



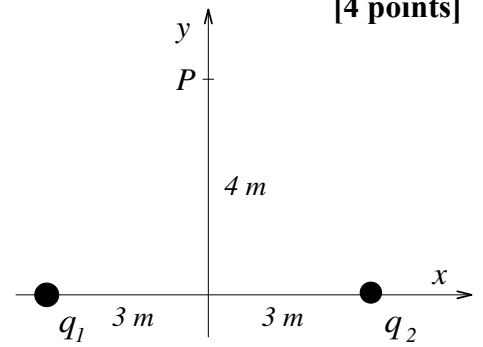
**PART I. Solve the following problems. Show your solutions in detail.**

1. Two point charges  $q_1 = 1 \mu\text{C}$ ,  $q_2 = -1 \mu\text{C}$  lie along the  $x$ -axis as shown. Find the magnitude of electric field at point  $P$ . [4 points]

$$\vec{E}_1 = \frac{k|q|}{r^2} \frac{3}{r} \hat{i} + \frac{k|q|}{r^2} \frac{4}{r} \hat{j} \quad \vec{E}_2 = \frac{k|q|}{r^2} \frac{3}{r} \hat{i} - \frac{k|q|}{r^2} \frac{4}{r} \hat{j}$$

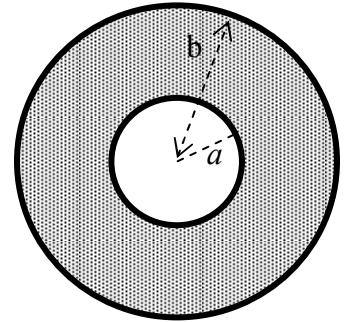
$$r = \sqrt{3^2 + 4^2} = 5\text{m}$$

$$\vec{E}_{net} = 2E_{1x} = 432 \text{ N/C}$$



2. A hollow spherical shell with inner radius  $a = 30\text{ cm}$  and outer radius  $b = 70\text{ cm}$  has uniform volume charge density  $\rho$ . The magnitude of electric field  $50\text{ cm}$  from the center of the shell is  $E = 2\text{ N/C}$ . Calculate the magnitude of  $\rho$ . [4 points]

$$\oint \vec{E} \cdot d\vec{A} = E4\pi r^2 = \frac{\rho \frac{4}{3} \pi (r^3 - a^3)}{\epsilon_0} \Rightarrow \rho = \frac{E3r^2 \epsilon_0}{(r^3 - a^3)} = 1.35 \times 10^{-10} \text{ C/m}^3$$



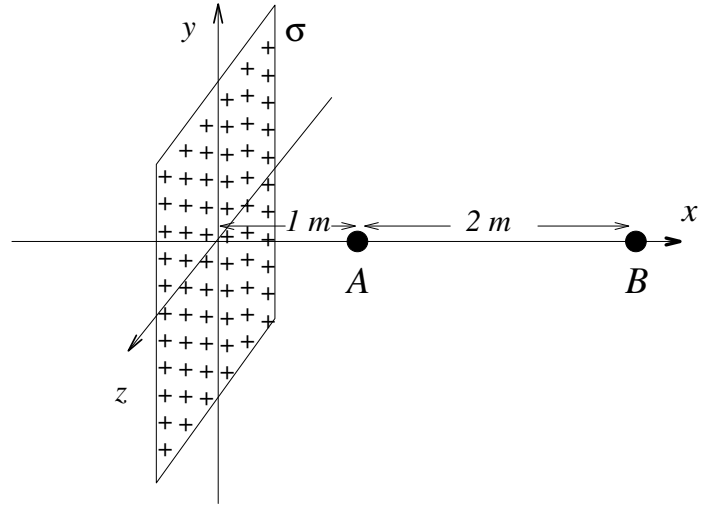
3. An infinite, non-conducting sheet with uniform surface charge density  $\sigma = 3.54 \times 10^{-11} \text{ C/m}^2$  is in the  $y$ - $z$ -plane. A particle with charge  $q = 2 \mu\text{C}$  and mass  $m = 4 \times 10^{-6} \text{ kg}$  is released from rest at point  $A$ .

What is the speed of the particle at point  $B$ ?

