



**PART I. Solve the following problems. Show your solutions in detail.**

1. An air-filled capacitor consists of two parallel plates, each with an area of  $7.6 \text{ cm}^2$ , separated by a distance of  $1.8 \text{ mm}$ . If a potential difference of  $20 \text{ V}$  is applied across the plates, calculate the energy stored in the capacitor.

$$C = \epsilon_0 A/d = 3.7 \text{ PF}$$

$$U = \frac{1}{2} CV^2 = 74.7 \text{ nJ}$$

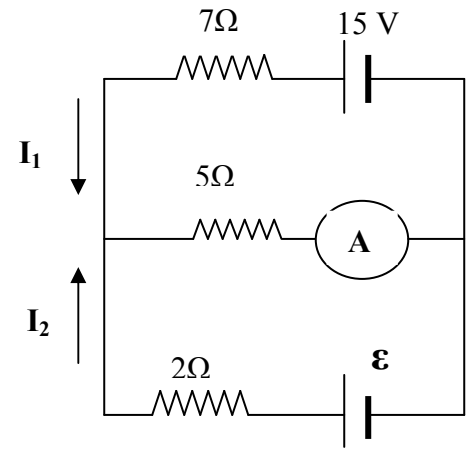
2. Suppose you want to install a heating wire that will convert electric energy to heat at a rate of  $300 \text{ W}$  for a current of  $1.5 \text{ A}$ . The resistivity of the wire is  $5 \times 10^{-7} \text{ } \Omega\text{m}$ , and its diameter is  $0.3 \text{ mm}$ . What must be the length of the wire?

$$P = RI^2 \rightarrow R = 133.3 \text{ } \Omega$$

$$A = \pi D^2/4 = 7.1 \times 10^{-8} \text{ m}^2$$

$$R = \rho l/A \rightarrow l = 18.9 \text{ m}$$

3. The ammeter in the circuit shown reads 2A. Find the values of  $I_1$ ,  $I_2$ , and  $\mathcal{E}$ .



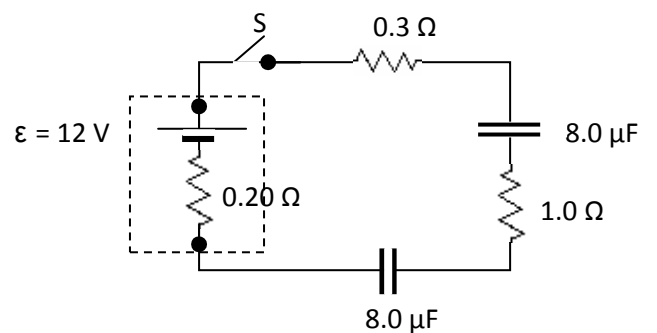
Current through 5-Ω resistor:  $I_3 = I_1 + I_2 = 2A$  to the right

Upper loop:  $15 - 7I_1 - 5I_3 = 0 \rightarrow I_1 = 5/7 A$

$I_2 = 9/7 A$

Lower loop:  $\mathcal{E} - 2I_2 - 5I_3 = 0 \rightarrow \mathcal{E} = 12.6 V$

4. At time  $t = 0$  the switch  $S$  is closed in the circuit shown. What is the terminal voltage of the battery at  $t = 12 \mu s$ ?

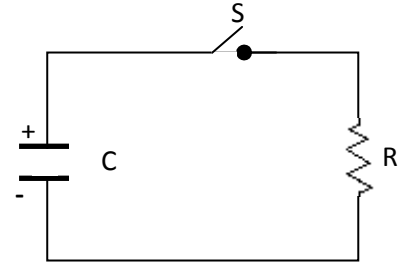


$$\tau = RC = 1.5 \times 4 \times 10^{-6} = 6 \mu s$$

$$I = \frac{\mathcal{E}}{R} e^{-t/\tau} = \frac{12}{1.5} e^{-2} = 1.1 A$$

$$V_T = \mathcal{E} - rI = 12 - 0.2 \times 1.1 = 11.8 V$$

5. In the circuit shown,  $C = 4.6 \mu\text{F}$  and  $R = 25 \Omega$ . The capacitor initially has a charge of magnitude  $3.5 \text{ nC}$  on its plates. After switch  $S$  is closed what will be the current in the circuit at the instant that 70% of the energy stored in the capacitors is lost?

