



**PART I: Solve the following problems. Show your solutions in detail.**

1. In the figure shown,  $C_1 = 1 \mu\text{F}$ ,  $C_2 = 6 \mu\text{F}$ , and  $C_3 = 3 \mu\text{F}$ . If the electric-field energy stored on  $C_1$  is  $8 \mu\text{J}$ , what is charge on  $C_2$ ? [4 points]

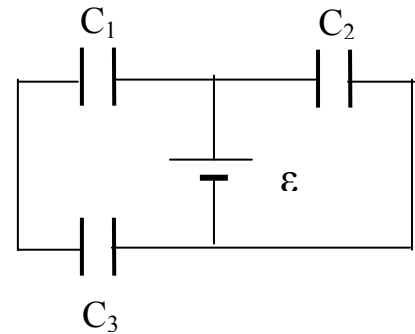
$$U_1 = \frac{q_1^2}{2C_1} \rightarrow q_1 = 4 \mu\text{C}$$

$$q_1 = q_3 = q_{eq} = C_{13} \varepsilon$$

$$C_{13} = \frac{C_1 C_3}{C_1 + C_3} = \frac{1 \times 3}{1 + 3} = \frac{3}{4} \mu\text{F}$$

$$\varepsilon = \frac{4 \mu\text{C}}{\frac{3}{4} \mu\text{F}} = \frac{16}{3} \text{ V}$$

$$q_2 = C_2 \varepsilon = 6 \mu\text{F} \times \frac{16}{3} \text{ V} = 32 \mu\text{C}$$



2.  $N = 5 \times 10^{13}$  electrons are removed from an isolated conducting ball of radius 2.5 cm. How much electrical energy is stored in the whole space around the ball? [3 points]

$$q = Ne = 8 \mu\text{C}$$

$$C = 4\pi \varepsilon_0 R = 2.78 \text{ pF}$$

$$U = \frac{q^2}{2C} = 11.5 \text{ J}$$

3. The figure shows a charged parallel-plate capacitor with the plate area  $A$ . A slab with dielectric constant  $K$  and thickness  $d_1$  partially fills the space between the plates, leaving an air gap of thickness  $d_2$ . The strength of the electric field in the air gap is  $E_0$ .

a) Show that the potential difference between the plates is  $V = (d_2 + d_1/K)E_0$ .

[2 points]

b) Obtain the capacitance of this capacitor from the above equation.

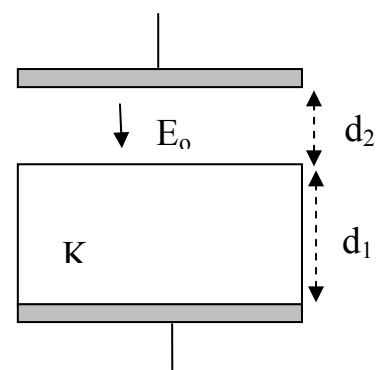
[2 points]

$$a) V = E_0 d_2 + E_1 d_1, \quad \text{where } E_1 = \frac{E_0}{K}$$

$$\Rightarrow V = \left( d_2 + \frac{d_1}{K} \right) E_0$$

$$b) E_0 = \frac{\sigma}{\varepsilon_0} = \frac{q}{A \varepsilon_0} \rightarrow q = \frac{A \varepsilon_0}{\frac{d_1}{K} + d_2} V$$

$$\rightarrow C = \frac{q}{V} = \frac{A \varepsilon_0}{\frac{d_1}{K} + d_2}$$



4. A current of 16 mA passes through a light bulb as soon as it is turned on. The current drops to a steady value of 2 mA after a short while. What is the temperature of the filament when it is hot? (Take its initial temperature as 20 °C and its temperature coefficient of resistivity  $4.5 \times 10^{-3} \text{ }^\circ\text{C}^{-1}$ ) **[3 points]**

$$R = R_0 [1 + \alpha \Delta T]$$

$$\frac{\varepsilon}{I} = \frac{\varepsilon}{I_0} [1 + \alpha \Delta T]$$

$$\frac{I_0}{I} = 1 + \alpha \Delta T$$

$$\Delta T = 1556 \text{ }^\circ\text{C} \rightarrow T = 15676 \text{ }^\circ\text{C}$$

