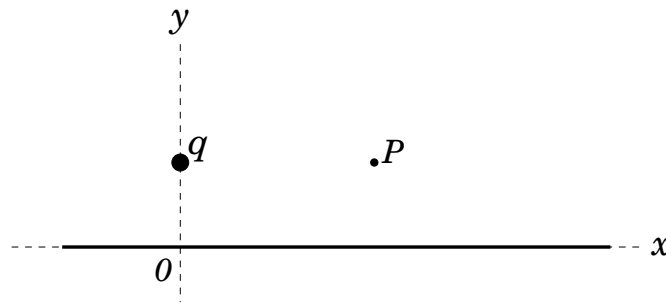


Part I: Solve the following problems. Show all your works

1. An infinitely long straight wire lying along the x -axis has a uniform linear charge density $\lambda = -4.0 \times 10^{-6}$ C/m. A point charge $q = 6.0 \mu\text{C}$ is at $x = 0$ cm, $y = 20.0$ cm. Find the net electric field \vec{E} at the point P with coordinates $x = 40.0$ cm, $y = 20.0$ cm. 3 points



Solution: The electric field at P is

$$\vec{E} = \vec{E}_q + \vec{E}_\lambda = \frac{kq}{0.4^2} \hat{i} + \frac{2k|\lambda|}{0.2} (-\hat{j})$$

$$\implies \vec{E} = 3.38 \times 10^5 \hat{i} - 3.60 \times 10^5 \hat{j} \text{ N/C}$$

2. The point charge q_1 is fixed and the point charge q_2 of mass $m = 2.0 \times 10^{-6}$ kg is held 50.0 cm from q_1 when their potential energy is $U = -0.32$ J. If q_2 is released from rest what will be its speed when it has travelled 30.0 cm from start? 4 points

Solution: Since potential energy is negative, the charges will attract each other, so the final distance will be 20.0 cm. Then

$$U_i = -0.32 = \frac{kq_1q_2}{0.5} \implies kq_1q_2 = -0.16 \text{ N} \cdot \text{m}^2$$

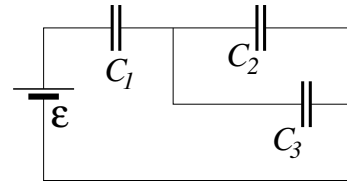
$$U_f = \frac{kq_1q_2}{0.2} = -0.80 \text{ J}$$

The work-energy principle gives

$$\frac{1}{2}mv^2 - 0 = U_i - U_f = 0.48 \implies v = 693 \text{ m/s}$$

3. In the circuit shown, $C_1 = C_3 = 12.0 \mu\text{F}$, $C_2 = 8.0 \mu\text{F}$ and the emf \mathcal{E} is unknown. The energy stored in C_1 is $U_1 = 1.35 \times 10^{-3} \text{ J}$. Calculate the energy stored in C_3 .

4 points

**Solution:**

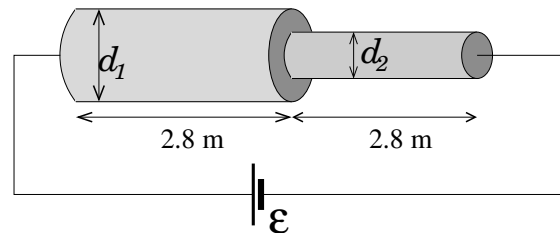
$$Q_1 = \sqrt{2C_1 U_1} = 1.8 \times 10^{-4} \text{ C}$$

$$Q_{23} = Q_1 \text{ and } V_3 = V_{23} = \frac{Q_{23}}{C_{23}} = 9.0 \text{ V}$$

$$\Rightarrow U_3 = \frac{1}{2} C_3 V_3^2 = 4.86 \times 10^{-4} \text{ J}$$

4. A cylindrical wire of length 2.8 m and diameter $d_1 = 3.2 \text{ mm}$ is joined with a cylindrical wire of the same material of length 2.8 m and diameter $d_2 = 1.4 \text{ mm}$ and an ideal battery as shown. The magnitude of the electric field in the *thicker* wire is $E_1 = 0.12 \text{ V/m}$. Find the emf \mathcal{E} of the battery.

