Problems

Please show *all* working

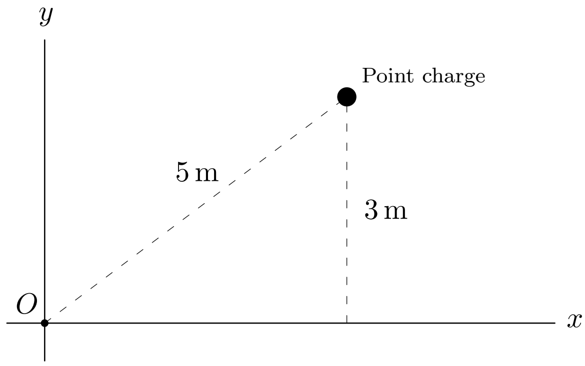
1. The ***point charge*** below is in the plane of the page. At the origin O, the electric field has y-component $E_y = -1.2 \times 10^3\,\mathrm{N/C}$. Find E_x at the origin. (2 points)

$$E_y = -|\vec{E}|\sin\theta = -0.6|\vec{E}|$$

$$E_x = -|\vec{E}|\cos\theta = -0.8|\vec{E}|$$

$$\therefore E_x/E_y = 0.8/0.6$$

$$\implies E_x = -1600\,\mathrm{N/C}$$

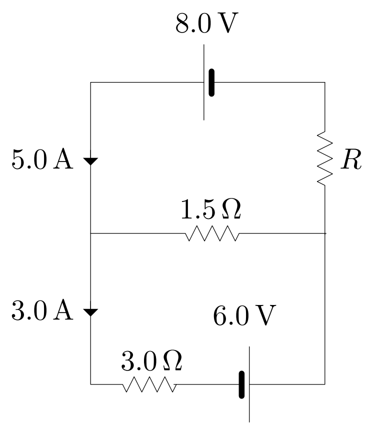


2. Find the power dissipated in the resistor of resistance R below. (3 points)

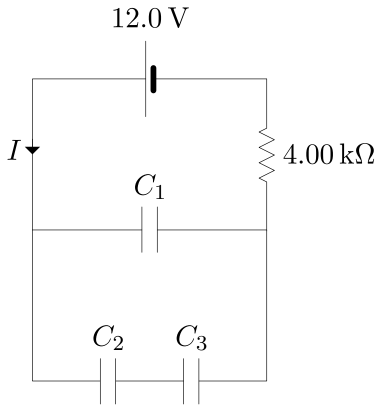
$$8.0\,\mathrm{V} - (3.0\,\mathrm{A})(3.0\,\Omega) + 6.0\,\mathrm{V} - (5.0\,\mathrm{A})R = 0$$

$$R = 1.0\,\Omega$$

$$P = I^2R = 25\,\mathrm{W}$$



3. In the circuit diagram below, $C_1 = 10.0\,\mu\text{F}$, $C_2 = 3.00\,\mu\text{F}$ and $C_3 = 6.00\,\mu\text{F}$. Find the energy stored in the capacitor of capacitance C_2 when the current $I = 2.00\,\text{mA}$. (The capacitors were uncharged before they were connected to the 12.0 V-emf.) (4 points)



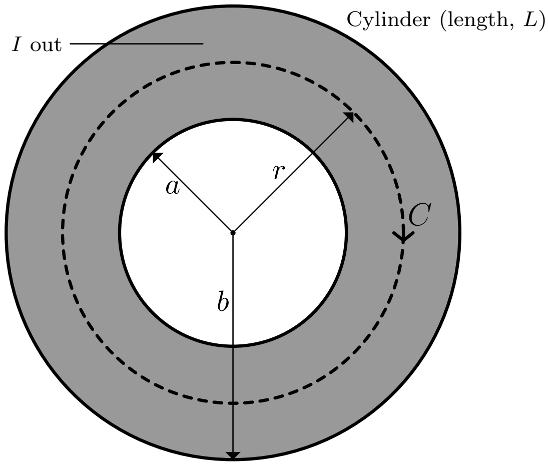
$$\frac{Q_2}{C_2} + \frac{Q_3}{C_3} = 12.0\,\text{V} - I(4.00\,\text{k}\Omega)$$

$$Q_2 = Q_3$$

$$\Longrightarrow Q_2 = 8.00\,\mu\text{C}$$

$$U_2 = \frac{Q_2^2}{2C_2} = 10.7\,\mu\text{J}$$

4. The ***cylinder*** of *resistivity* $\rho = 1.72 \times 10^{-8}\,\Omega \cdot \text{m}$ shown in cross-section below (inner radius $a = 2.00\,\text{mm}$, outer radius $b = 4.00\,\text{mm}$) has length $L = 25.0\,\text{cm}$ and carries an *outward* current, uniformly distributed over the cylinder's cross section; the voltage across the length of the cylinder is 12.0 mV. Find $\oint_C \vec{B} \cdot d\vec{l}$ for the *clockwise* circular path C of radius $r = 3.00\,\text{mm}$. (4 points)



