Kuwait University — Department of Physics

Physics 102

First Midterm Examination (Fall Semester, 2008-2009)

November 16, 2008
Time: 6:00 PM to 7:30 PM

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Name: ....Solution.............. **Student No.:** ....

Fundamental Constants

$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N·m^2/C^2	Coulomb's constant
$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/(N·m^2)	Permitivity of free space
$\mu_0 = 4\pi \times 10^{-7}$ T·m/A	Permeability of free space
$e = 1.6 \times 10^{-19}$ C	Elementary unit of charge
$m_e = 9.11 \times 10^{-31}$ kg	Mass of an electron
$m_p = 1.67 \times 10^{-27}$ kg	Mass of a proton
1 eV = 1.6×10^{-19} J	Conversion from eV to J
$N_A = 6.022 \times 10^{23}$ /mol	Avogadro's number
$g = 9.8$ m/s^2	Acceleration due to gravity

Prefixes of units

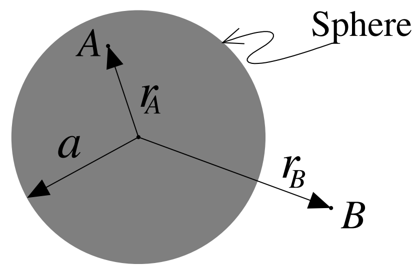
m = 10^{-3}	$\mu = 10^{-6}$	n = 10^{-9}	p = 10^{-12}
k = 10^{3}	M = 10^{6}	G = 10^{9}	T = 10^{12}

For use by instructors only

Problems	1	2	3	4	5	6	7	8	Questions	Total
Marks										

Part I: Solve the following problems

1. The sphere (radius $a = 5.0$ cm) below contains a uniformly distributed charge Q. The points A and B are such that $r_A = 2.0$ cm and $r_B = 6.0$ cm. At A, the electric field has magnitude $E_A = 2.0 \times 10^4$ N/C. Calculate the magnitude E_B of the electric field at B. **2 points**



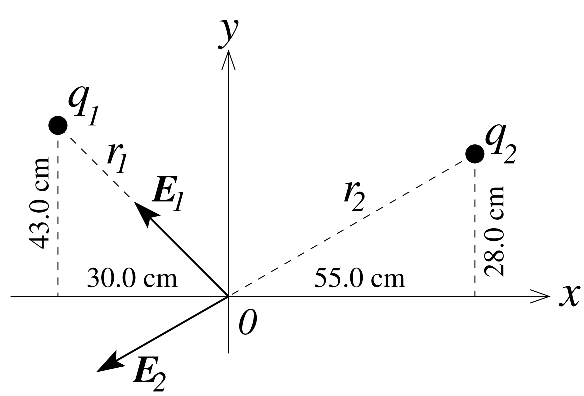
Solution: The magnitude of electric field at A is given by

$$E_A = \frac{kQr_A}{a^3} \implies Q = \frac{a^3 E_A}{kr_A} = 1.4 \times 10^{-8} \text{ C}$$

The magnitude of electric field at B is

$$E_B = \frac{kQ}{r_B^2} = 3.5 \times 10^4 \text{ N/C}$$

2. Two point charges are located in the xy-plane as follows: charge $q_1 = -4.6$ nC is at the point $x = -30.0$ cm, $y = 43.0$ cm, and $q_2 = +7.3$ nC is at the point $x = 55.0$ cm, $y = 28.0$ cm. Calculate the x-component of the net electric field at the origin. **3 points**

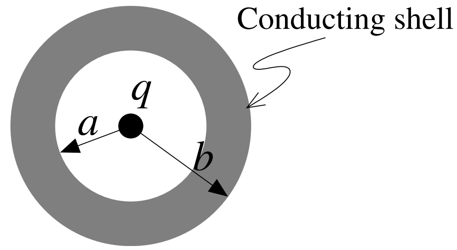


Solution: The electric fields $\vec{E}_1$ and $\vec{E}_2$ are as shown.

