points**



Solution: Choose a cylindrical Gaussian surface of radius $r = 5.0$ cm and length L. Then $\Phi_E = 0$ (Since $E = 0$ everywhere on this surface) and so

$$Q_{enc} = 2\pi a L \sigma_1 + 2\pi b L \sigma_2 = 0$$

$$\Longrightarrow \sigma_1 = \frac{-b\sigma_2}{a} = +6.24 \times 10^{-8} \text{ C/m}^2$$

At $r = 2.0$ cm, $\vec{E}$ will be **outward** because σ_1 is positive. Choose a cylindrical Gaussian surface of radius $r = 2.0$ cm and length L. Then Φ_E and Q_{enc} are

$$\Phi_E = (2\pi r L)\, E \qquad\qquad Q_{enc} = 2\pi a L \sigma_1$$

Applying the Gauss' law

$$E = \frac{\sigma_1 a}{\varepsilon_0 r} = 5.29 \times 10^3 \text{ N/C, and is } \textbf{outward}$$

6. Three charged particles, $q_1 = -3.0\ \mu$C, $q_2 = -6.0\ \mu$C, and $q_3 = -9.0\ \mu$C, are arranged at the vertices of an equilateral triangle of side $a = 2.5 \times 10^{-3}$ m. Calculate the total electric potential energy stored in the system (Take $U = 0$ when the charges are very far apart.). **2 points**

Solution: The total potential energy is

$$U = \frac{kq_1q_2}{a} + \frac{kq_2q_3}{a} + \frac{kq_3q_1}{a} = +3.6 \times 10^2 \text{ J}$$

7. Two charged particles, q_1 and q_2, are released from rest when they were 40.0 cm apart. When the distance between them becomes 60.0 cm, the total kinetic energy of the particles becomes 0.35 J. Find the total kinetic energy of the particles when the distance between them is 90.0 cm. **4 points**

Solution: When the distance between them is 60.0 cm, the work-energy principle gives us

$$0.35 - 0 = \frac{kq_1q_2}{0.4} - \frac{kq_1q_2}{0.6} = kq_1q_2\left(\frac{1}{0.4} - \frac{1}{0.6}\right)$$
$$\Longrightarrow kq_1q_2 = 0.42 \text{ J} \cdot \text{m}$$

When the distance is 90.0 cm, again the work-energy principle gives us

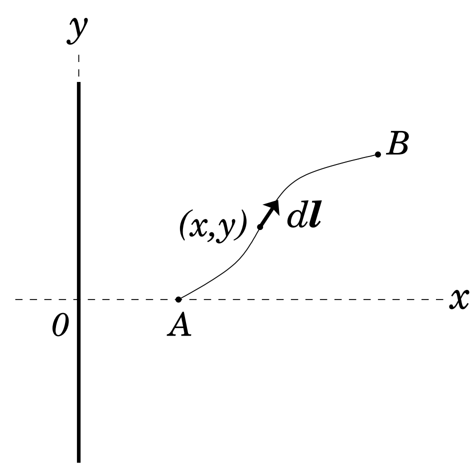
$$\begin{aligned} K - 0 &= \frac{kq_1q_2}{0.4} - \frac{kq_1q_2}{0.9} \\ \Longrightarrow K &= kq_1q_2\left(\frac{1}{0.4} - \frac{1}{0.9}\right) = 0.58 \text{ J} \end{aligned}$$

8. A long thin wire with a *positive* uniform charge density λ lies along the y-axis. Point A with coordinates $(x_A,\ 0)$ and point B with coordinates $(x_B,\ y_B)$ are as shown below. We want to calculate the potential difference $(V_A - V_B)$ by using the formula

$$V_A - V_B = \int_A^B \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{\ell}}$$

We choose a small lenth element $d\vec{\boldsymbol{\ell}}$ at the point with coordinates $(x,\ y)$ on the line AB as shown. Then

$$d\vec{\boldsymbol{\ell}} = dx\ \hat{\boldsymbol{i}} + dy\ \hat{\boldsymbol{j}}$$



