Kuwait University **Department of Physics**

Physics 102

Final Examination (Spring Semester, 2008-2009)

June 16, 2009
Time: 11:00 AM to 1:00 PM

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Name: ....Solution.............. **Student No.:** ....

Fundamental Constants

$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N·m^2/C^2	Coulomb's constant
$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/(N·m^2)	Permitivity of free space
$\mu_0 = 4\pi \times 10^{-7}$ T·m/A	Permeability of free space
$e = 1.6 \times 10^{-19}$ C	Elementary unit of charge
$m_e = 9.11 \times 10^{-31}$ kg	Mass of an electron
$m_p = 1.67 \times 10^{-27}$ kg	Mass of a proton
1 eV = 1.6×10^{-19} J	Conversion from eV to J
$N_A = 6.022 \times 10^{23}$ /mol	Avogadro's number
$g = 9.8$ m/s^2	Acceleration due to gravity

Prefixes of units

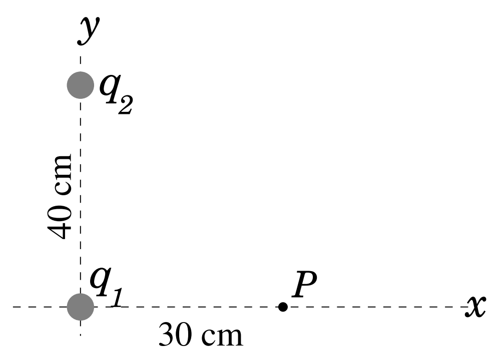
m = 10^{-3}	$\mu = 10^{-6}$	n = 10^{-9}	p = 10^{-12}
k = 10^{3}	M = 10^{6}	G = 10^{9}	T = 10^{12}

For use by instructors only

Problems	1	2	3	4	5	6	7	8	Questions	Total
Marks										

Part I: Solve the following problems

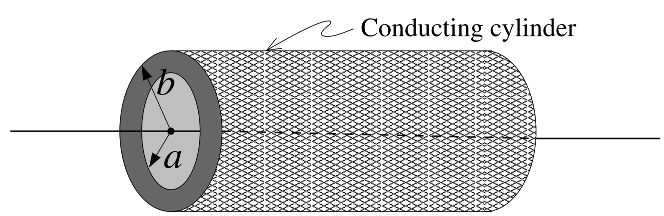
1. Two point charges, q_1 and q_2 are located in the xy-plane as shown. The net electric field at the point P on the x-axis is $\vec{E} = \left(4.76 \times 10^2\ \hat{i} + 2.52 \times 10^2\ \hat{j}\right)$ N/C. Find the charge q_2 (both its magnitude and its sign). **3 points**



Solution: E_y is due to $\vec{E}_2$ only. Since $E_y > 0 \implies \boxed{q_2 < 0}$, and

$$\frac{k\,|q_2|}{0.5^2}\,\frac{0.4}{0.5} = 2.52 \times 10^2 \implies |q_2| = 8.75 \times 10^{-9}\ \text{C} \implies \boxed{q_2 = -8.75 \times 10^{-9}\ \text{C}}$$

2. The magnitude of the electric field at a distance of $r = 12.0$ cm from the axis of a very long charged **conducting** hollow cylinder is $E = 7.5$ N/C and it is **inward**. The inner and outer radii of the cylinder are $a = 3.0$ cm and $b = 5.0$ cm. Calculate the surface charge density σ on the outer surface of the cylinder. **4 points**



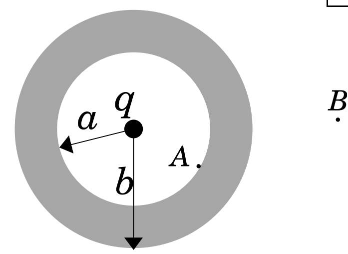
Solution: We have a cylindrical symmetry. So we choose a cylindrical Gaussian surface of radius r and length L. All the charge is on the outer surface. Since $\vec{E}$ is inward, the flux is negative, so

$$\Phi_E = -\left(2\pi r L\right) E \qquad\qquad Q_{enc} = \left(2\pi b L\right) \sigma$$

Using Gauss' law

$$-\left(2\pi r L\right) E = \frac{\left(2\pi b L\right) \sigma}{\varepsilon_0} \implies \sigma = -\frac{\varepsilon_0 E r}{b} = -1.6 \times 10^{-10}\ \text{C/m}^2$$

3. A **conducting** shell of inner radius $a = 4.0$ cm and outer radius $b = 9.0$ cm contains no net charge. A point charge $q = -5.0$ nC is placed at the centre of the shell. Find the electric potential difference, $(V_A - V_B)$, if point A is on the inner surface of the shell and point B is such that $r_B = 12.0$ cm. **3 points**



