Kuwait University Department of Physics

Physics 102

First Midterm Examination
Summer Semester (2008-2009)

July 19, 2009
Time: 6:00 PM to 7:30 PM

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Name: .... **Student No.:** ....

Fundamental Constants

$k = \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N·m^2/C^2	Coulomb's constant
$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/(N·m^2)	Permitivity of free space
$\mu_0 = 4\pi \times 10^{-7}$ T·m/A	Permeability of free space
$e = 1.6 \times 10^{-19}$ C	Elementary unit of charge
$m_e = 9.11 \times 10^{-31}$ kg	Mass of an electron
$m_p = 1.67 \times 10^{-27}$ kg	Mass of a proton
1 eV = $1.6{\times}10^{-19}$ J	Conversion from eV to J
$g = 9.8$ m/s^2	Acceleration due to gravity

Prefixes of units

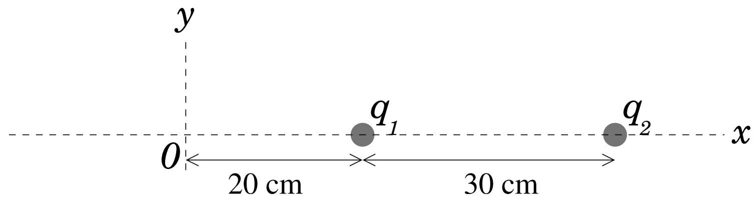
m = 10^{-3}	$\mu = 10^{-6}$	n = 10^{-9}	p = 10^{-12}
k = 10^3	M = 10^6	G = 10^9	T = 10^{12}

For use by instructors only

Problems	1	2	3	4	5	6	7	8	Total	G. Total
Marks										
Questions	1	2	3	4	5	6	7	8	Total	
Marks										

Part I: Solve the following problems

1. The point charge $q_1 = -5.0\ \mu\text{C}$ is at $x = 20.0$ cm and the point charge $q_2 = +17.0\ \mu\text{C}$ is at $x = 50.0$ cm on the x-axis. Find the x-coordinate of a third point charge $q_3 = -13.0\ \mu\text{C}$ on the x-axis such that the net force on q_2 is zero. **3 points**



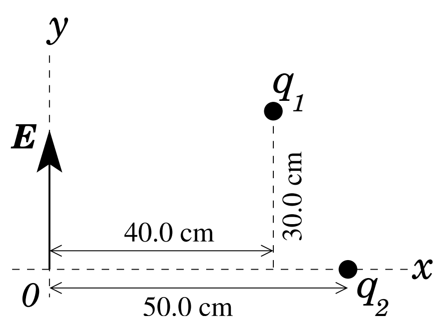
> **Note:** The numerical values for some quantities may differ from those on your answering sheet. The solution below is consistent with the values given above.

Solution: Since $\vec{F}_{21}$ is in the $-x$-direction, $\vec{F}_{23}$ must be in $+x$-direction so that the net force is zero. So q_3 must be placed at a point $x > 0.5$. Let the distance between q_1 and q_3 be r, then

$$\frac{k\,|q_1 q_2|}{0.3^2} = \frac{k\,|q_2 q_3|}{r^2} \implies r = 0.48\ \text{m} = 48.0\ \text{cm}$$

But q_3 must be to the right of q_2, so the coordinate of q_3 is $\underline{98.0\ \ \text{cm}}$.

2. Two point charges, q_1 and q_2, are placed in the xy-plane as shown. q_1 is unknown and $q_2 = +3.64\ \mu\text{C}$. The net electric field at the origin is along the positive y-axis as shown. Find its magnitude. **3 points**