(a) Write the expression for the electric field $\vec{\boldsymbol{E}}$ at the point $(x,\ y)$ in terms of λ, x, y, and the unit vectors $\hat{\boldsymbol{i}}$ and $\hat{\boldsymbol{j}}$. **1 point**

$$\vec{\boldsymbol{E}} = \frac{2k\lambda}{x}\ \hat{\boldsymbol{i}}$$

(b) Find the product $\vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{\ell}}$ **1 point**

$$\vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{\ell}} = \frac{2k\lambda}{x}\ dx$$

(c) Find $(V_A - V_B)$ by integrating $\vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{\ell}}$ obtained in part (b) from A to B **2 points**

$$\begin{aligned} V_A - V_B &= \int_{x_A}^{x_B} \frac{2k\lambda}{x}\ dx = 2k\lambda \int_{x_A}^{x_B} \frac{dx}{x} \\ &= 2k\lambda\ \ln x\Big|_{x_A}^{x_B} = 2k\lambda\ \ln\left(\frac{x_B}{x_A}\right) \end{aligned}$$

Part II: Conceptual Questions

In the following, tick (√) the best answer. Each question carries 1 point.

1. A point charge of mass m and charge Q and another point charge of mass $3m$ and charge $2Q$ are released on a frictionless table when they were separated by a distance d. If the acceleration of charge Q is a at some instant, what is the acceleration of charge $2Q$ at that instant?

   (a) $2a$.

   (b) a.

   (c) $\frac{2a}{3}$.

   (d) $\boxed{\frac{a}{3}}$.

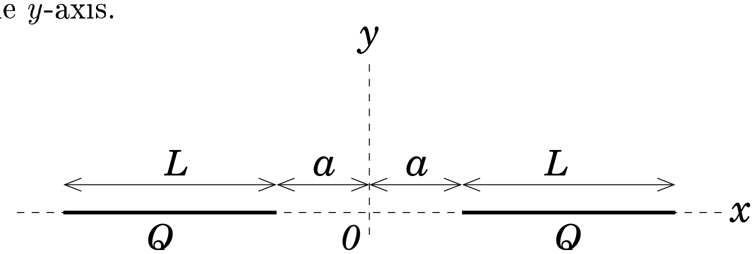
2. Two identical lines with equal charges lie along the x-axis as shown. Which statement about the net electric field is correct?

   (a) $\vec{\boldsymbol{E}} = 0$ at all points on the x-axis for which $-a < x < a$.

   (b) $\vec{\boldsymbol{E}} = 0$ everywhere on the y-axis.

   (c) $\boxed{\vec{\boldsymbol{E}} = 0 \text{ at the origin}}$.

   (d) None of the above.



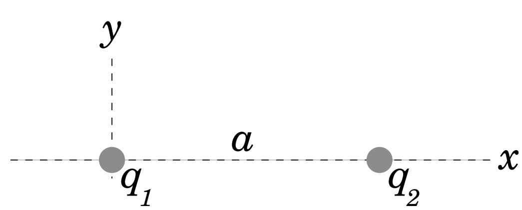
3. The point charge q_1 (negative) is at the origin and the point charge q_2 (positive) is at $x = a$ on the x-axis. A third charge q_3 (positive) with $q_3 > |q_1|$ is placed on the x-axis such that the net force on q_2 is zero. If x_3 is the x-coordinate of q_3, then

   (a) $x_3 = 0$.

   (b) $x_3 > a$.

   (c) $\boxed{x_3 < 0}$.

   (d) $0 < x_3 < a$.



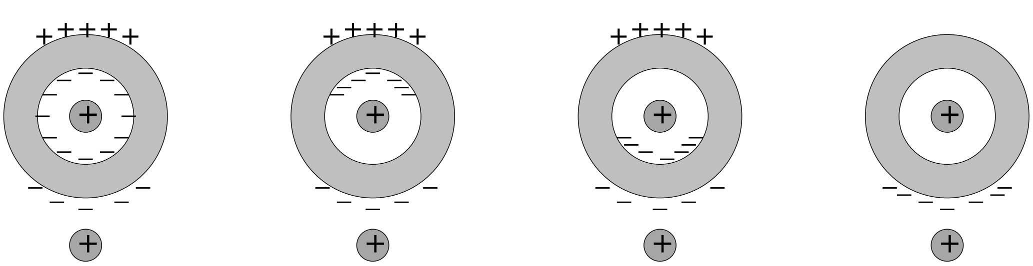
4. A positive charge is located in the cavity of an electrically neutral **conducting** spherical shell. If another positive charge is brought close to the shell without touching it, which of the figures best represents the charge distribution in the shell?