5 points



Solution: The electric field in the thinner wire is (since the current is the same in the two parts)

$$E_2 = \rho J_2 = \frac{\rho I}{A_2} = \frac{\rho J_1 A_1}{A_2} = \frac{E_1 A_1}{A_2} = \frac{E_1 d_1^2}{d_2^2} = 0.63 \text{ V/m}$$

So the emf of the battery is

$$\mathcal{E} = V_1 + V_2 = E_1 L + E_2 L = 2.1 \text{ V}$$

5. A proton, with the initial velocity $\vec{v} = (-3.2 \times 10^5 \hat{i} + 8.7 \times 10^5 \hat{k})$ m/s, enters a region of uniform magnetic field $\vec{B} = 0.23 \hat{i}$ T. Find the pitch of the helical path of the proton.

3 points

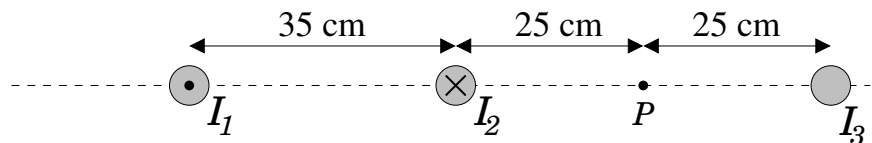
Solution: We have

$$v_{\parallel} = 3.2 \times 10^5 \text{ m/s}$$

$$P = v_{\parallel} T = v_{\parallel} \frac{2\pi m}{Be} = 0.09 \text{ m}$$

6. Three long wires perpendicular to the plane of paper carry currents $I_1 = 6.0$ A out-of-the-plane, $I_2 = 7.5$ A into the plane, and an unknown current I_3 . The net magnetic field is zero at the point P shown in the figure. Find the current I_3 and its direction.

5 points



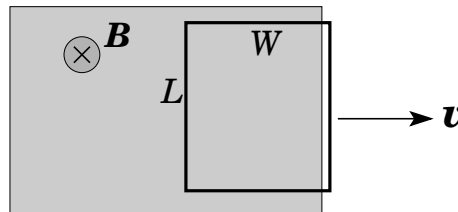
Solution: At P , \vec{B}_1 is upward and \vec{B}_2 is downward. So

$$0 = \frac{\mu_0 I_1}{2\pi(0.60)} \hat{j} + \frac{\mu_0 I_2}{2\pi(0.25)} (-\hat{j}) + \vec{B}_3 \implies \vec{B}_3 = 4.0 \times 10^{-6} \hat{j}$$

So I_3 is into-the-plane, and

$$\frac{\mu_0 I_3}{2\pi(0.25)} = 4.0 \times 10^{-6} \implies I_3 = 5.0 \text{ A}$$

7. A rectangular loop of length $L = 35.0$ cm and width $W = 12.0$ cm has a total resistance of 4.2Ω . It moves with a constant velocity \vec{v} perpendicular to a uniform magnetic field $B = 1.2$ T which is into-the-plane. As it starts moving out of the field region the rate of dissipation of energy in the loop is 2.0 mW. (a) What is the direction of the induced current in the loop? (b) What is the magnitude of the velocity \vec{v} ? 4 points



Solution: (a) By Lenz's law, the induced current is clockwise.

(b) The rate of energy dissipation is the power

$$P = \frac{\mathcal{E}^2}{R} = \frac{B^2 L^2 v^2}{R} \implies v = \sqrt{\frac{PR}{B^2 L^2}} = 0.22 \text{ m/s}$$

8. Consider a very long hollow cylindrical wire of inner radius a and outer radius b . It carries a current I parallel to its axis.

