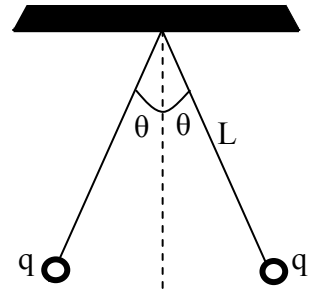


PART I. Solve the following problems. Show your solutions in detail.

1. Two identical negatively charged spheres each having a mass of 3×10^{-2} kg and charge q , hang in equilibrium as shown below. The length of each string is $L = 0.15$ m and the angle $\theta = 5^\circ$. How many extra electrons exist on each sphere?

[4 points]



$$F_e = mg \tan \theta = (3 \times 10^{-2} \text{ kg})(9.8 \text{ m/s}^2) \tan(5^\circ) = 2.6 \times 10^{-2} \text{ N}$$

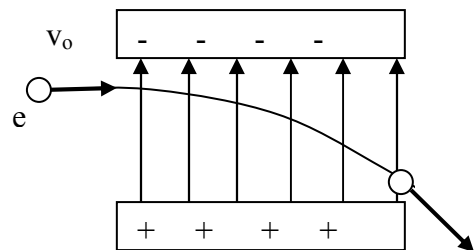
$$a = L \sin \theta = 0.15 \text{ m} (\sin 5^\circ) = 0.013 \text{ m}$$

$$r = 2a = 0.026 \text{ m}$$

$$F = k \frac{q^2}{r^2} ; \quad q = 4.4 \times 10^{-8} \text{ C}$$

$$N = q/e = 2.75 \times 10^{11}$$

2. An electron enters the region of a uniform electric field 200 N/C with an initial velocity of magnitude $v_0 = 3 \times 10^6$ m/s, as shown in the figure. Find the time it takes the electron to travel inside the electric field. The vertical displacement of the electron while it is inside the electric field is -1.95 cm. **[3 points]**



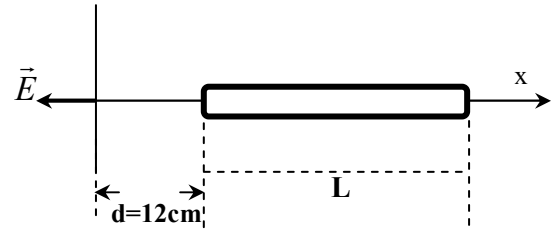
$$a_y = eE/m = 3.51 \times 10^{13} \text{ m/s}^2$$

$$y = a_y t^2 / 2$$

$$t = \sqrt{\frac{2y}{a_y}} = 3.33 \times 10^{-8} \text{ s}$$

3. A rod of length L , as shown below, has a uniformly distributed charge Q . Show that the magnitude of electric field at $x = 0$ is $E = k \frac{Q}{d(L+d)}$. [2 points]

If $Q = 1.1 \times 10^{-11}$ C and the magnitude of the electric field produced at $x = 0$ is $E = 2.58$ N/C, what is the length of the rod? [1 point]



$$E = k \int_d^{L+d} \frac{\lambda dx}{x^2} = k\lambda \left[-\frac{1}{x} \right]_d^{L+d} = k\lambda \left(\frac{1}{d} - \frac{1}{L+d} \right) = k \frac{Q}{d(L+d)}$$

$$E = 2.58 \text{ N/C} = \frac{9 \times 10^9 \times 1.1 \times 10^{-11}}{12 \text{ cm}(L + 12 \text{ cm})}$$

$$0.3127 / \text{m} = \frac{1}{L + 12 \text{ cm}}$$

$$0.32 \text{ cm} = L + 12 \text{ cm}$$

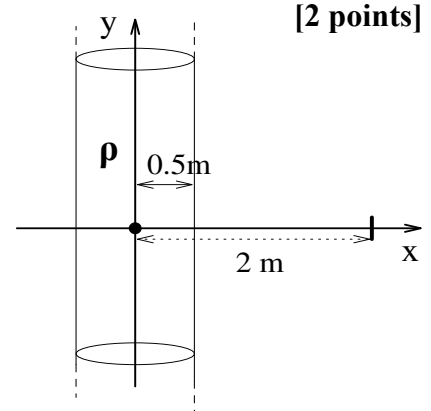
$$32 \text{ cm} - 12 \text{ cm} = L$$

$$L = 0.2 \text{ m}$$

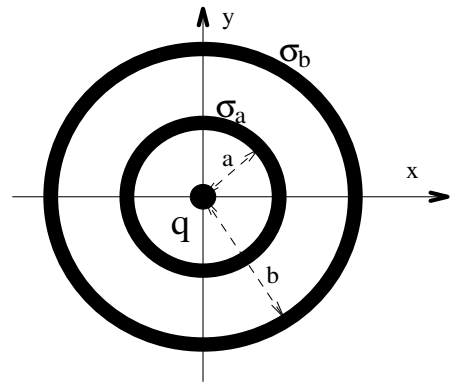
4. An infinitely long cylinder of radius $R = 0.5$ m has its axis along the y -direction, as shown. The cylinder has a uniform volume charge density ρ . The electric field at $x = 2$ m along the x -axis is $E = 100$ N/C. Calculate the absolute value of ρ . [2 points]