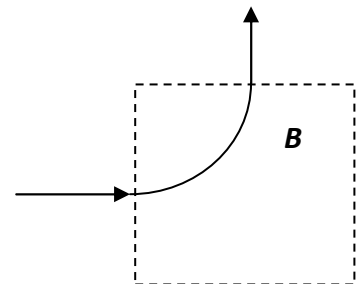


$$U = \frac{q^2}{2C} = U_o e^{\frac{-2t}{\tau}}$$

$$0.3 U_o = U_o e^{\frac{-2t}{\tau}} \rightarrow e^{\frac{-t}{\tau}} = \sqrt{0.3}$$

$$I = \frac{q_o}{\tau} e^{-t/\tau} = 16.7 \mu\text{A}$$

6. A proton enters and then leaves a region of the uniform magnetic field  $\mathbf{B}$ , which is perpendicular to the plane of paper, as shown below. The proton travels a distance of  $2.0 \text{ cm}$  in  $10 \mu\text{s}$  inside the field. Find the magnitude and the direction (outward or inward) of  $\mathbf{B}$ .



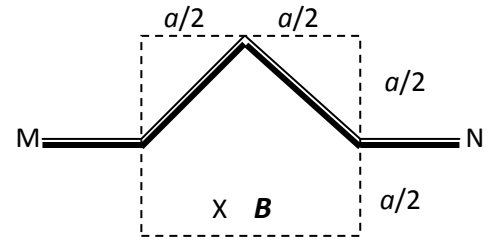
$\vec{B}$  is inward

$$v = l/t = 2000 \text{ m/s}$$

$$R = \frac{2}{\pi} l = 1.27 \times 10^{-2} \text{ m}$$

$$B = mv/eR = 1.64 \text{ mT}$$

7. Wire MN with resistance  $0.2 \Omega$  carrying a current  $I$  passes through a region of uniform magnetic field as shown ( $B = 0.7 \text{ T}$ , into the page;  $a = 20 \text{ cm}$ ). It experiences a net downward magnetic force of  $2.1 \text{ N}$ . Calculate the potential difference  $V_M - V_N$ .



$$\vec{F} = I(\vec{L} \times \vec{B})$$

Since the net force is downward, the direction of current is from N to M

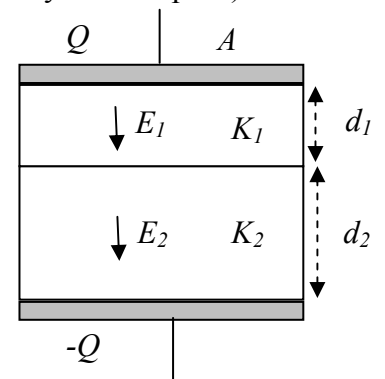
$$I = \frac{F}{aB} = 15 \text{ A}$$

$$V_M - V_N = -RI = -3 \text{ V}$$

8. The figure shows a parallel-plate capacitor with plate area  $A$  and charge  $Q$ . It is filled with two slabs of different dielectric constants  $K_1$  and  $K_2$  and thicknesses  $d_1$  and  $d_2$ .

a) Express the electric fields  $E_1$  and  $E_2$  in terms of these parameters and  $\epsilon_0$  (permittivity of free space).

$$E_1 = \frac{Q}{A\epsilon_0 K_1} \quad , \quad E_2 = \frac{Q}{A\epsilon_0 K_2}$$



b) Use the results in part a) to obtain the potential difference  $V$  across the plates.