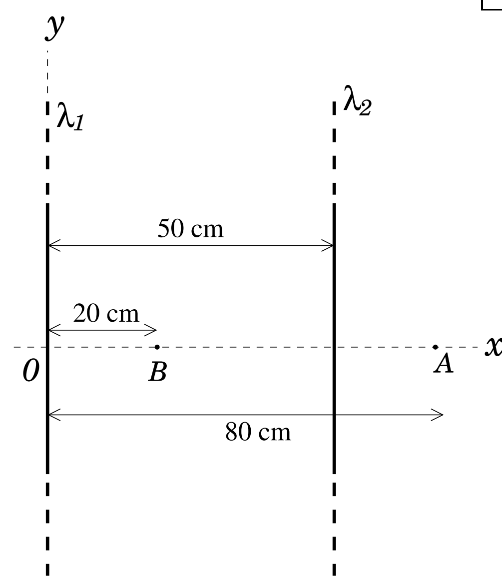
Solution: Obviously q_1 is negative. The x-component of the net electric field is zero, so

$$\frac{k\,|q_1|}{0.5^2}\,\frac{0.4}{0.5} = \frac{kq_2}{0.5^2} \implies |q_1| = 4.55 \times 10^{-6}\ \text{C}$$

The magnitude of the electric field is the y-component of $\vec{E}_1$, so

$$E = E_{1y} = \frac{k\,|q_1|}{0.5^2}\,\frac{0.3}{0.5} = 9.83 \times 10^4\ \text{N/C}$$

3. Two infinitely long thin wires with uniform linear charge densities lie in the xy-plane perpendicular to the x-axis as shown. $\lambda_1 = +12.0$ nC/m but λ_2 is unknown. The net electric field $\vec{\boldsymbol{E}} = 0$ at point A in the figure. Calculate $\vec{\boldsymbol{E}}$ (both magnitude and direction) at point B. **4 points**



Solution: At point A

$$\frac{2k\lambda_1}{0.8}\,\hat{\boldsymbol{\imath}} + \frac{2k\lambda_2}{0.3}\,\hat{\boldsymbol{\imath}} = 0 \implies \lambda_2 = -\frac{0.3}{0.8}\,\lambda_1 \implies \lambda_2 = -4.5 \times 10^{-9}\ \text{C}$$

At point B

$$\vec{\boldsymbol{E}} = \frac{2k\lambda_1}{0.2}\,\hat{\boldsymbol{\imath}} + \frac{2k\lambda_2}{0.3}\left(-\hat{\boldsymbol{\imath}}\right) \implies \vec{\boldsymbol{E}} = 1.35 \times 10^3\,\hat{\boldsymbol{\imath}}\ \text{N/C}$$

4. An unknown charge Q is uniformly distributed over the volume of a sphere of radius $a = 4.0$ cm. The magnitude of the electric field at a distance of 1.5 cm from the centre is $E = 3.80{\times}10^3$ N/C. Calculate the magnitude of the electric field at a distance of 6.0 cm from the centre. **2 points**

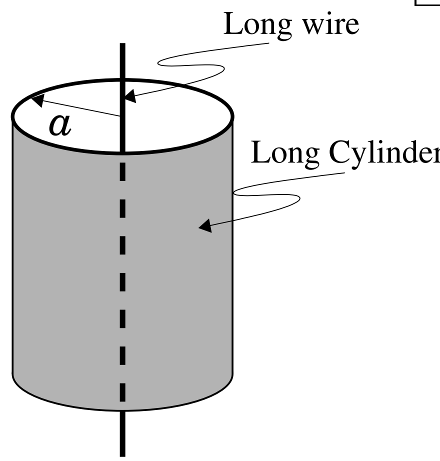
Solution: The electric field at $r = 1.5$ cm $(r < a)$ is given by

$$3.80 \times 10^3 = \frac{kQr}{a^3} \implies Q = 1.80 \times 10^{-9}\ \text{C}$$

The electric field at $r = 6.0$ cm $(r > a)$ is

$$E = \frac{kQ}{r^2} = 4.50 \times 10^3\ \text{N/C}$$

5. A long wire with an unknown linear charge density λ passes through the axis of a thin long hollow conducting cylinder of radius $a = 3.0$ cm with a uniform surface charge density $\sigma = -2.37 \times 10^{-8}$ C/m^2. The net electric field at $r = 6.0$ cm from the wire is zero. Calculate the net electric field $\vec{\boldsymbol{E}}$ (magnitude as well as the direction) at $r = 1.5$ cm from the wire. **3 points**