(a) Assuming the current density to be uniform, write the expression for the current density in the wire. 1 point

$$J = \frac{I}{\pi(b^2 - a^2)}$$

(b) We want to apply Ampere's law to calculate the magnitude of the magnetic field inside the wire ($a < r < b$). Choose a circular loop of radius r around the axis. Obtain the expression for the current passing through this loop. Express your answer in terms of I , r , a and b . 1 point

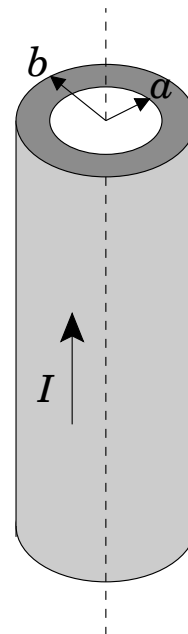
$$I_{enc} = J \pi (r^2 - a^2) = \frac{I(r^2 - a^2)}{(b^2 - a^2)}$$

(c) Write the line integral of \vec{B} around the loop in terms of B and r . 1 point

$$\oint \vec{B} \cdot d\vec{\ell} = 2\pi r B$$

(d) Use Ampere's law and the results from (b) and (c) to derive the expression for B at $a < r < b$. Express your answer in terms of I , r , a , b and μ_0 . 1 point

$$B = \frac{\mu_0 I_{enc}}{2\pi r} = \frac{\mu_0 I (r^2 - a^2)}{2\pi r (b^2 - a^2)}$$

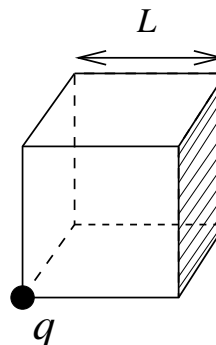


Part II: Conceptual Questions

In the following, tick (\checkmark) the best answer. Each question carries 1 point.

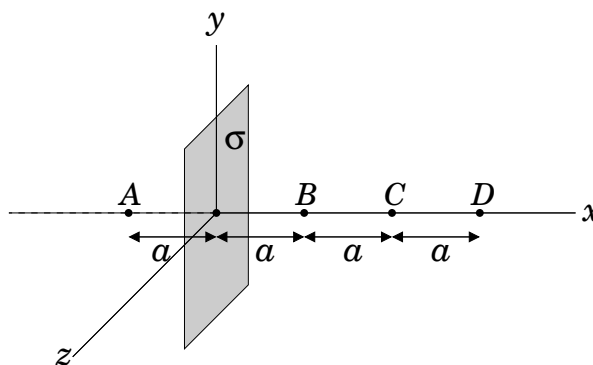
1. A positive charge q is at one of the corners of a cube of side L as shown. The electric flux through the shaded face of the cube is

- (a) $\Phi_E = \frac{q}{\epsilon_0}$
 (b) $\Phi_E = \frac{q}{3\epsilon_0}$
 (c) $\Phi_E = \frac{q}{6\epsilon_0}$
 (d) $\Phi_E = \frac{q}{24\epsilon_0}$



2. An infinitely large sheet with **positive** uniform surface charge density σ lies in the yz -plane. If we choose the electric potential at the origin to be zero, the electric potentials at the points A , B , C and D can be ranked as

- (a) $V_B > V_C > V_D > V_A$.
 (b) $V_D > V_C > V_B > V_A$.
 (c) $V_D > V_C > V_B = V_A$.
 (d) $V_A = V_B > V_C > V_D$.