Solution: Since all the points in a conductor have the same electric potential, the potential at A is the same as that on the outer surface, so

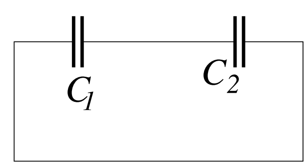
$$V_A = \frac{kq}{b} \qquad V_B = \frac{kq}{r_B} \qquad \text{So} \qquad V_A - V_B = \frac{kq}{b} - \frac{kq}{r_B} = -1.25 \times 10^2 \text{ V}$$

OR
The electric field $\vec{\boldsymbol{E}} = 0$ inside the conducting shell (for $a < r < b$), and for all other points, according to Gauss' law, it is

$$\vec{\boldsymbol{E}} = \frac{kq}{r^2}\,\hat{\boldsymbol{r}}$$

$$\text{So} \qquad V_A - V_B = \int_{0.09}^{0.12} \frac{kq}{r^2}\,dr = \frac{kq}{0.09} - \frac{kq}{0.12} = -1.25 \times 10^2 \text{ V}$$

4. Two air-filled capacitors, $C_1 = 13.5$ μF and C_2 (unknown), connected as shown are charged. The energy stored in C_1 is 4.32×10^{-4} J. When the space between the plates of C_1 is filled with a dielectric ($K = 3$), the energy in C_1 becomes $7.29{\times}10^{-4}$ J. Calculate the capacitance of capacitor C_2. **4 points**



Solution: The two capacitors are **parallel**.

When both C_1 and C_2 are air-filled, let V be the potential across each capacitor. Then

$$U_1 = \frac{1}{2}C_1V^2 \implies V = \sqrt{\frac{2U_1}{C_1}} = 8.0 \text{ V}$$

After the dielectric ($K = 3$) is inserted in C_1, $C_1' = 3C_1 = 40.5$ μF. Let V' be the potential across each capacitor. Then

$$U_1' = \frac{1}{2}C_1'V'^2 \implies V' = \sqrt{\frac{2U_1'}{C_1'}} = 6.0 \text{ V}$$

The total charge remains constant. So

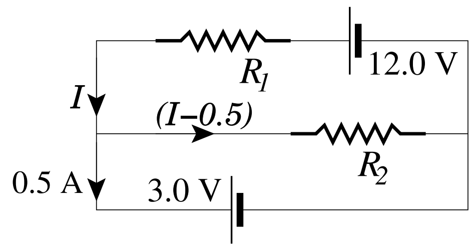
$$(C_1 + C_2) \times 8 = (C_1' + C_2)\,6 \implies C_2 = 6.75 \times 10^{-5} \text{ F}$$

5. A hollow cylindrical wire of length $L = 55.0$ cm, inner radius $a = 1.0$ mm and outer radius $b = 2.0$ mm, and resistivity of $\rho = 1.72 \times 10^{-8}$ Ω·m is connected across a potential difference of 3.0 V. Calculate the power being dissipated by the wire. **3 points**

Solution:

$$P = \frac{V^2}{R} = \frac{V^2 A}{\rho L} = \frac{V^2 \pi \left(b^2 - a^2\right)}{\rho L} = 9.0 \times 10^3 \text{ W}$$

6. In the circuit shown, $R_2 = 2R_1$. The current through the 3.0 V battery is 0.5 A as shown. What is the resistance of the resistor R_1? **4 points**



Solution: Let the current in R_1 be I as shown, then the current in R_2 is $(I - 0.5)$ as shown. Then

$$\text{Big loop:} \quad 3 + IR_1 - 12 = 0 \implies IR_1 = 9$$

$$\text{lower loop:} \quad 2R_1 (I - 0.5) - 3 = 0 \implies 18 - R_1 - 3 = 0 \implies R_1 = 15\ \Omega$$

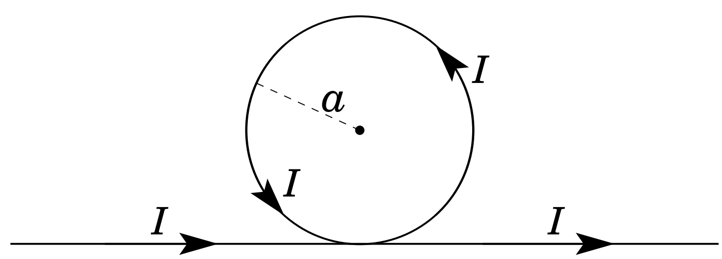
7. An electron with a speed of $v = 4 \times 10^6$ m/s moves in a helical path of radius $R = 4.8$ cm in a region of uniform magnetic field of magnitude $B = 4.5 \times 10^{-4}$ T. Calculate the pitch of its helical path. **4 points**