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow$$

$$E 2\pi r L = \frac{\rho \pi R^2 L}{\epsilon_0} \Rightarrow \rho = \frac{2\epsilon_0 E r}{R^2} = 1.4 \times 10^{-8} \text{ C/m}^3$$



5. A point charge $q = 6 \mu\text{C}$ is at the origin. Spherical surfaces with charge densities $\sigma_a = -0.3 \mu\text{C}/\text{m}^2$ and $\sigma_b = 0.3 \mu\text{C}/\text{m}^2$ and radii $a = 2 \text{ m}$ and $b = 4 \text{ m}$ are centered at the origin. Find the magnitude and direction of the electric field at a point along the x-axis at $x = 3 \text{ m}$. **[3 points]**



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E = \frac{Q_{enc}}{4\pi\epsilon_0 r^2} \Rightarrow$$

$$E = \frac{k(6\mu\text{C} + 4\pi a^2 \sigma_a)}{9\text{m}^2} = -9.1 \times 10^3 \text{ N/C}$$

So inward.

6. Two electrons are released from rest at a distance of 2 m from each other. What is the speed of each electron when they are 4 m apart? **[3 points]**

$$\Delta K + \Delta U = 0 \Rightarrow$$

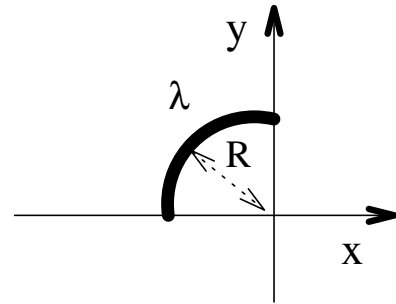
$$2 \frac{mv^2}{2} = \frac{ke^2}{2m} - \frac{ke^2}{4m} \Rightarrow v = \sqrt{\frac{9 \cdot (1.6)^2 100}{4 \cdot 9.11}} \text{ m/s} = 8 \text{ m/s}$$

7. A quarter-circular wire of radius $R = 2 \text{ m}$ and linear charge density $\lambda = 5 \text{ nC/m}$ is in the x - y plane and centered at the origin, as shown. If $V = 0$ at infinity, find the value of the electric potential V at the origin.

[3 points]

$$Q = \lambda L = \lambda 2\pi R / 4 = \lambda \pi R / 2$$

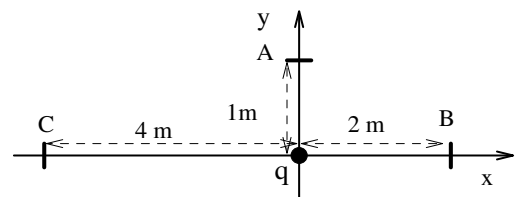
$$V = \frac{kQ}{R} = \frac{k\lambda \pi R / 2}{R} = \frac{k\pi\lambda}{2} = 70.7 \text{ V}$$



8. A point charge q is held at the origin. The potential difference between points A and B is $V_A - V_B = 2 \text{ V}$.

What is the potential difference $V_B - V_C$?

[2 points]



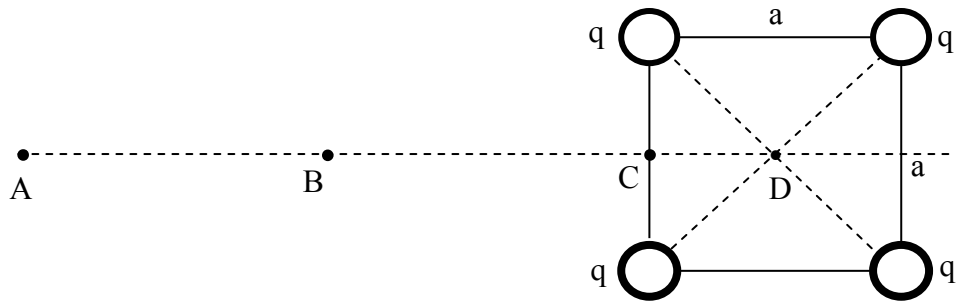
$$V_{A-B} = kq \left(\frac{1}{r_a} - \frac{1}{r_b} \right) = 2 \text{ V} \Rightarrow kq = 4Vm$$

$$V_{B-C} = kq \left(\frac{1}{r_B} - \frac{1}{r_c} \right) = \frac{4Vm}{4m} = 1V$$

PART II: Conceptual Questions (each carries 1 mark). Tick the most proper answer:

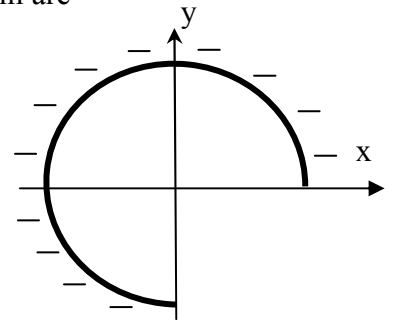
1. Equal charges are placed at the vertices of a square. The electric field is the smallest at point

- a) A.
- b) B.
- c) C.
- d) D.**



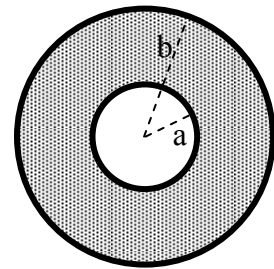
2. A negative charge is distributed uniformly on a circular wire shown below. The wire is in the x-y plane and centered at the origin. The two components of the electric field at the origin are

- a) $E_x > 0$ and $E_y < 0$.
- b) $E_x > 0$ and $E_y > 0$.
- c) $E_x < 0$ and $E_y > 0$.**
- d) $E_x < 0$ and $E_y < 0$.

