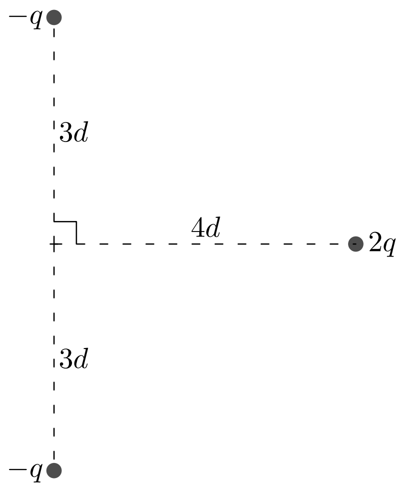
Problems

Please show ALL working.

1. The system shown below consists of three point charges; $d = 2\times10^{-16}$ m, $q = 5.34\times10^{-20}$ C. Find the electrostatic potential energy U of the system in MeV (if $U = 0$ when the charges are infinitely far apart).

(4 points)



$$U = \frac{k(-q)^2}{r_{(-q)(-q)}} + 2\frac{k(-q)(2q)}{r_{(-q)(2q)}}$$

$$r_{(-q)(-q)} = 6d = 1.2\times10^{-15}\,\text{m}$$

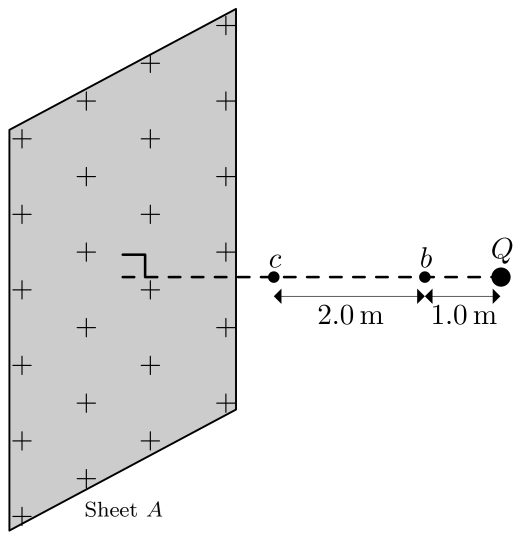
$$r_{(-q)(2q)} = 5d = 1\times10^{-15}\,\text{m}$$

$$U = -\frac{19}{30}\frac{kq^2}{d} = -8.1\times10^{-14}\,\text{J}$$

$$= -0.51\,\text{MeV}$$

2. Below, the point charge $Q = +0.75\,\mu$C is 5.0 m from the very large sheet A of uniform surface charge density $\sigma_A = +17.7\,\text{nC/m}^2$. Find the potential difference $V_b - V_c$.

(4 points)



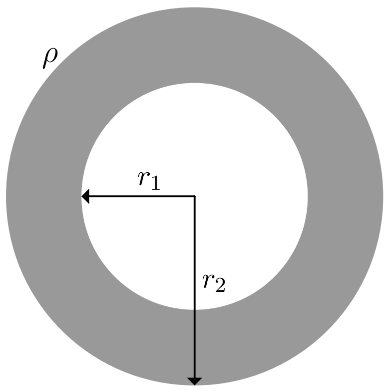
$$E_A = \frac{|\sigma_A|}{2\epsilon_0} = 1.00\times10^3\,\text{N/C}$$

$$V_b - V_c = -E_A(2\,\text{m}) + kQ\left(\frac{1}{1\,\text{m}} - \frac{1}{3\,\text{m}}\right)$$

$$= +2.5\times10^3\,\text{V}$$

3. The spherical ***shell*** of charge shown in cross-section below (of inner radius $r_1 = 3.0\,\text{cm}$ and outer radius $r_2 = 5.0\,\text{cm}$) has uniform volume charge density $\rho = -3.5\,\mu\text{C/m}^3$. Find the ***magnitude and direction*** of the electric field 4.0 cm from the center of the shell.