Solution

$$R = \frac{mv_\perp}{Be} = \frac{mv\sin\theta}{Be} \implies \theta = \sin^{-1}\left(\frac{BeR}{mv}\right) = 71.52^o$$

$$P = \frac{2\pi m}{Be}\, v\cos\theta = 0.10 \text{ m}$$

8. A long wire with insulating cover is bent to make a circular loop of radius $a = 8.0$ cm as shown. The wire carries a current of $I = 40.0$ A. Calculate the magnetic field $\vec{B}$ (both its magnitude as well as its direction) at the centre of the loop. **4 points**



Solution: The two fields, one by the long wire and the other by the circular loop, are both **out-of-the-plane**, so

$$B = \frac{\mu_0 I}{2\pi a} + \frac{\mu_0 I}{2a} = 4.1 \times 10^{-4} \text{ T} \qquad \textbf{out-of-the-plane}$$

9. A long cylindrical wire of radius $a = 2.0$ mm carries a current $I = 15.0$ A with a uniform current density. Calculate the magnitude of the magnetic field at a distance of $r = 0.5$ mm from the axis. **4 points**

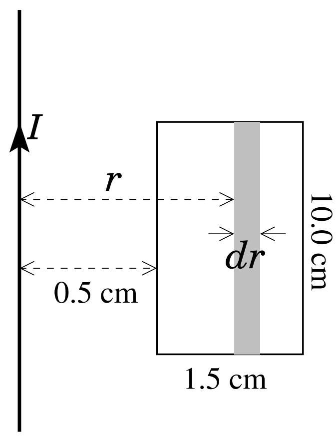
Solution: Take an Amperean loop of radius $r = 0.5$ mm with centre at the axis. Then

$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = 2\pi r B \qquad \text{and} \qquad I_{enc} = \pi r^2 J = \frac{Ir^2}{a^2}$$

Then Ampere's law gives

$$2\pi r B = \mu_0 \frac{Ir^2}{a^2} \implies B = \frac{\mu_0 I r}{2\pi a^2} = 3.75 \times 10^{-4} \text{ T}$$

10. A rectangular loop of wire with one of its sides parallel to a long straight wire are in one plane as shown. The long wire carries a current I in the direction shown. This current decreases at the rate of 0.2 A/s. Find the magnitude of the induced emf and the direction of the induced current in the loop. **5 points**



Solution: The magnetic field is not uniform, neither constant. Take a thin strip of width dr parallel to the wire as dA (the shaded area), then $\vec{\boldsymbol{B}}$ and $d\vec{\boldsymbol{A}}$ are parallel. So $\vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = B\, dA$ with $dA = (0.1)dr$. So

$$\Phi_B = \int_{0.005}^{0.020} \frac{\mu_0 I}{2\pi r} \left[(0.1)dr\right] = \frac{\mu_0 I (0.1)}{2\pi} \ln\left(\frac{0.020}{0.005}\right) = \left(2.773 \times 10^{-8}\right) I$$

Then

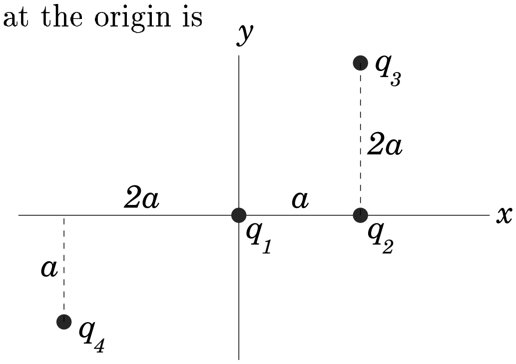
$$\mathcal{E} = \frac{d\Phi_B}{dt} = \left(2.773 \times 10^{-8}\right) \frac{dI}{dt} = 5.5 \times 10^{-9} \text{ V}$$

Since the current is decreasing (so the flux is decreasing), according to Lenz's law, the induced current is **clockwise**.

Part II: Conceptual Questions

In the following, tick (√) the best answer. Each question carries 1 point.

1. Four point charges are placed in the xy-plane as shown. The net electric flux through a spherical surface of radius $R = 2a$ centred at the origin is



   (a) $\Phi_E = \dfrac{q_1 + q_2 + q_3 + q_4}{\varepsilon_0}$

   (b) $\Phi_E = \dfrac{q_1}{\varepsilon_0}$

   (c) $\boxed{\Phi_E = \dfrac{q_1 + q_2}{\varepsilon_0}}$

   (d) $\Phi_E = \dfrac{q_2 + q_3 + q_4}{\varepsilon_0}$

2. Three identical conducting spheres, A, B and C contain charges $+Q$, $+Q$, and $-Q$ respectively. First A and C were connected by a thin wire for a short time, then B and C were connected for a short time, and finally A and B were connected for a short time. What is the total charge remaining in the three spheres after this?