Solution: For the calculations at $r = 6.0$ cm, choose a cylindrical Guassian surface of radius $r = 6.0$ cm and length L. Then the total charge enclosed by this surface is zero,

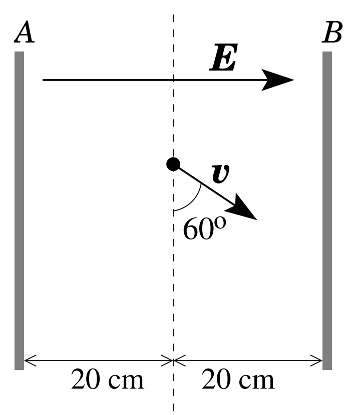
$$\lambda L + (2\pi a L)\,\sigma = 0 \implies \lambda = +4.47 \times 10^{-9}\ \text{C/m}$$

For $r = 1.5$ cm, choose a cylindrical Guassian surface of radius $r = 1.5$ cm and length L. Then

$$\Phi_E = (2\pi r L E) \qquad \text{and} \qquad Q_{enc} = \lambda L$$

$$(2\pi r L E) = \frac{\lambda L}{\varepsilon_0} \implies E = \frac{\lambda}{2\pi\varepsilon_0 r} = 5.36 \times 10^3\ \text{N/C} \qquad \underline{\textbf{outward}}$$

6. Two infinitely large sheets A and B are separated by 40.0 cm. A uniform electric field $E = 1.2 \times 10^3$ V/m in the direction shown is produced by the charges on the sheets. A proton is projected from a plane midway between the sheets with initial kinetic energy $K = 300$ eV in the direction shown in the figure. Find the kinetic energy of the proton (in eV) as it hits the sheet B. **3 points**

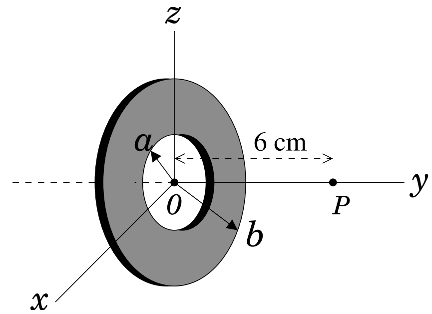


Solution: The work-energy principle gives us

$$\Delta K = -\Delta U \implies K_f - K_i = -e\,[-E(0.2)] = 3.84 \times 10^{-17}\ \text{J} = 240\ \text{eV}$$

$$\implies K_f = K_i + 240\ \text{eV} = 540\ \text{eV}$$

7. A thin disc with a circular hole at its centre has inner radius $a = 3.0$ cm and outer radius $b = 8.0$ cm. The disc is perpendicular to the y-axis as shown and has a uniform surface charge density $\sigma = -1.56 \times 10^{-7}$ C/m^2. Calculate the electric potential at a point P on its axis which is 6.0 cm from its centre. **3 points**



Solution: Using the superposition principle, the electric potential at P will be the potential due to the disc of radius b minus the potential due to the disc of radius a

$$V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{y^2+b^2} - y\right] - \frac{\sigma}{2\varepsilon_0}\left[\sqrt{y^2+a^2} - y\right] = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{y^2+b^2} - \sqrt{y^2+a^2}\right]$$

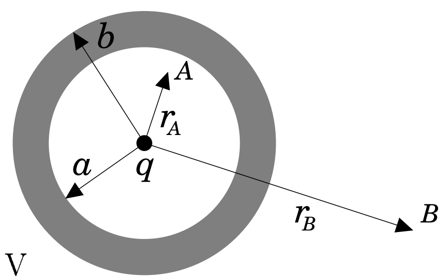
$$\Longrightarrow V = -2.90 \times 10^2 \text{ V}$$