[4 points]

$$-\Delta U = \Delta K \Rightarrow -q\Delta V = \frac{mv^2}{2} \Rightarrow v = \sqrt{\frac{2q|\Delta V|}{m}}$$

$$\text{with } \Delta V = -\frac{\sigma d}{2\epsilon_0} \quad \text{so } v = 2 \text{ m/s}$$



4. Two conducting spheres have radii  $R_1 = 60 \text{ cm}$ ,  $R_2 = 30 \text{ cm}$  and charges  $Q_1 = 120 \mu\text{C}$  and  $Q_2 = 0 \text{ C}$ . If the two spheres are connected with a thin wire, what will be the electric potential energy stored on the smaller sphere after the equilibrium is established?

[4 points]

$$Q_1 = Q_1^{fin} + Q_2^{fin} \text{ (Eq.1)} \quad \text{and} \quad V_1 = V \Rightarrow \frac{Q_1^{fin}}{C_1} = \frac{Q_2^{fin}}{C_2} = \frac{Q_1^{fin}}{R_1} = \frac{Q_2^{fin}}{R_2} \text{ (Eq.2)}$$

$$\text{From (Eq.1) and (Eq.2)} \quad Q_2^{fin} = 40 \mu\text{C} \Rightarrow U = \frac{Q^2}{2C} = 24 \text{ J}$$

5. Find the current supplied by the 13-V battery to the circuit, if the current in the middle resistance is  $I$  A, as shown. **[6 points]**

Currents, as shown, and junction rules can be assigned:

$$I = I_1 + I_2$$

$$I_1 + 1A = I_4$$

$$I_2 - 1A = I_3$$

Loops rules, for instance in the middle left and right loops, by using the above junction rules:

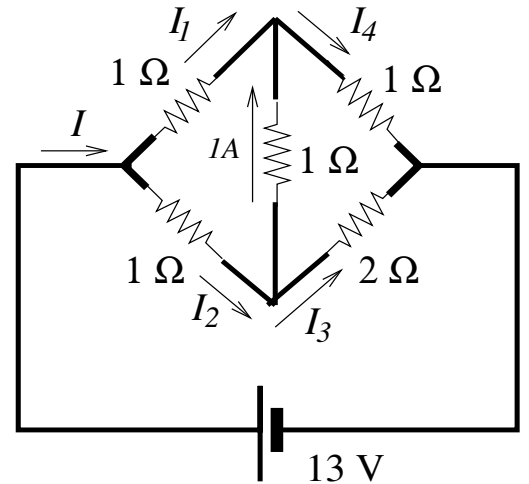
$$-I_1 1\Omega + 1A 1\Omega + I_2 1\Omega = 0$$

$$-1A 1\Omega - (I_1 + 1A) 1\Omega + (I_2 - 1A) 2\Omega = 0$$

⇓

$$I_1 = 6A \quad I_2 = 5A.$$

$$\Rightarrow I = I_1 + I_2 = 11A$$



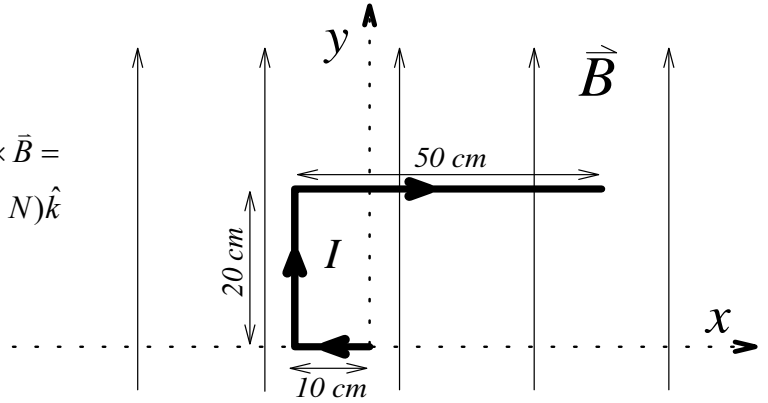
6. A point charge  $q = 2 \mu C$  enters a region of a uniform magnetic field of  $\vec{B} = (0.5 \text{ T})\hat{j}$  with velocity  $\vec{v} = (4 \times 10^5 \text{ m/s})\hat{i} + (5 \times 10^4 \text{ m/s})\hat{j}$ . Find the electric field  $\vec{E}$  that keeps the particle on a straight path inside this region. **[3 points]**

$$\vec{F}_{net} = \vec{F}_E + \vec{F}_B = 0 \Rightarrow \vec{F}_E = -\vec{F}_B \Rightarrow \vec{E} = -\vec{v} \times \vec{B} \Rightarrow$$