The x-component of $\vec{E}$ is

$$\begin{aligned} E_x &= E_{1x} + E_{2x} \\ &= \frac{k|q_1|}{0.30^2 + 0.43^2}\left(\frac{-0.30}{\sqrt{0.30^2 + 0.43^2}}\right) + \frac{k|q_2|}{0.55^2 + 0.28^2}\left(\frac{-0.55}{\sqrt{0.55^2 + 0.28^2}}\right) \\ &= -2.40 \times 10^2 \text{ N/C} \end{aligned}$$

3. A **conducting** shell (of inner radius $a = 4.0$ cm and outer radius $b = 7.0$ cm) contains a net charge $Q = 6.0$ nC. When a point charge q is placed at its centre, the electric potential at $r = 9.0$ cm from the centre is -300.0 V (taking $V = 0$ as $r \to \infty$). Calculate the electric potential at $r = 3.0$ cm. **4 points**



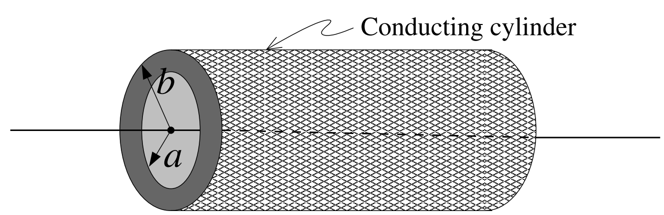
Solution: To satisfy Gauss' law, the charge on the inner surface of the shell must be $Q_{inner} = -q$ and the charge on the outer surface of the shell is $Q_{outer} = Q + q$. So

$$\frac{k(Q+q)}{0.09} = -300.0 \implies Q + q = -3.0 \times 10^{-9} \implies q = -9.0 \times 10^{-9}$$

The electric potential at $r = 3.0$ cm ($r < a$) is

$$V = \frac{kq}{r} + \frac{k(-q)}{a} + \frac{k(Q+q)}{b} = -1.061 \times 10^{3} \text{ V}$$

4. A long thin wire (with linear charge density $\lambda = -20.0$ pC/m) is on the axis of a long **conducting** cylinder of inner radius $a = 3.0$ cm and outer radius $b = 5.0$ cm as shown. The electric field at $r = 8.0$ cm from the wire is **outward** and has magnitude $E = 13.5$ N/C. Calculate the surface charge density σ_{outer} on the outer surface of the cylinder. **3 points**



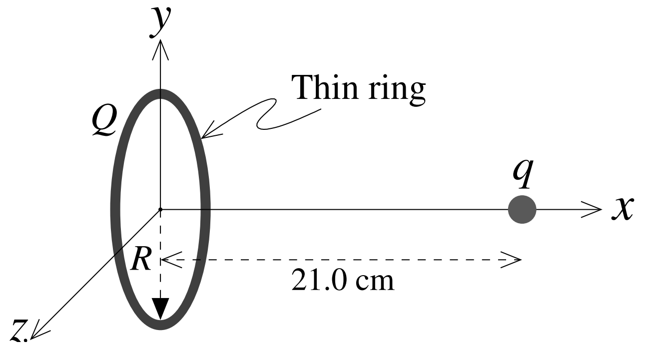
<u>**Solution:**</u> We have a cylindrical symmetry. So we choose a cylindrical Gaussian surface of radius r and length L. Since the electric field inside the cylinder must be zero, the charge on the wire and the inner surface of the cylinder add up to zero. Then

$$\Phi_E = (2\pi r L)\, E \qquad Q_{enc} = (2\pi b L)\, \sigma_{outer}$$

and $Q_{enc} > 0$ since $\vec{E}$ is **outward**. Using Gauss' law

$$(2\pi r L)\, E = \frac{(2\pi b L)\, \sigma_{outer}}{\varepsilon_0} \implies \sigma_{outer} = \frac{\varepsilon_0 E r}{b} = 1.91 \times 10^{-10} \text{ C/m}^2$$