Or alternatively

$$V_{big} = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{y^2+b^2} - y\right] = -3.52 \times 10^2 \text{ V}$$

$$V_{small} = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{y^2+a^2} - y\right] = -62.4 \text{ V} \quad \Longrightarrow \quad V = V_{big} - V_{small} = -2.90 \times 10^2 \text{ V}$$

8. A conducting shperical shell of inner radius $a = 4.0$ cm and outer radius $b = 9.0$ cm has a net charge $Q = 6.0$ nC. A point charge $q = -14.0$ nC is placed at the centre. Calculate the potential difference $(V_A - V_B)$ where the distances, $r_A = 2.0$ cm and $r_B = 15.0$ cm. **4 points**



Solution: Due to the point charge q at the centre, the inner surface of the shell will have charge $-q$ and the outer surface will have charge $(Q+q)$. Then

$$V_A = \frac{kq}{r_A} + \frac{k(-q)}{a} + \frac{k(Q+q)}{b} = -3.95 \times 10^3 \text{ V}$$

$$V_B = \frac{k(Q+q)}{r_B} = -4.80 \times 10^2 \text{ V} \qquad \Longrightarrow \qquad V_A - V_B = -3.47 \times 10^3 \text{ V}$$

Alternative Solution: According to Gauss' law, $\vec{\boldsymbol{E}} = 0$ for $0.04 < r < 0.09$,

$$\vec{\boldsymbol{E}} = \frac{kq}{r^2}\,\hat{\boldsymbol{r}} \quad \text{for} \quad 0 \le r \le 0.04 \qquad \text{and} \qquad \vec{\boldsymbol{E}} = \frac{k(Q+q)}{r^2}\,\hat{\boldsymbol{r}} \quad \text{for} \quad r > 0.09$$

$$\text{So} \qquad V_A - V_B = \int_A^B \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{r}} = \int_{0.02}^{0.04} \frac{kq}{r^2}\,dr + \int_{0.09}^{0.15} \frac{k(Q+q)}{r^2}\,dr$$

$$\Longrightarrow V_A - V_B = \frac{kq}{0.02} - \frac{kq}{0.04} + \frac{k(Q+q)}{0.09} - \frac{k(Q+q)}{0.15} = -3.47 \times 10^3 \text{ V}$$

Part II: Conceptual Questions

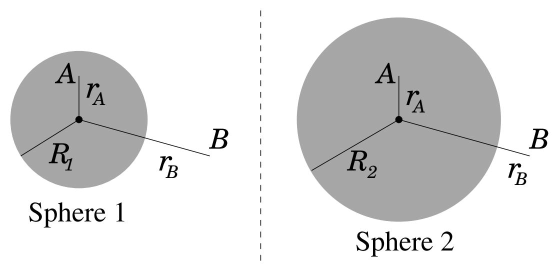
In the following, tick ($\surd$) the best answer. Each question carries 1 point.

1. An infinitely long wire with a uniform positive charge density λ lies along the z-axis. The magnitude of the electric field at a point with coordinates $(x,\ y,\ z)$ is

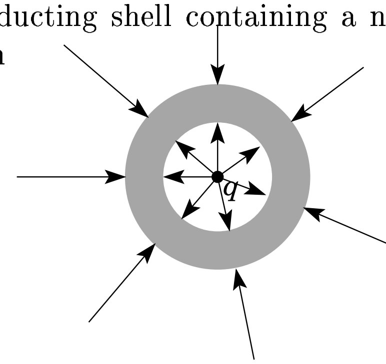
   (a) $E = \dfrac{2k\lambda}{|x|}$
   (b) $E = \dfrac{2k\lambda}{|y|}$
   (c) $E = \dfrac{2k\lambda}{|z|}$
   (d) $E = \dfrac{2k\lambda}{\sqrt{x^2+y^2}}$
   (e) $E = \dfrac{2k\lambda}{\sqrt{x^2+y^2+z^2}}$

2. Two isolated spheres with radii R_1 and R_2 ($R_2 = 2R_1$) have uniform volume charge densities. In each case, point A is at a distance of r_A and point B is at a distance of r_B from the centre. The magnitudes of the electric fields at A for the two spheres are equal, $E_{1A} = E_{2A}$. The magnitudes of the electric fields at B will be related as

   (a) $E_{2B} = E_{1B}$.
   (b) $E_{2B} = 2E_{1B}$.
   (c) $E_{2B} = 4E_{1B}$.
   (d) $E_{2B} = 8E_{1B}$.



