

Part I: Solve the following problems

1. Three charges are located in the x-y plane as shown in the figure. What is the magnitude of the resultant force on the charge $+q = 1 \text{ nC}$? **[3 points]**

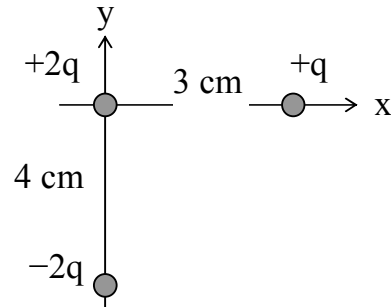
$$\vec{F}_1 = \frac{k|q \cdot 2q|}{(0.03)^2} \hat{i} = 2.0 \times 10^{-5} \hat{i} \text{ N}$$

$$\vec{F}_2 = \frac{k|q \cdot (-2q)|}{(0.05)^2} [-0.6 \hat{i} - 0.8 \hat{j}]$$

$$= [-4.32 \times 10^{-6} \hat{i} - 5.76 \times 10^{-6} \hat{j}] \text{ N}$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = [1.568 \times 10^{-5} \hat{i} - 5.76 \times 10^{-6} \hat{j}] \text{ N}$$

$$F = 1.67 \times 10^{-5} \text{ N}$$



2. A point charge ($q = -1 \text{ mC}$ and mass $m = 2 \times 10^{-6} \text{ kg}$) moving with a velocity $v = 2.0 \times 10^3 \text{ m/s}$ enters the region between two oppositely charged parallel plates through a hole in the positive plate. If the plates are 0.10 m apart and the charge stops just before hitting the negative plate, what is the magnitude of the electric field between the plates? **[2 points]**

$$0 - v^2 = -2ax \rightarrow a = \frac{v^2}{2x} = 2.0 \times 10^7 \text{ m/s}^2$$

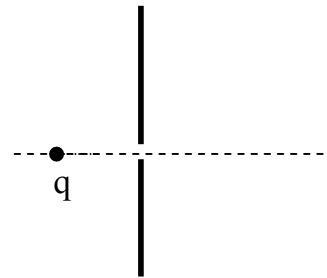
$$|q|E = ma \rightarrow E = \frac{ma}{|q|}$$

$$= 4.0 \times 10^4 \frac{\text{N}}{\text{C}}$$

OR

$$0 - \frac{1}{2}mv^2 = -q\Delta V = -q(-Ed)$$

$$\rightarrow E = -\frac{mv^2}{2q} = 4.0 \times 10^4 \frac{\text{N}}{\text{C}}$$



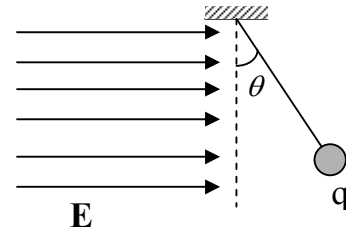
3. A small ball of mass 1.02 g and charge q is suspended by a light string and placed in a uniform electric field $E = 10^6$ N/C. If the ball is in equilibrium at $\theta = 45^\circ$, find the charge on the ball. **[2 points]**

Obviously $q > 0$

$$T \sin \theta = qE$$

$$T \cos \theta = mg$$

$$\tan \theta = \frac{qE}{mg} \rightarrow q = \frac{mg \tan \theta}{E} = 1.0 \times 10^{-8} \text{ C}$$



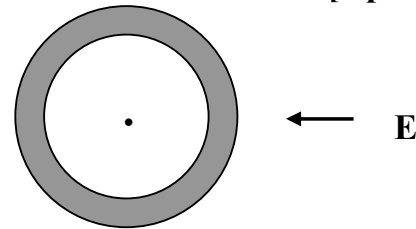
4. A conducting spherical shell of outer radius 18 cm has a surface charge density $\sigma = 7.35 \mu\text{C}/\text{m}^2$. When a point charge Q is brought to the centre of the shell, the electric field at a distance of 30 cm from the centre is 4×10^5 V/m pointing radially inwards. What is the value of Q ? **[3 points]**