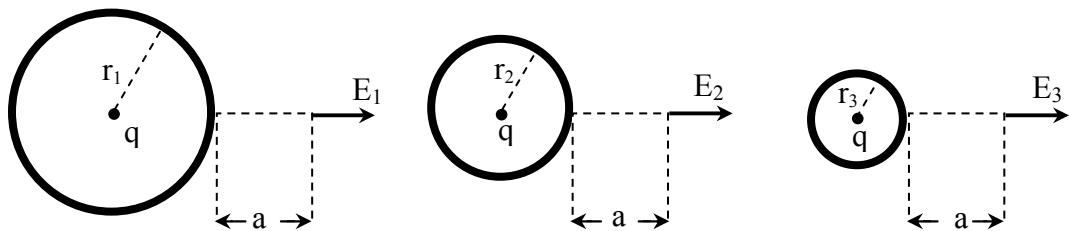


3. A conducting spherical shell with inner and outer radii a, b carries a net charge Q . If $V = 0$ at infinity, the electric potential at the inner surface of the shell is

- a) kQ/a .
- b) kQ/b .**
- c) $kQ/(a - b)$.
- d) zero.



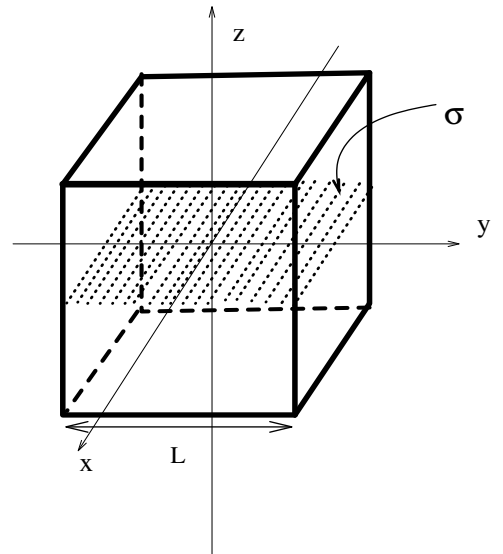
4. The figure shows three uncharged conducting spherical shells of radii $r_1 > r_2 > r_3$. Charge q is placed at the center of shells. The electric fields E_1, E_2 and E_3 are related as



- b) $E_1 > E_2 > E_3$
- c) $E_1 < E_2 < E_3$**
- d) $E_1 = E_2 = E_3 \neq 0$
- e) $E_1 = E_2 = E_3 = 0$

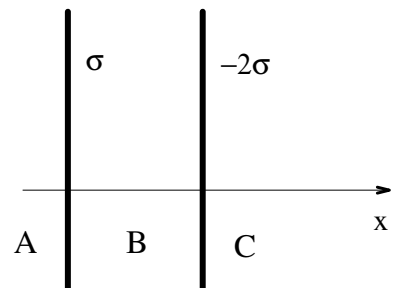
5. A cube of side L is centered at the origin, as shown. The xy -plane has a uniform surface charge density σ . The electric flux through the cube is

- a) $\sigma 6L^2$.
 b) $\frac{\sigma L^2}{6\epsilon_0}$.
 c) σL^2 .
d) $\frac{\sigma L^2}{\epsilon_0}$.



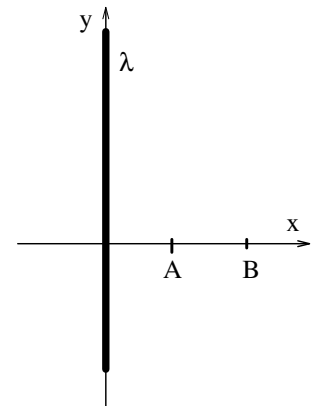
6. Two parallel planes with surface charge densities σ and -2σ are perpendicular to the x -axis, as shown. In which of the three regions (A, B, C) the magnitude of electric field has the largest value?

- a) A
b) B
 c) C
 d) Electric field is the same in all regions.



7. The entire y -axis is charged uniformly with a linear charge density λ (>0), as shown. An electron is moved from point A to point B. The work done on the electron by the electric field is

- a) positive.
b) negative.
 c) zero.
 d) path dependent.



8. Some equipotential surfaces which are perpendicular to the x -axis are shown. What is the direction of electric field at point P?

- a) $+\hat{i}$
 b) $+\hat{j}$
c) $-\hat{i}$
 d) $-\hat{j}$