5. Copper has  $8.5 \times 10^{28}$  free electron per cubic meter. A copper wire of length 90 cm and radius 1.0 mm carries 5.0 A of current. How much time does it take for an electron to travel the length of the wire? **[3 points]**

$$I = JA = J \pi r^2$$

$$I = nev_d \cdot \pi r^2$$

$$v_d = 1.17 \times 10^{-4} \text{ m/s}$$

$$t = \frac{L}{v_d} = 7691 \text{ s}$$

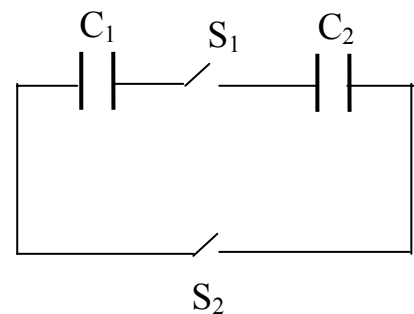
6. A capacitor  $C_1 = 2 \text{ } \mu\text{F}$  with charge  $Q = 20 \text{ } \mu\text{C}$  is connected to the initially uncharged capacitor  $C_2 = 3 \text{ } \mu\text{F}$  by closing switches  $S_1$  and  $S_2$  shown below. How much charge is on  $C_2$ ? **[2 points]**

$C_1$  and  $C_2$  are in parallel (think about it).

$$C_{eq} = C_1 + C_2 = 5 \text{ } \mu\text{F}$$

$$V = \frac{Q}{C_{eq}} = 4 \text{ V}$$

$$q_2 = C_2 V = 12 \text{ } \mu\text{C}$$



7. In the circuit shown,  $\varepsilon = 12 \text{ V}$ ,  $r = 1 \Omega$  (internal resistance of the battery),  $R = 5 \Omega$ ,  $C_1 = 3 \mu\text{F}$  and  $C_2 = 6 \mu\text{F}$ . What is the terminal voltage of the battery,  $6 \mu\text{s}$  after the switch S is closed? **[4 points]**

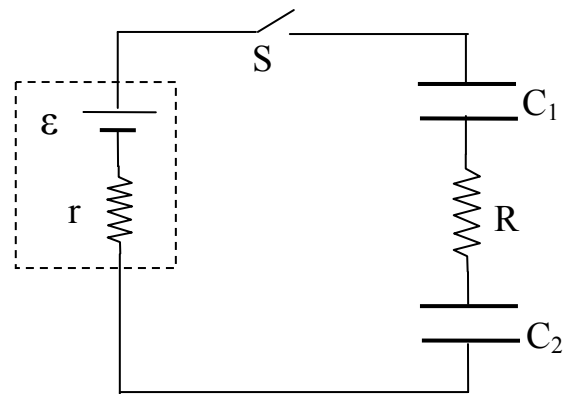
$$R_{eq} = R + r = 6 \Omega$$

$$C_{eq} = \frac{C_1 + C_2}{C_1 + C_2} = 2 \mu\text{F}$$

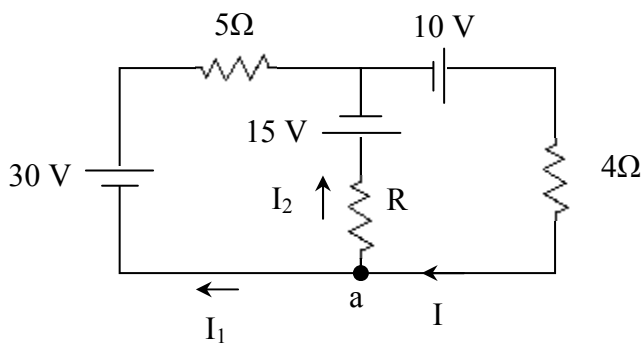
$$\tau = R_{eq} C_{eq} = 12 \mu\text{s}$$