$$V = E_1 d_1 + E_2 d_2 = Q \left( \frac{d_1}{A\epsilon_0 K_1} + \frac{d_2}{A\epsilon_0 K_2} \right)$$

c) Use the result in part b) to obtain the capacitance of the capacitor

$$C = \frac{Q}{V} = \frac{\epsilon_0 A}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$

**PART II: Conceptual Questions (each carries 1 mark). Circle the most proper answer:**

1. Consider a charged and isolated air-filled parallel-plate capacitor. If the distance between the plates is increased by a factor of three, the energy stored in the capacitor

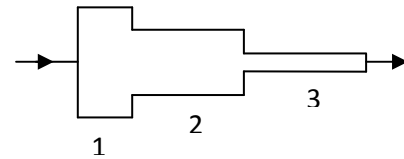
- a) **Increases three times.**
- b) Increases nine times.
- c) Decreases three times.
- d) Does not change.

2. The Kirchhoff's loop theorem is the consequence of the conservation law for

- a) Momentum.
- b) Charge.
- c) **Energy.**
- d) Rotational momentum.

3. A steady current passes through a cylindrical conductor with a cross sectional view as shown. If the electron drift velocity is denoted by  $v_1$ ,  $v_2$  and  $v_3$  in regions 1, 2 and 3, then

- a)  $v_1 = v_2 = v_3$
- b)  **$v_1 < v_2 < v_3$**
- c)  $v_1 > v_2 > v_3$
- d)  $v_1 < v_2 = v_3$

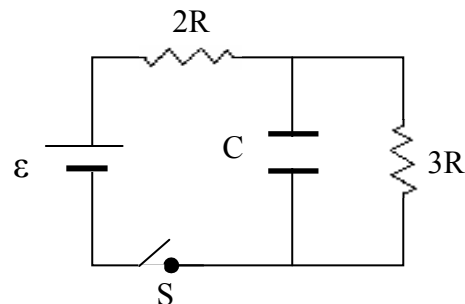


4. Four slabs with different dielectric constant  $K$  are placed between the plates of a charged and isolated parallel-plate capacitor. The induced surface charge density on the dielectric slab is the smallest for the slab with

- a)  $K = 5$
- b)  **$K = 2$**
- c)  $K = 6$
- d)  $K = 4$

5. At time  $t = 0$ , when the switch  $S$  is closed, the current through the battery is  $I_0$ . A long time later ( $t \rightarrow \infty$ ) the current is  $I$ . The ratio  $I_0/I$  is

- a)  $3/2$
- b)  **$5/2$**
- c)  $1/3$
- d)  $1/5$

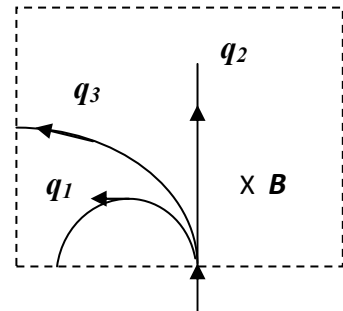


6. The electric field  $E$ , the magnetic field  $B$  and the speed  $v$  of the charged particles which pass straight through a velocity selector are related as

- a)  $B = E/v$
- b)  $2E = v/B$
- c)  $B = 2vE$
- d)  $v = EB$

7. Three particles with identical mass and velocity but different charge ( $q_1, q_2, q_3$ ) enter perpendicular to a uniform magnetic field  $B$  (into the page) and move in different paths as shown. One can conclude that

- a)  $q_3 > q_1$
- b)  $q_2 > q_3$
- c)  $q_3 < q_1$
- d)  $q_3 = q_1$



8. A wire carries current  $I$  in the x-y plane as shown. In the presence of a uniform magnetic field

$\vec{B} = B\hat{j}$ , the direction of the magnetic force acting on the wire is

- a)  $\hat{i}$
- b)  $\hat{j}$
- c)  $\hat{k}$
- d)  $\hat{i} + \hat{j}$