5. A thin ring of radius $R = 8.0$ cm in the yz-plane shown below contains a uniformly distributed charge $Q = -20.0\ \mu$C. A point charge $q = -2.5\ \mu$C is released from rest on the x-axis at $x = 21.0$ cm. Find the x-coordinate of the point where the kinetic energy of q is 0.6 J. **4 points**



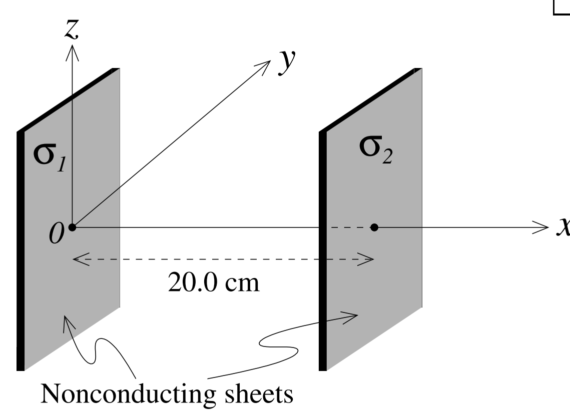
Solution: Let the point be at a distance of x from the centre of the ring. Clearly $x > 21.0$ cm. Now

$$\Delta K = -\Delta U \implies K_f = -\left[\frac{kQq}{\sqrt{x^2+0.08^2}} - \frac{kQq}{\sqrt{0.21^2+0.08^2}}\right]$$

$$\implies 0.6 = -\frac{kQq}{\sqrt{x^2+0.08^2}} + 2.0 \implies \frac{kQq}{\sqrt{x^2+0.08^2}} = 1.4$$

$$\implies x^2 = 0.097 \implies x = \pm 0.31 \quad \text{But} \quad x > 0.21 \implies \boxed{x = 0.31\ \text{m}}$$

6. The two large thin non-conducting sheets below are perpendicular to the x-axis and have uniform charge densities σ_1 and σ_2; $\sigma_1 = -30.0$ nC/m^2 but σ_2 is not known. The net electric field at $x = 12.0$ cm is $\vec{E} = -6.40 \times 10^2\ \hat{i}$ N/C. Find the net electric field $\vec{E}$ at $x = 45.0$ cm. **3 points**



Solution: At $x = 12.0$ cm,

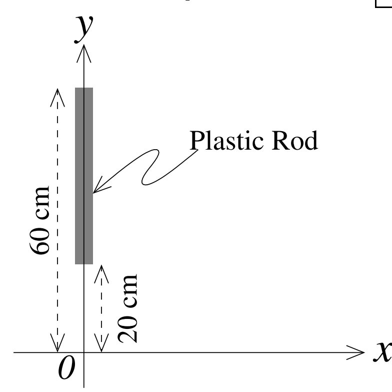
$$\vec{E} = \frac{\sigma_1}{2\varepsilon_0}\ \hat{i} + \frac{\sigma_2}{2\varepsilon_0}\left(-\hat{i}\right)$$

$$\implies \frac{\sigma_2}{2\varepsilon_0}\ \hat{i} = \frac{\sigma_1}{2\varepsilon_0}\ \hat{i} - \vec{E} = -1.05\times10^3\ \hat{i} \implies \sigma_2 = -1.86\times10^{-8}\ \text{C/m}^2$$

At $x = 45.0$ cm, the net electric field is

$$\vec{E} = \frac{\sigma_1}{2\varepsilon_0}\ \hat{i} + \frac{\sigma_2}{2\varepsilon_0}\ \hat{i} \implies \vec{E} = -2.75 \times 10^3\ \hat{i}\ \text{N/C}$$

7. A plastic rod with a uniformly distributed charge $Q = -50.0$ nC lies along the y-axis, from $y = 20.0$ cm to $y = 60.0$ cm. Find the electric potential at the origin. **3 points**