\vec{E} inward, so $\phi_E < 0$

$$-4\pi r^2 E = \frac{Q + 4\pi b^2 \sigma}{\epsilon_0}$$

$$\rightarrow Q = -4\pi \epsilon_0 r^2 E - 4\pi \epsilon_0 b^2 \sigma$$

$$= -7.0 \times 10^{-6} \text{ C}$$



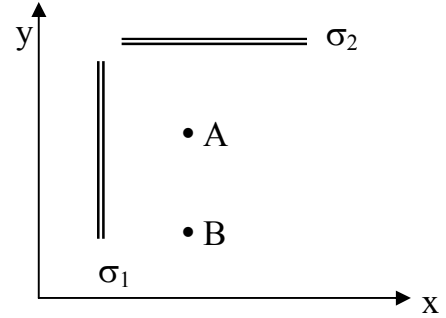
5. Two large sheets with surface charge densities $\sigma_1 = +1.77 \text{ nC/m}^2$ and $\sigma_2 = +0.885 \text{ nC/m}^2$ are perpendicular to the xy-plane as shown in the figure. How much work is done on a charge $q = 2.5 \text{ }\mu\text{C}$ to move a distance 20 cm from the point A to B in the xy-plane? **[3 points]**

\vec{E}_1 has no contribution to the work

$$\vec{E}_2 = \frac{\sigma_2}{2\epsilon_0} (\hat{j})$$

$$\Delta\vec{l} = -0.2 \hat{j}$$

$$W = -\Delta V = q \vec{E} \cdot \Delta\vec{l} = q \frac{\sigma_2}{2\epsilon_0} (0.2) = 2.5 \times 10^{-5} \text{ J}$$

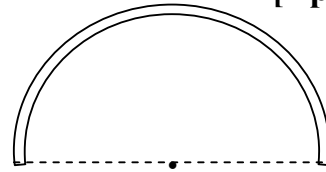


6. A plastic rod with a linear charge density $-0.5 \text{ }\mu\text{C/m}$ is bent into a semicircle of radius 30 cm as shown in the figure. What is the electric potential at the centre of the semicircle, if $V=0$ is at infinity. **[2 points]**

$$V = \frac{kQ}{a}$$

$$= \frac{k(\lambda\pi a)}{a}$$

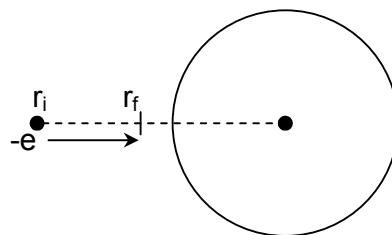
$$= k\lambda\pi = -1.4 \times 10^4 \text{ V}$$



7. An electron outside a conducting sphere (radius = 1.0 m) with a charge +10 C has an electric potential energy of -10 nJ. When the electron is released from rest, what is its kinetic energy after it has moved by 20 cm? **[3 points]**

$$v_i = \frac{-kQe}{r_i}$$

$$\rightarrow r_i = -\frac{kQe}{V_i} = 1.44 \text{ m}$$



Upon release the electron moves towards sphere.

$$K = -(U_f - U_i) \quad \rightarrow r_f = 1.24 \text{ m}$$

$$= -\frac{kQe}{r_f} + U_i = 1.6 \times 10^{-9} \text{ J}$$

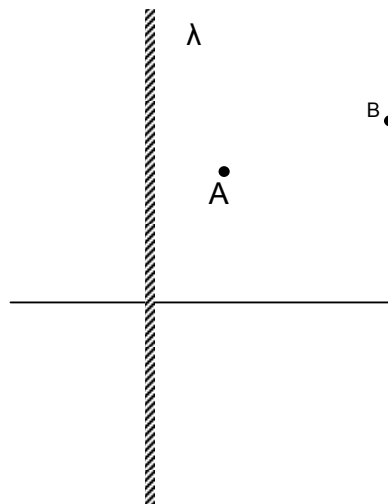
8. An infinite line of charge with the linear charge density $\lambda = +4$ nC/m is along the y-axis. What is the potential difference $V_A - V_B$ between the two points A (1 m, 2 m) and B (5 m, 4 m)? **[3 points]**