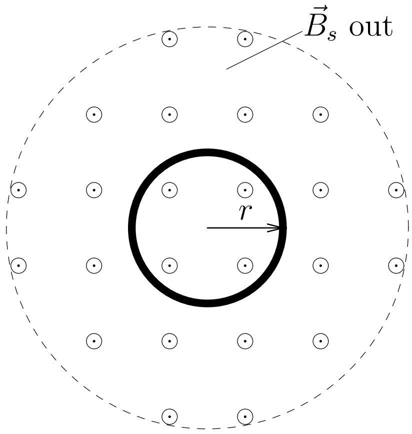
$$J = \frac{1}{\rho}\frac{\Delta V}{L} = 2.79 \times 10^6\,\text{A/m}^2$$

$$|I_{\text{encl}}| = J\pi(r^2 - a^2)$$

$$\left|\oint_C \vec{B} \cdot d\vec{l}\right| = +\mu_0\,|I_{\text{encl}}| = 5.51 \times 10^{-5}\,\text{T·m}$$

$$\oint_C \vec{B} \cdot d\vec{l} = -5.51 \times 10^{-5}\,\text{T·m}$$

5. The ***conducting loop*** below (radius $r = 1.5\,\text{cm}$, resistance $R = 80\,\text{m}\Omega$) is in the uniform magnetic field $\vec{B}_s$ of a solenoid with 250 turns *per unit length*; $\vec{B}_s$ is perpendicular to the plane of the loop. The current in the solenoid $I_s = (5.0\,\text{A})e^{-t/\tau}$, where $\tau = 15\,\text{s}$. Find the *magnitude* and *direction* of the current in the ***loop*** when $I_s = 3.0\,\text{A}$. (4 points)



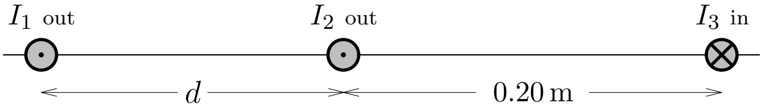
$$|\Phi_{B_s}| = \mu_0 n I_s \pi r^2$$

$$|\mathcal{E}| = \left|\frac{d\Phi_{B_s}}{dt}\right| = \mu_0 n \pi r^2 \left|\frac{dI_s}{dt}\right| = \mu_0 n \pi r^2 \frac{I_s}{\tau}$$

$$I_{\text{Loop}} = \frac{|\mathcal{E}|}{R} = 5.6 \times 10^{-7}\,\text{A}$$

$$\frac{d\Phi_{B_s}}{dt} < 0 \implies I_{\text{Loop}} \text{ counter-clockwise}$$

6. Three infinitely long *straight* parallel wires are shown in the figure below; the currents are $I_1 = 4.5\,\text{A}$, $I_2 = 3.0\,\text{A}$ and $I_3 = 5.0\,\text{A}$. The *net* force *per unit length* on the wire carrying current I_2 is $33\,\mu\text{N/m}$. What is d? (3 points)

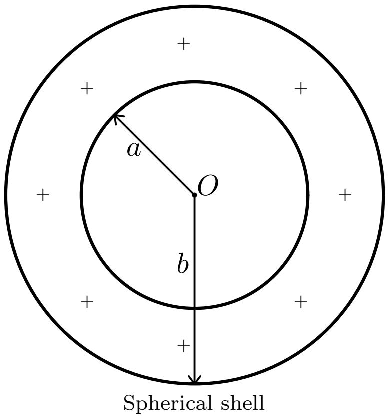


$$\frac{F_2}{L} = I_2 B \implies B = 11\,\mu\text{T}$$

$$B = B_1 + B_3 \implies B_1 = B - B_3 = 6.0\,\mu\text{T}$$

$$d = \frac{\mu_0 I_1}{2\pi B_1} = 0.15\,\text{m}$$

7. The ***spherical shell*** of charge drawn (in cross section) below has constant *volume* charge density $\rho > 0$, inner radius a and outer radius b. Let r be the distance from O.