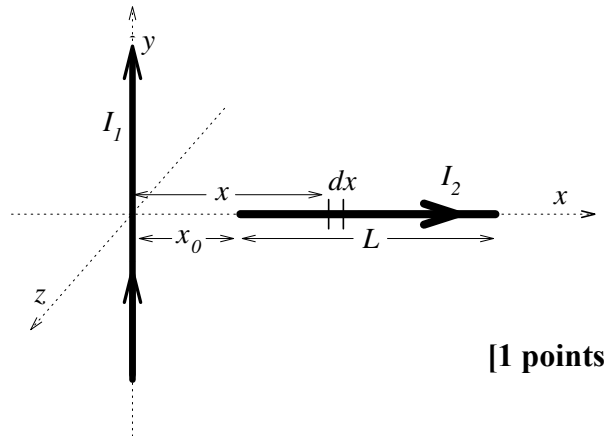
$$\vec{E} = -(4 \times 10^5 \text{ m/s})\hat{i} + (5 \times 10^4 \text{ m/s})\hat{j} \times 0.5T\hat{j} = (-2 \times 10^5 \text{ N/C})\hat{k}$$

7. A bent wire, as shown in the figure, carrying a current of 3 A is in uniform magnetic field  $\vec{B} = (4 \text{ T}) \hat{j}$ . Find the magnetic force vector acting on the wire. **[3 points]**

$$\begin{aligned}\vec{F}_{net} &= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = I\vec{L}_1 \times \vec{B} + I\vec{L}_2 \times \vec{B} + I\vec{L}_3 \times \vec{B} = \\ &= (-IL_1B + 0 + IL_3B)\hat{k} = I(L_3 - L_1)B\hat{k} = (4.8 \text{ N})\hat{k}\end{aligned}$$



8. An infinitely long straight wire carrying a current  $I_1$  lies along the  $y$ -axis. A straight wire of length  $L$  carrying a current  $I_2$  lies along the  $x$ -axis as shown.



- (a) Write down a formula for the magnetic field vector  $\vec{B}_1$  due to  $I_1$  at distance  $x$  from the wire. Use the unit vectors  $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$ .

$$\vec{B}_1 = -\frac{\mu_0 I_1}{2\pi x} \hat{k}$$

**[1 points]**

- (b) Use the general expression  $d\vec{F} = I d\vec{L} \times \vec{B}$  and the result in (a) to find a vector expression for the force acting on segment  $dx$  of the wire along the  $x$ -axis.

**[1 points]**

$$d\vec{F} = I_2 (dx \hat{i}) \times \left( -\frac{\mu_0 I_1}{2\pi x} \hat{k} \right) = \frac{\mu_0 I_2 I_1 dx}{2\pi x} \hat{j}$$

- (c) Write down an integral expression of the total force  $\vec{F}$  acting on the wire with  $I_2$  and integrate it.

**[2 points]**

$$\vec{F} = \int d\vec{F} = \int_{x_0}^{x_0+L} \frac{\mu_0 I_2 I_1 dx}{2\pi x} \hat{j} = \frac{\mu_0 I_2 I_1}{2\pi} \ln\left(\frac{x_0+L}{x_0}\right) \hat{j}$$

**PART II: Conceptual Questions (each carries 1 mark). Tick the most proper answer:**

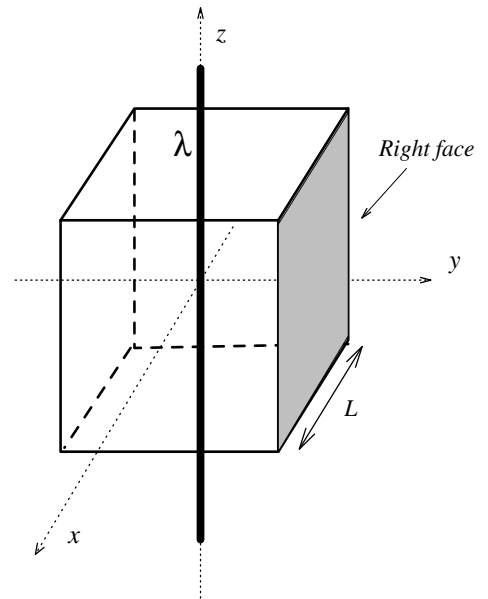
1. The magnitude of the electric force between two point charges is  $F$ , if they are a distance  $d_1$  apart.

If the distance is changed to  $d_2$ , the force decreases to  $F/2$ . The ratio  $\frac{d_1}{d_2}$  is

- (a) 2.  
 (b)  $\sqrt{2}$ .  
 (c)  $1/2$ .  
 (d)  $1/\sqrt{2}$ . Ans.