(4 points)

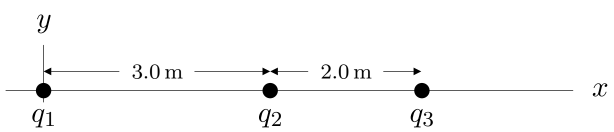


$$\begin{aligned} E &= \frac{k}{r^2}\,|(\text{Charge inside sphere of radius } r)| \\ &= \frac{k}{r^2}\left|\rho\frac{4\pi}{3}(r^3 - r_1^3)\right| \\ &= \frac{|\rho| r_1}{3\epsilon_0}\left(\frac{r}{r_1} - \frac{r_1^2}{r^2}\right) \\ &= 3.0 \times 10^3\,\text{N/C} \end{aligned}$$

Direction: radially inward

4. After a point charge $q_4 = +10.0\,\text{nC}$ is added to the three point charge system below, the net electric force on q_2 is zero. If $q_1 = +2.50\,\text{nC}$ and $q_3 = -4.40\,\text{nC}$, find the x coordinate of the position of q_4.

(4 points)



At the position of q_2, the net electric field

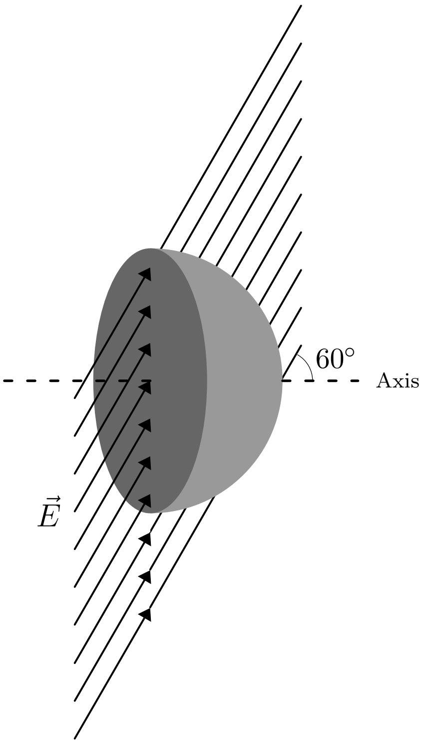
$$\begin{aligned} \vec{E} = \vec{E}_{q_1} + \vec{E}_{q_3} + \vec{E}_{q_4} &= 0 \\ \Longrightarrow \vec{E}_{q_4} = -E_{q_1}\hat{\imath} - E_{q_3}\hat{\imath} &= -\frac{k|q_1|}{(3.0\,\text{m})^2}\hat{\imath} - \frac{k|q_3|}{(2.0\,\text{m})^2}\hat{\imath} \\ &= (-12.4\,\text{N/C})\hat{\imath} \\ \Longrightarrow \text{Distance of } q_4 \text{ from } q_2 &= \sqrt{\frac{kq_4}{E_{q_4}}} = 2.69\,\text{m} \end{aligned}$$

Since q_4 is to the right of q_2,

$$x\text{-coordinate of } q_4 = 3.0 + 2.7 = +5.7\,\text{m}.$$

5. A uniform electric field of magnitude 5.0×10^3 N/C makes an angle of 60° with the axis of a closed hemispherical surface of radius 0.50 m (see below). Calculate the electric flux through the ***curved*** part of the closed surface.

(3 points)



$$\Phi_{\text{Total}} = \Phi_{\text{Flat surface}} + \Phi_{\text{Curved surface}} = 0$$