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 x} \hat{i}, \quad d\vec{l} = dx\hat{i} + dy\hat{j}$$

$$V_A - V_B = \int_A^B \vec{E} \cdot d\vec{l}$$

$$\int_A^B \frac{\lambda}{2\pi\epsilon_0 x} dx$$

$$= \frac{\lambda}{2\pi\epsilon_0} \ln x \Big|_1^5 = 1.16 \times 10^2 \text{ V}$$

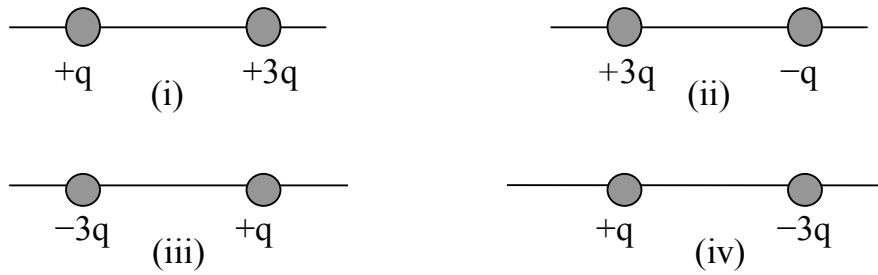


Part II: Conceptual questions. Tick the best answer.
Each question carries 1 point.

1. Three identical conducting spheres A, B, and C initially have charges of $+8Q$, $+4Q$, and $-8Q$, respectively. Thin conducting wire connects two balls at a time in the following sequence: (1) balls A and B, then (2) balls A and C. The final charge on ball A is:

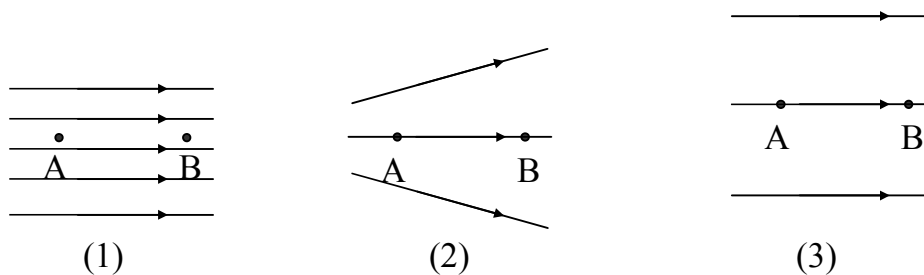
- (a) zero
- ✓ (b) $-Q$
- (c) $-4Q$
- (d) $-8Q$.

2. The figure shows four different arrangements of two point charges. In which situation is there a point to the left of the arrangement where an electron will be in equilibrium?