3. The electric potential is **positive** at the midpoint between two point charges of equal magnitude, $|q_1| = |q_2|$. If the electric field magnitude due to q_1 is E_1 at the midpoint, the magnitude of the net electric field at the midpoint

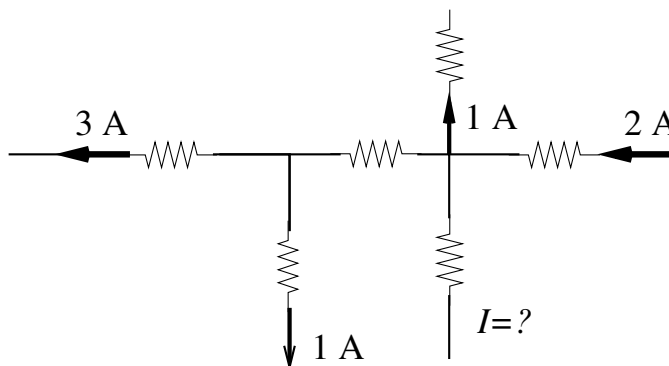
- (a) is $2E_1$.
 (b) is $4E_1$.
 (c) is zero.
 (d) cannot be determined from the information provided.

4. Two cylindrical wires made of the same material have equal resistances. If the lengths are related as $L_2 = 2L_1$, the radii must be such that

- (a) $r_2 = 2r_1$.
 (b) $r_1 = 2r_2$.
 (c) $r_2 = \sqrt{2}r_1$.
 (d) $r_1 = \sqrt{2}r_2$.

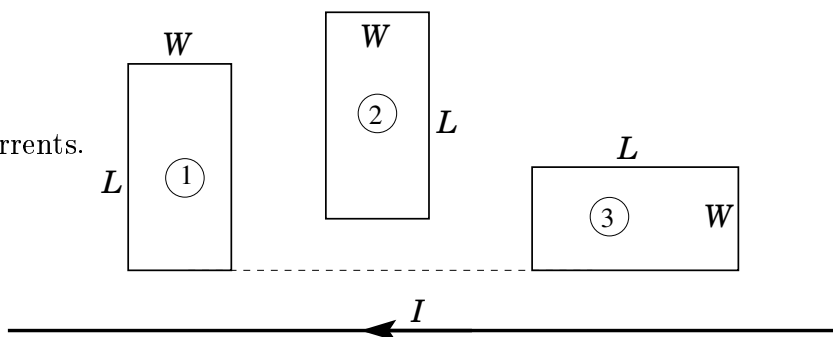
5. A part of a circuit is shown below with some currents shown. What is the current I ?

- (a) 3 A downward.
 (b) 3 A upward.
 (c) 1 A downward.
 (d) 2 A upward.



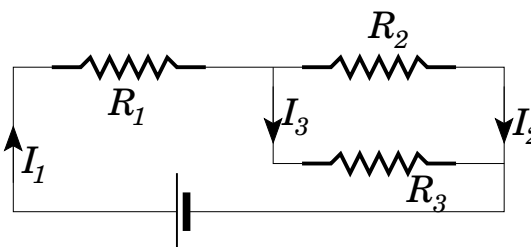
6. The figure shows a long wire and three identical rectangular loops (with $L > W$), all lying in one plane. If the current I in the wire is increasing at a constant rate, rank the loops according to the magnitude of the induced currents, largest first.

- (a) Loop 3, Loop 1, Loop 2
 (b) Loop 1, Loop 3, Loop 2
 (c) Loop 1, Loop 2, Loop 3
 (d) They have equal induced currents.



7. In the circuit, $R_1 > R_2 > R_3$. The currents I_1 , I_2 and I_3 are such that

- (a) $I_3 > I_2 > I_1$.
 (b) $I_1 > I_2 > I_3$.
 (c) $I_1 > I_3 > I_2$.
 (d) $I_1 > I_2 = I_3$.



8. A wire of the shape shown carrying a current I lies perpendicular to a uniform magnetic field in the $+z$ -direction. What is the direction of the magnetic force on the wire?

- (a) along the $+x$ -direction.
 (b) along the $-x$ -direction.
 (c) along the $+y$ -direction.
 (d) along the $-y$ -direction.