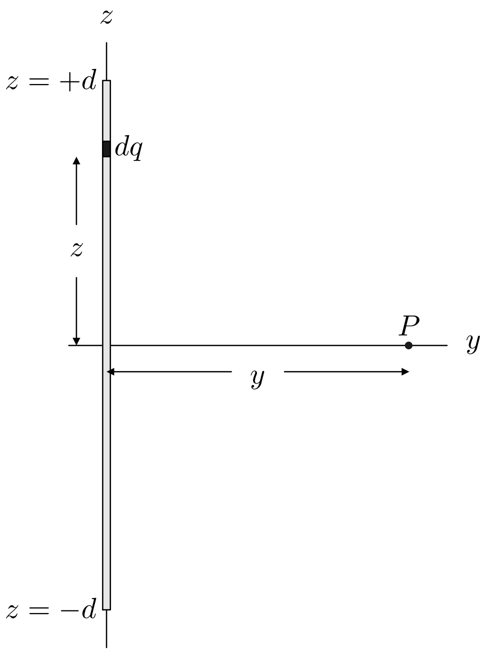
$$\implies \Phi_{\text{Curved surface}} = -\Phi_{\text{Flat surface}}$$

$$\Phi_{\text{Flat surface}} = EA\cos\theta$$

$$A = \pi(0.50\,\text{m})^2, \ \theta = 120°$$

$$\Phi_{\text{Curved surface}} = +2.0 \times 10^3\,\text{V·m}$$

6. Below, a ***uniform line*** of ***negative*** charge $Q\,(< 0)$ is on the z-axis from $z = -d$ to $z = +d$.



The "point" charge dq is of length dz. Give dq **in terms of dz and Q.** *(1 point)*

$$dq = \frac{Q}{2d}dz$$

Charge dq has electric field $d\vec{E}$ at point P. Give the magnitude dE **in terms of y, z and Q**. *(1 point)*

$$dE = \frac{k|Q|}{2d}\frac{dz}{y^2 + z^2}$$

Give the scalar component dE_y of $d\vec{E}$ **in terms of y, z and Q**. *(2 points)*

$$dE_y = -\frac{k|Q|y}{2d}\frac{dz}{(y^2 + z^2)^{\frac{3}{2}}}$$

Write down an **integral** for the component E_y of the ***total*** electric field at P. (Do ***not*** evaluate this integral.) *(1 point)*

$$E_y = -\frac{k|Q|y}{2d}\int_{z=-d}^{z=+d}\frac{dz}{(y^2 + z^2)^{\frac{3}{2}}}$$

Conceptual Questions (each 1 point) **Tick (✓) the best answer.**

1. Hollow charged conducting spheres are shown below. The charges are uniformly distributed over the outer surfaces of the spheres; the radii of these outer surfaces are given.

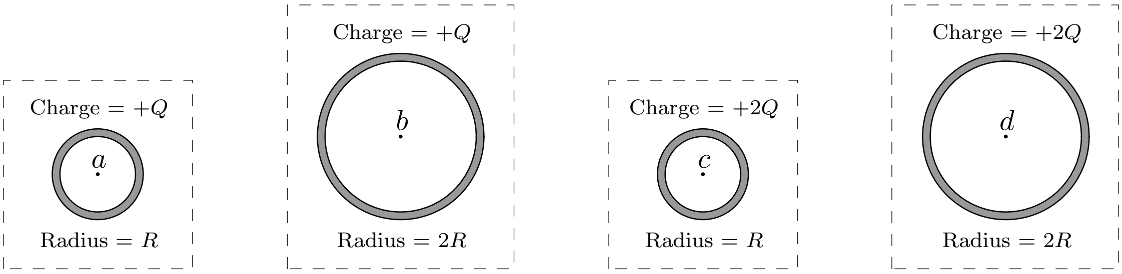


Fig. (i) Fig. (ii) Fig. (iii) Fig. (iv)

In each figure, the electric potential $V = 0$ at infinity. The potentials at the ***centers*** of the spheres are such that:

a) $V_a = V_b < V_c = V_d$;