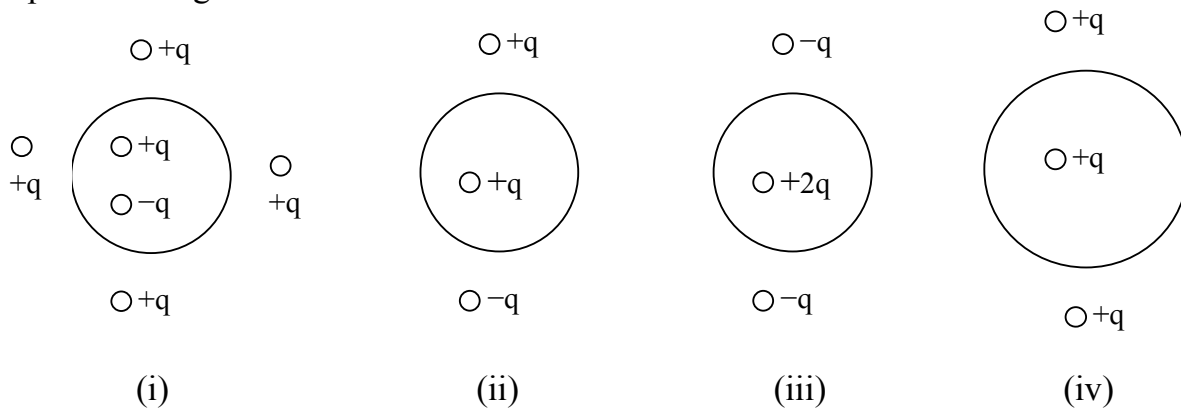
- (a) i
- (b) ii
- (c) iii
- ✓ (d) iv

3. The figure shows three arrangements (1), (2), and (3) of the electric field lines. In each arrangement a proton is released from rest at point A. It reaches point B with a speed v_1 , v_2 , and v_3 , respectively. If the separation between A and B is the same in all the arrangements, then which of the following relations is true?



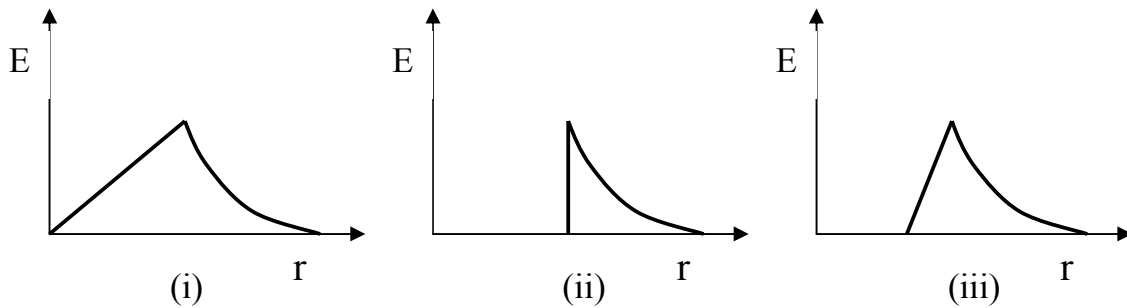
- (a) $v_2 > v_3 > v_1$
- (b) $v_3 > v_2 > v_1$
- ✓ (c) $v_1 > v_2 > v_3$
- (d) $v_1 = v_2 = v_3$

4. In the following figures, in which configuration is the electric flux through the sphere the highest?



- (a) i
- (b) ii
- ✓ (c) iii
- (d) iv

5. A sphere has a uniform charge density ρ . Which of the following diagram represents the electric field of the shell as a function of the distance r from the centre?

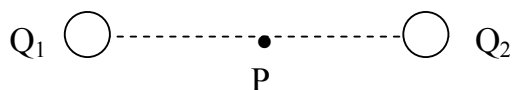


- ✓ (a) i
- (b) ii
- (c) iii

6. If an electron and a proton are released inside an electric field, which statement is correct?

- ✓ (a) Only the proton moves towards the lower electric potential.
- (b) Only the electron moves towards the lower electric potential.
- (c) Both of them move to the higher electric potential.
- (d) Both of them move to the lower electric potential.

7. In the following figure, if the electric field at the point P midway between the two electric charges Q_1 and Q_2 is zero, then which of the following conclusions is *definitely* correct?



- ✓ (a) Electric potential at point P is not zero.
- (b) Electric potential at point P is positive.
- (c) Electric potential at point P is negative.
- (d) Electric potential at point P is zero.

8. A *conducting* sphere of radius R has a charge Q . A test charge q_0 is moved from the surface of the sphere to a half-radius point inside the sphere. The work done on the charge is:

- (a) $k \frac{Qq}{R}$
- (b) $k \frac{Qq}{(R/2)}$
- (c) $k \frac{Qq}{2R}$

- ✓ (d) zero.