   (a) zero

   (b) $\boxed{Q}$

   (c) $\dfrac{Q}{2}$

   (d) $\dfrac{3Q}{4}$

3. When a point charge moves on an equipotential surface, the electric field does not do any work, because the electric field is

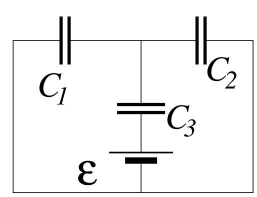
   (a) zero at all points on the equipotential surface.

   (b) same at all points on the equipotential surface.

   (c) [perpendicular to equipotential surface at all points on the equipotential surface.]

   (d) parallel to equipotential surface at all points on the equipotential surface.

4. In the circuit shown, $C_1 = C_2 = C_3$. Then, in the steady state situation



   (a) $Q_1 = Q_2 = Q_3$.

   (b) $\boxed{Q_1 = Q_2,\ Q_3 = Q_1 + Q_2}$.

   (c) $Q_1 = Q_3,\ Q_2 = Q_1 + Q_3$.

   (d) $Q_2 = Q_3,\ Q_1 = Q_2 + Q_3$.

5. Two cylindrical wires of same material and of equal lengths are connected in parallel to a source of emf. Wire 1 is a solid cylinder of radius a and wire 2 is a hollow cylinder of inner radius a and outer radius $2a$. The current densities in the two wires are related as

   (a) $\boxed{J_1 = J_2}$

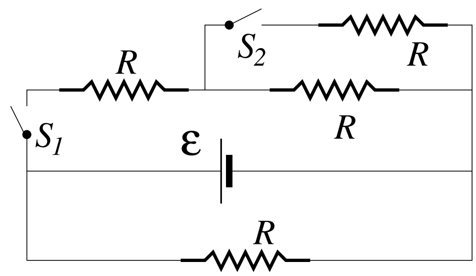
   (b) $J_2 = 2J_1$

   (c) $J_2 = 3J_1$

   (d) $J_2 = 4J_1$

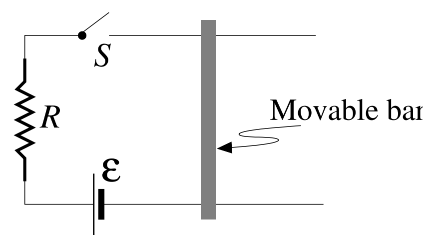
6. In the circuit shown, the current passing through the battery is largest when

   (a) S_1 is open and S_2 is closed.
   (b) S_1 is closed and S_2 is open.
   (c) [both S_1 and S_2 are closed.]
   (d) both S_1 and S_2 are open.



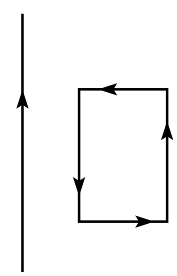
7. The circuit shown is in a region of uniform magnetic field $\vec{B}$. When the switch S is closed, the movable conducting bar jumps directly out of the page. The magnetic field **must** have a component

   (a) out of the page.
   (b) into the page.
   (c) [from left to right.]
   (d) from right to left.



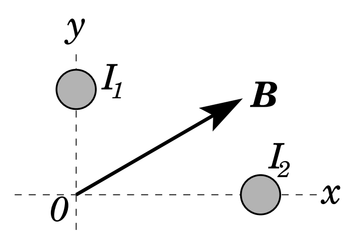
8. The direction of the currents in the long straight wire and the rectangular loop in the figure are as shown. The magnetic force exerted by the wire on the loop is

   (a) towards the top of the page.
   (b) towards the bottom of the page.
   (c) [from left to right.]
   (d) from right to left.



9. The net magnetic field at the origin due to the two long wires carrying currents perpendicular to the xy-plane is as shown. Then

   (a) both I_1 and I_2 are out of the page.
   (b) [I_1 is out of the page and I_2 is into the page.]
   (c) I_1 is into the page and I_2 is out of the page.
   (d) both I_1 iand I_2 are into the page.



10. Two rectangular loops are placed side by side in one plane as shown. The loop 1 is connected to a source that supplies a current which is incresing in time. The induced current in loop 2 is

   (a) clockwise.
   (b) [anticlockwise.]
   (c) zero.
   (d) clockwise if loop 2 is bigger, else anticlockwise.