   (a) $\boxed{1}$.

   (b) 2.

   (c) 3.

   (d) 4.



(1) (2) (3) (4)

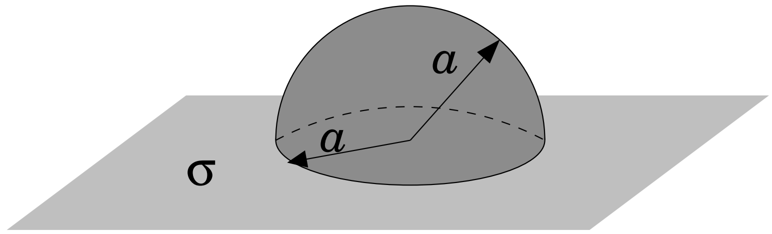
5. The flat sheet in the figure has a uniform surface charge density σ. A hemispherical surface of radius a has its base on the sheet. What is the total electric flux through the hemispherical surface?

(a) $\Phi_E = \dfrac{4\pi a^2\sigma}{\varepsilon_0}$

(b) $\Phi_E = \dfrac{2\pi a^2\sigma}{\varepsilon_0}$

(c) $\Phi_E = \dfrac{\pi a^2\sigma}{\varepsilon_0}$

(d) $\boxed{\Phi_E = \dfrac{\pi a^2\sigma}{2\varepsilon_0}}$



6. When a **negative** charge is released in the region of electric field, it moves towards a region of

(a) higher electric potential and its potential energy increases.

(b) $\boxed{\text{higher electric potential but its potential energy decreases}}$.

(c) lower electric potential but its potential energy increases.

(d) lower electric potential and its potential energy decreases.

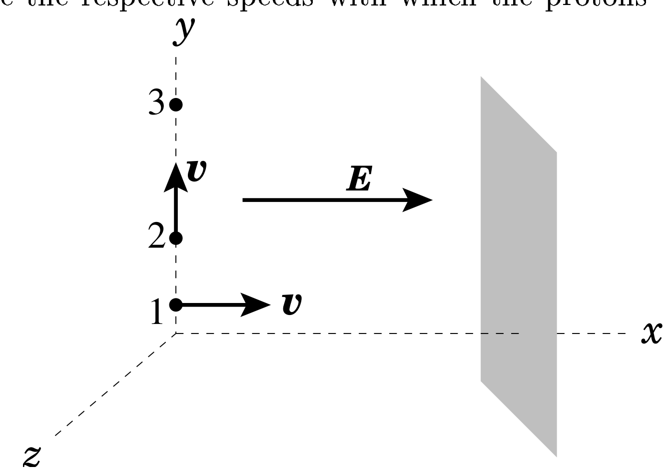
7. A uniform electric field is maintained along the $+x$-direction in the region. Three protons, marked 1, 2 and 3, start from points on the y-axis. Proton 1 starts with speed v in $+x$-direction, proton 2 starts with speed v in $+y$-direction and proton 3 starts from rest. Let v_1, v_2 and v_3 be the respective speeds with which the protons pass the shaded plane. Then

(a) $v_1 > v_2 = v_3$.

(b) $v_1 = v_2 = v_3$.

(c) $\boxed{v_1 = v_2 > v_3}$.

(d) $v_1 > v_3 > v_2$.



8. The solid lines are the electric field lines in the shaded region. Points A and B are marked in the figure. Let V_A and V_B are the electric potentials respectively, and E_A and E_B are the magnitudes of the electric fields at A and B. Then

(a) $\boxed{E_A < E_B,\text{ but } V_A > V_B}$.

(b) $E_A < E_B$, and $V_A < V_B$.

(c) $E_A > E_B$, and $V_A > V_B$.

(d) $E_A > E_B$, but $V_A < V_B$.