3. A positive point charge q is at the centre of a conducting shell containing a net charge Q. The electric field lines are as shown. Then



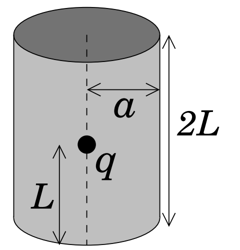
   (a) $Q < 0$, with $|Q| < q$.
   (b) $Q < 0$, with $|Q| > q$.
   (c) $Q < 0$, with $|Q| = q$.
   (d) $Q > 0$, with $Q < q$.

4. Two identical point charges held a distance d apart store an electrostatic potential energy U. A third identical charge is now placed at equal distance d from the two charges. If the three charges are released from rest, the maximum total kinetic energy they would gain is

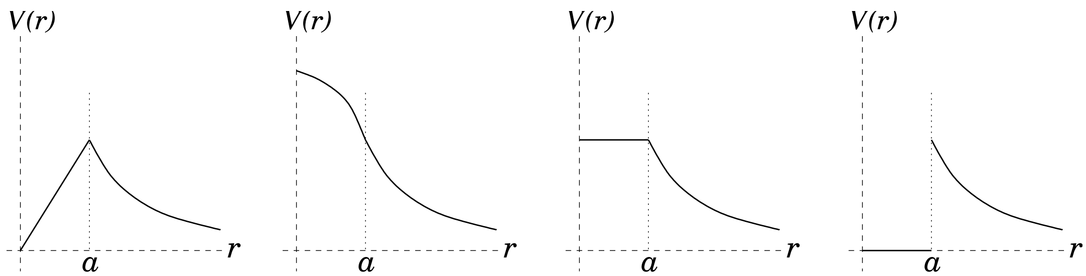
   (a) $1.5\,U$.
   (b) $2.0\,U$.
   (c) $3.0\,U$.
   (d) It is impossible to tell without knowing their masses.

5. A cylindrical Gaussian surface of radius a and length $2L$ has a point charge q at its centre (midpoint of its axis) as shown. If the electric flux through the curved cylindrical surface is Φ_c, the electric flux through the top circular face is

(a) $\Phi_{top} = \frac{kq}{L^2}(\pi a^2)$.
(b) $\Phi_{top} = \left(\frac{q}{\varepsilon_0} - \Phi_c\right)$.
(c) $\Phi_{top} = \frac{1}{2}\left(\frac{q}{\varepsilon_0} - \Phi_c\right)$.
(d) $\Phi_{top} = 0$.



6. A conducting sphere of radius a carries a positive charge. Which one of the following figures best represents the electric potential (relative to infinity) produced by this sphere as a function of the distance r from the centre of the sphere?



(a) (b) (c) (d)

7. Two large parallel sheets have uniform surface charge densities. When the separation between the plates is d, the magnitude of the electric field between them is E and the potential differnce between them is V. If they are now moved closer until the separation between them becomes $\frac{d}{2}$, the electric field and the potential difference will become

(a) $\frac{E}{2}$ and $2V$.
(b) E and $2V$.
(c) E and $\frac{V}{2}$.
(d) $\frac{E}{2}$ and $\frac{V}{2}$.

8. In the figure below, the vertical dashed lines represent equipotential surfaces perpendicular to the plane of the paper. A **proton** is moved from one equipotential surface to another through different paths 1, 2, and 3 as shown. The work done by the electric force along these paths is W_1, W_2 and W_3. Then

(a) $W_1 < W_2 = W_3$.
(b) $W_3 < W_1 < W_2$.
(c) $W_1 = W_2 = W_3$.
(d) $W_2 > W_1 = W_3$.