$$I = \frac{\varepsilon}{R_{eq}} e^{-\frac{t}{\tau}} = 1.2 \text{ A}$$

$$V_T = \varepsilon - Ir = 10.8 \text{ V}$$



8. In the circuit shown, the current  $I = 3 \text{ A}$ . What is the value of the resistance R? **[4 points]**



- Arbitrary directions are taken for  $I_1$  and  $I_2$  as shown.
- Outer loop:  

$$10 - 12 + 30 - 5I_1 = 0$$

$$\rightarrow I_1 = 5.6 \text{ A}$$
- Junction a:  

$$I = I_1 + I_2$$

$$\rightarrow I_2 = -2.6 \text{ A}$$
- Left loop:  

$$-RI_2 - 15 + 5I_1 - 30 = 0$$

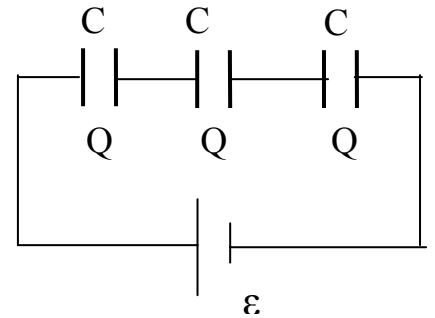
$$\rightarrow R = 6.5 \Omega$$

**PART II: Conceptual Questions (each carrying 1 point). Tick the best answer.**

1. A capacitor  $C_0$  connected to a battery has charge  $Q_0$ . The space between the plates of the capacitor is now filled with the dielectric  $K=3$ . Charge on the capacitor becomes
2. Three identical uncharged capacitors are connected in series to a battery. The charge passing through the battery during the charging is:

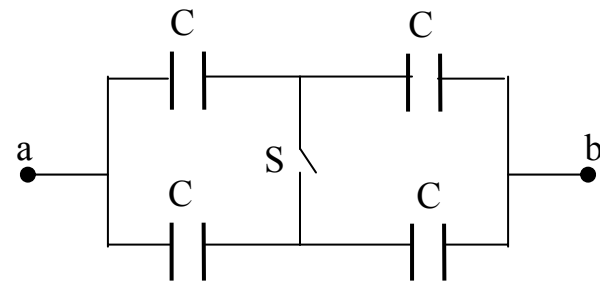
- a.  $Q_0$   
 b.  $Q_0/3$   
 c.  $3Q_0$   
 d.  $4Q_0$

- a.  $Q$   
 b.  $2Q$   
 c.  $3Q$   
 d.  $Q/3$



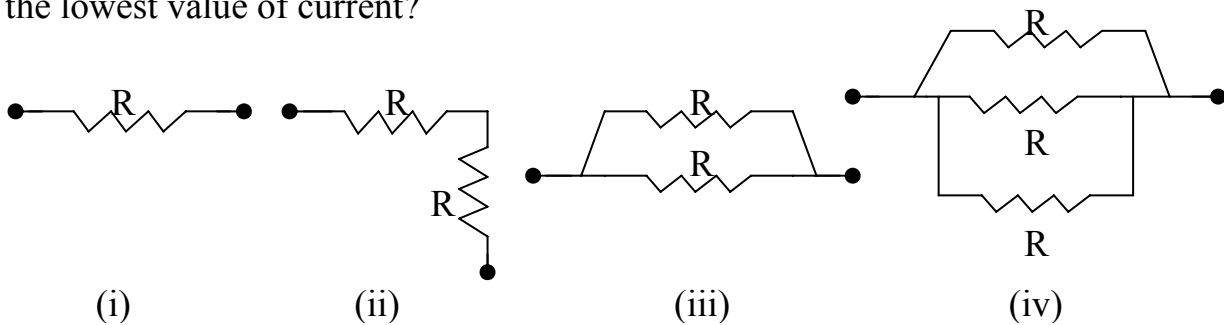
3. A potential difference is applied between the points a and b of the system of capacitors as shown. If the switch S is closed, the charge stored on each capacitor

- a. will become double.  
 b. will become half.  
 c. **will not change.**  
 d. will become zero.