a) Show that the electric field has magnitude

$$E = \frac{\rho r}{3\epsilon_0}\left(1 - \frac{a^3}{r^3}\right)$$

when $a < r < b$. (2 points)

b) What is the *maximum* value of E? (2 points)

c) If the electric potential V is zero at infinity, ***what is V on the outer surface*** $(r = b)$ of the shell? (Give your answer in terms of ρ and ϵ_0.) (1 point)

a)
$$Q_{\text{encl}} = \rho\frac{4\pi}{3}(r^3 - a^3)$$
$$4\pi r^2 E = \frac{Q_{\text{encl}}}{\epsilon_0} \Longrightarrow E = \frac{\rho r}{3\epsilon_0}\left(1 - \frac{a^3}{r^3}\right)$$

b)
$$\frac{dE}{dr} = \frac{\rho}{3\epsilon_0}\left(1 + 2\frac{a^3}{r^3}\right) > 0$$
$$\Longrightarrow \text{Maximum at r=b:} \quad E_{\text{max}} = \frac{\rho b}{3\epsilon_0}\left(1 - \frac{a^3}{b^3}\right)$$

c)
$$V = \frac{kQ}{b} \text{ with } Q = \rho\frac{4\pi}{3}(b^3 - a^3)$$
$$\Longrightarrow V = \frac{\rho b^2}{3\epsilon_0}\left(1 - \frac{a^3}{b^3}\right)$$

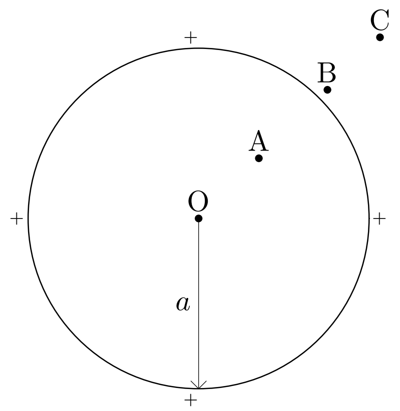
8. An ***electron*** moves in the fields $\vec{E} = (6x^2 + 2)\hat{\imath} + 4y^2\hat{\jmath}$ and $\vec{B} = 3y\hat{\imath} + (x^2 + 2)\hat{\jmath}$, where $\vec{E}$ is in N/C, $\vec{B}$ is in tesla and x, y are in meters. Let $\vec{F}$ be the *net* force on the electron. Find $\boldsymbol{\vec{F} \cdot \vec{v}}$ when the electron moves with velocity $\vec{v} = (2 \times 10^4\,\text{m/s})\hat{\imath}$ through the point $(x = 1\,\text{m}, y = 0\,\text{m})$. (2 points)

$$\vec{F} \cdot \vec{v} = q(\vec{E} + \vec{v} \times \vec{B}) \cdot \vec{v} = q\vec{E} \cdot \vec{v} = qE_x v$$

$$E_x v = (6 \cdot 1^2 + 2)(2 \times 10^4) = 1.6 \times 10^5\,\text{W/C}$$
$$\Longrightarrow \vec{F} \cdot \vec{v} = -2.6 \times 10^{-14}\,\text{W}$$

Conceptual Questions   Please choose the *best* answer

1. The ***spherical metal shell*** (of radius a) shown has a positive surface charge. The ***rate of change of the potential*** is greatest at:



a) O

b) A

c) B ✓

d) C

2. A student claims that, if the voltage across a *parallel-plate* capacitor is ***fixed***, then the *energy in the capacitor* is ***increased*** by: (**i**) increasing the plate area A; (**ii**) increasing the plate separation d, and; (**iii**) inserting a dielectric. The correct claims are:

a) (i) and (ii)

b) (i) and (iii) ✓

c) (ii) and (iii)

d) (i), (ii) and (iii)

3. An electron moves in the direction *opposite* to an electric field $\vec{E}$. The potential energy U of the electron and the potential V are such that:

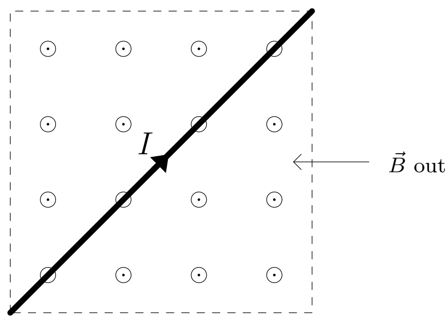
a) U increases and V decreases

b) U decreases and V decreases

c) U increases and V increases

d) U decreases and V increases ✓

4. The ***segment*** (to the right) of a current-carrying wire is in a uniform magnetic field (out of the page). The ***direction*** of the magnetic force on the wire segment is:



a) ↑

b) ↖

c) ↘ ✓

d) ↓

5. ***Wire loops*** 1 and 2 carry the *same current* but have *different diameters* d_1 and d_2: $d_2 = 2d_1$. If the magnetic field at the center of loop 2 is B, then ***the magnetic field at the center of loop 1*** is:

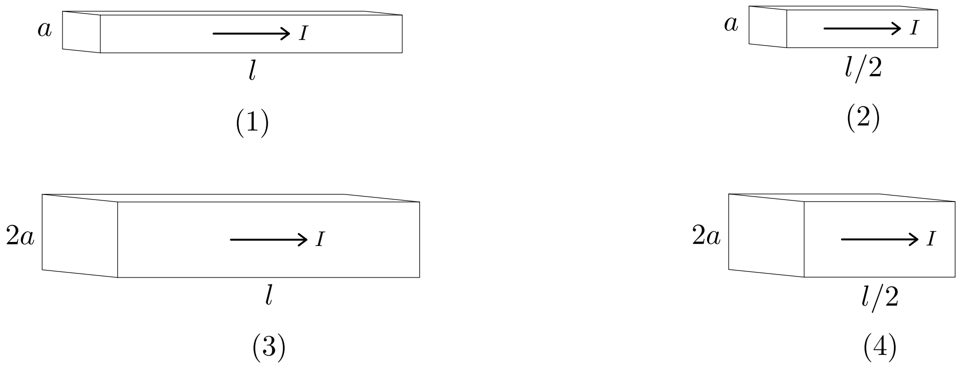
a) $2^{\frac{3}{2}}B$

b) $2B$ ✓

c) $B/2$

d) $B/2^{\frac{3}{2}}$

6. The four conducting rods below (with ***square*** cross sections) are all made from the ***same material*** and carry the ***same current*** I. In figures (1) and (2), the *square cross section* has sides of length a; in figures (3) and (4), the *square cross section* has sides of length $2a$.



The *potential differences* across the rods are such that:

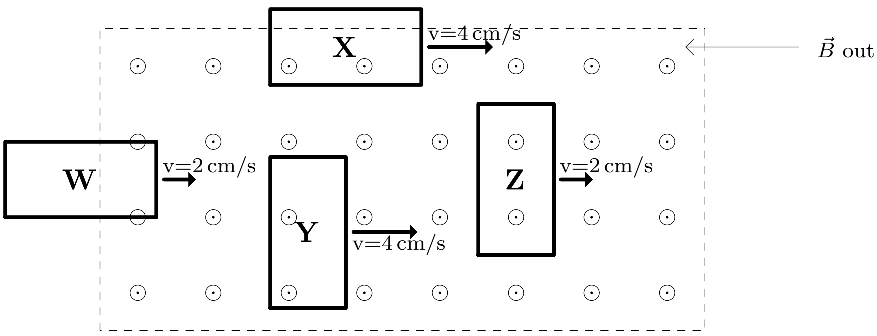
a) $V_{(1)} < V_{(2)} < V_{(3)} < V_{(4)}$

b) $V_{(1)} = V_{(3)} < V_{(2)} = V_{(4)}$

c) $V_{(2)} = V_{(4)} < V_{(3)} = V_{(1)}$

d) $V_{(4)} < V_{(3)} < V_{(2)} < V_{(1)}$ ✓

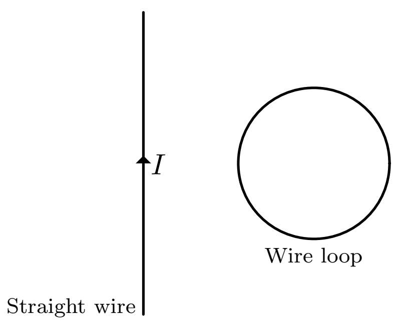
7. The four rectangular wire loops W, X, Y and Z below are *identical* but move with *different* speeds v perpendicular to a uniform magnetic field $\vec{B}$.

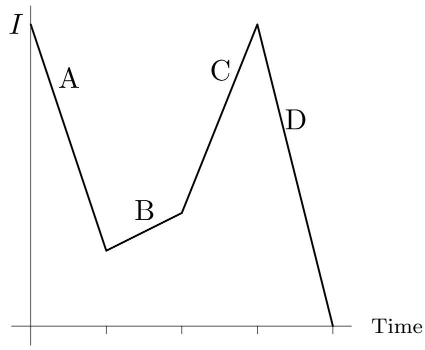


The rate of change of magnetic flux is *biggest* through loop:

a) W ✓ b) X c) Y d) Z

8. The *change with time* of the current I in the infinite ***straight wire*** below is plotted in the graph to the right.





The magnitude of the average *induced emf* in the ***circular wire loop*** is *smallest* during interval:

a) A b) B ✓ c) C d) D

End of Examination