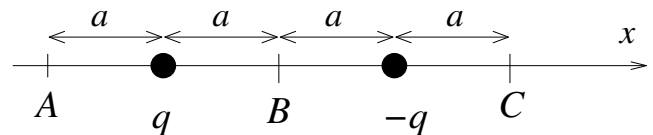
2. An infinite line of charge with uniform charge density  $\lambda$  lies along the symmetry axis of a cube of side length  $L$ , as shown. The electric flux through the right face of the cube is

- (a)  $\frac{\lambda L}{4\epsilon_0}$ . Ans.  
 (b)  $\frac{6\lambda L}{\epsilon_0}$ .  
 (c)  $\frac{\lambda L}{6\epsilon_0}$ .  
 (d)  $\frac{\lambda L}{\epsilon_0}$ .



3. Point charges  $q(>0)$  and  $-q$  are placed along the x-axis, as shown. Which relation is correct?

- (a)  $V_C > V_B > V_A$ .  
 (b)  $V_A > V_B > V_C$ . Ans.  
 (c)  $V_C > V_A > V_B$ .  
 (d)  $V_A > V_C > V_B$ .



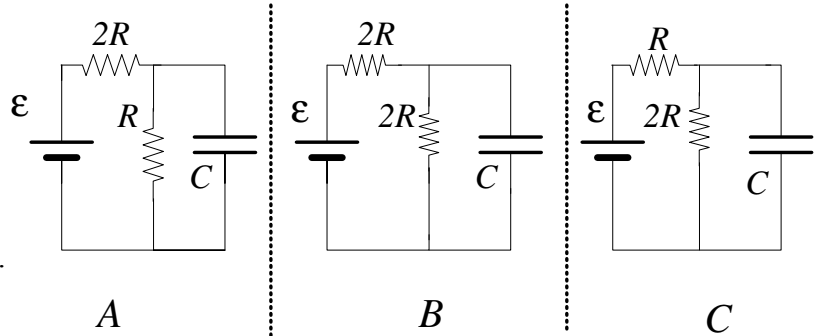
4. A capacitor is connected to a battery and has charge  $Q$  when it is fully filled with a material of dielectric constant  $K$ . If the battery remains connected, but the dielectric material is taken out, what will be the charge on the plates?

- (a)  $KQ$ .  
 (b)  $Q$ .  
 (c)  $Q/K$ . Ans.  
 (d)  $Q(K+1)$ .

5. A voltage is applied to the two ends of a cylindrical wire. If the diameter of the wire is doubled and the applied voltage is halved, the current through the wire
- will be double the initial current. Ans.
  - will not change.
  - will be half the initial current.
  - will be four times the initial current.

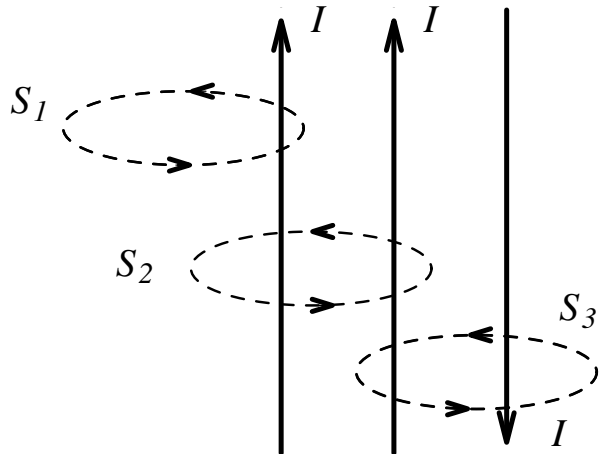
6. In which circuit is the final charge of the capacitor the highest?

- A.
- B.
- C. Ans
- All capacitors have the same final charge.



7. The three wires in this figure are infinitely long and carrying identical magnitude of current  $I$  as shown.  $S_1, S_2, S_3$  are circular loops. The integral  $\oint \vec{B} \cdot d\vec{l}$  is the largest on circle

- $S_1$ .
- $S_2$ . Ans.
- $S_3$ .
- The curve integrals are the same on each circles.



8. Two charged particles  $q_1$  and  $q_2$  with identical masses and speeds separately enter a region of a uniform magnetic field. In the magnetic field, both particles follow semicircular paths with radii  $R$  and  $2R$ , as shown. The relation of the charges is

- $q_1 = -2q_2$  Ans..
- $q_1 = 2q_2$ .
- $-2q_1 = q_2$ .
- $2q_1 = q_2$ .