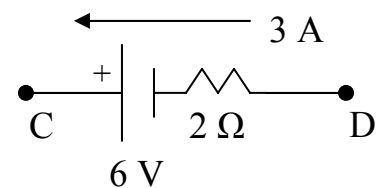
4. The resistors shown below are connected to a same potential difference. Which circuit has the lowest value of current?

- a. i  
 b. **ii**  
 c. iii  
 d. iv

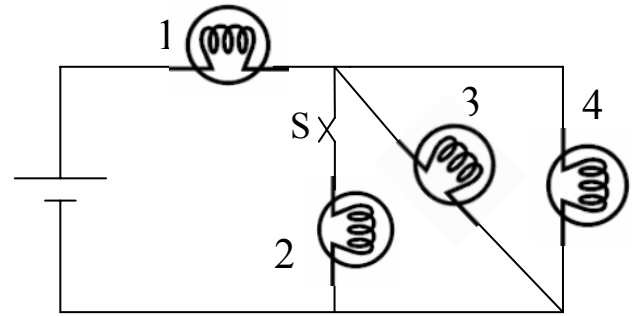


5. In a section of a circuit, the battery, the resistance and the current are shown in the figure. The potential of the point C

- a. is greater than that of the point D.  
 b. is less than that of the point D.  
 c. **is the same as that of the point D.**  
 d. cannot be given without the full circuit.

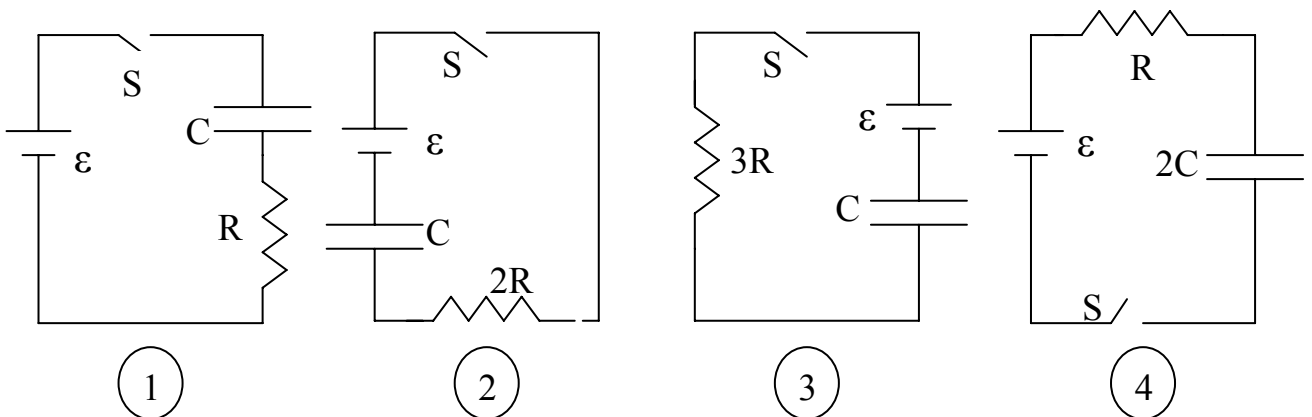


6. Four identical light bulbs are connected to a battery as shown. If the switch S is opened, which statement is correct?



- a. All the bulbs become brighter.  
 b. All the bulbs become dimmer  
 c. Bulb 1 becomes brighter, but bulbs 3 and 4 become dimmer.  
**d. Bulbs 3 and 4 become brighter, but bulb 1 will be dimmer.**
7. For the four RC circuits given, the time required to charge the capacitor to the 50% of the maximum charge is given by  $t_1$ ,  $t_2$ ,  $t_3$  and  $t_4$ , respectively. These are related to each other as

- a.  $t_1 < t_2 < t_3 < t_4$   
 b.  $t_2 < t_3 < t_1 < t_4$   
 c.  $t_1 < t_2 = t_3 < t_4$   
**d.  $t_1 < t_2 = t_4 < t_3$**



8. A potential difference  $V$  is applied between the ends of cylindrical conductors shown in the figure. For which conductor is the current density highest? Shown are the radii and lengths of the conductors.