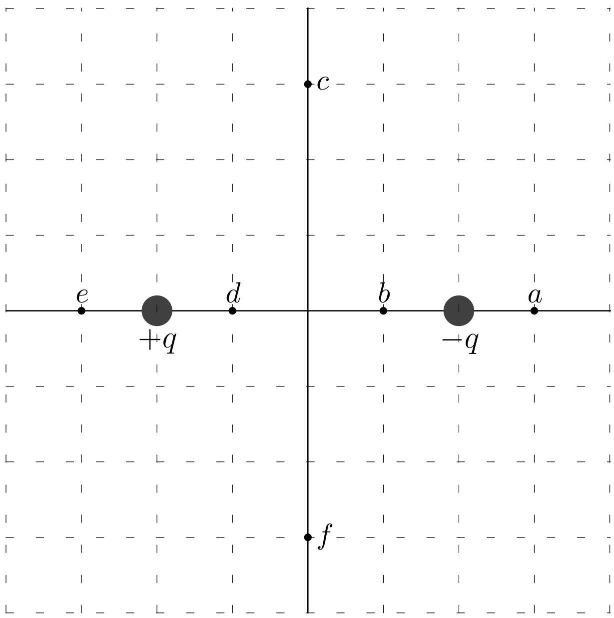
b) $V_b < V_a = V_d < V_c$; ✓

c) $V_b = V_d < V_a = V_c$;

d) $V_a = V_b = V_c = V_d$;

e) none of the above.

2. Points a to f below are in field of two point charges (q and $-q$).



If the electric potential is zero at infinity, the electric potential V is greatest at:

a) a;

b) b;

c) c and f;

d) d;

e) e. ✓

3. The spherical conducting shell in the figure below is charged. The electric field is zero in:

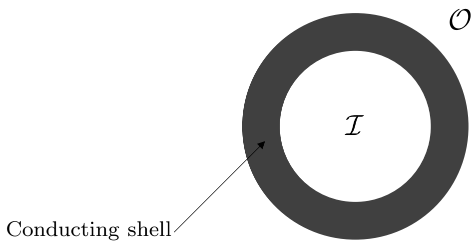
a) region $\mathcal{I}$.

b) in the shell.

c) region $\mathcal{O}$.

d) region $\mathcal{I}$ and in the shell. ✓

e) region $\mathcal{O}$ and in the shell.



4. A neutral metal ball hangs from an insulating thread. When a charged insulating rod is brought close, the ball:

a) is repelled by the rod if the rod is positive.

b) is attracted by the rod if the rod is positive. ✓

c) is repelled by the rod regardless of its charge.

d) does not feel any force because it is neutral.

e) None of the above.

5. A cavity inside a charged conductor contains the point charge $q = -1.0\,\text{nC}$ (see diagram). If the net charge of the conductor is $+8.0\,\text{nC}$, the electric flux through the Gaussian surface in the figure is:

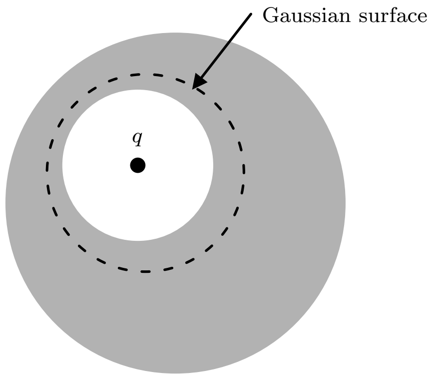
a) positive.

b) negative.

c) zero. ✓

d) dependent on the size of the gaussian surface.

e) not possible to determine with the information given.



6. The velocities of a negative point charge in a uniform electric field are shown below. The direction of the electric field is:

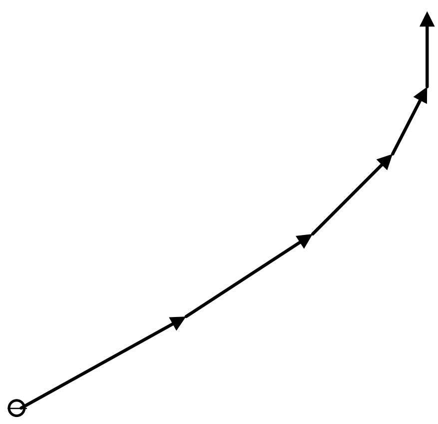
a) $\uparrow$.

b) $\downarrow$.

c) $\rightarrow$. ✓

d) $\leftarrow$.

e) $\nearrow$.



End of exam