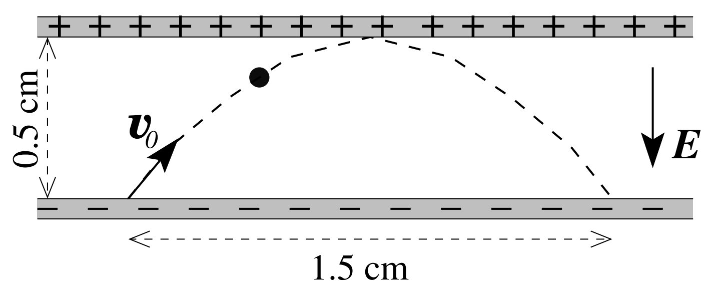
Solution: We choose a charge element $dq = (\lambda\, dy)$ at a distance of y from the origin, where λ is the linear charge density. The electric potential due to this charge element is

$$dV = \frac{k\lambda\, dy}{y} = \left(\frac{kQ}{0.4}\right)\frac{dy}{y}$$

The electric potential due to the whole of the charge is then

$$V = \left(\frac{kQ}{0.4}\right)\int_{0.2}^{0.6}\frac{dy}{y} = \left(\frac{kQ}{0.4}\right)\ln y\Big|_{0.2}^{0.6} = \left(\frac{kQ}{0.4}\right)\ln 3 = -1.24\times 10^{3}\text{ V}$$

8. A proton launched at the negative plate shown below with initial velocity $\vec{\boldsymbol{v}}_0$ just misses the positive plate and lands on the negative plate 1.5 cm from its starting position. If $E = 6.00\times 10^{2}$ N/C calculate the initial speed v_0 of the proton. **4 points**



Solution: We assume x-axis is horizontal and y-axis as vertical. Then

$$\vec{\boldsymbol{a}} = -\frac{eE}{m}\,\hat{\boldsymbol{j}} = -5.75\times 10^{10}\,\hat{\boldsymbol{j}}\text{ m/s}^2$$

At the top of the trajectory, $v_y = 0$, so

$$v_{0_y}^2 = 2ay \implies v_{0_y} = \sqrt{2\,(5.75\times 10^{10})\times 0.005} = 2.40\times 10^{4}\text{ m/s}$$

Also $$v_{0_y} = at \implies t = \frac{v_{0_y}}{a} = 4.17\times 10^{-7}\text{ s}$$

Then $$x = v_{0_x}(2t) \implies v_{0_x} = \frac{1.5\times 10^{-2}}{2\times 4.2\times 10^{-7}} = 1.80\times 10^{4}\text{ m/s}$$

Then $$v_0 = \sqrt{v_{0_x}^2 + v_{0_y}^2} = 3.00\times 10^{4}\text{ m/s}$$

Part II: Conceptual Questions

In the following, tick ($\surd$) the best answer. Each question carries 1 point.

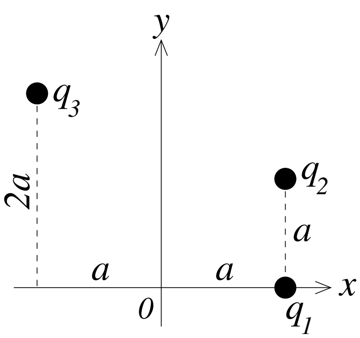
1. Three point charges are located in the xy-plane as shown. The net electric flux through a spherical surface of radius $R = 2a$ centred at the origin is

   (a) $\boxed{\Phi_E = \dfrac{q_1 + q_2}{\varepsilon_0}}$

   (b) $\Phi_E = \dfrac{q_1 + q_2 - q_3}{\varepsilon_0}$

   (c) $\Phi_E = \dfrac{q_1 + q_2 + q_3}{\varepsilon_0}$

   (d) $\Phi_E = \dfrac{q_1}{\varepsilon_0}$



2. A study of the electrostatic interactions between pairs of the 3 objects A, B and C yields the following results: B is attracted to A and C; A and C attract each other. Which inference is **not** justified?

   (a) A and C are oppositely charged.

   (b) B is neutral.

   (c) $\boxed{B \text{ is a spherical conductor.}}$

   (d) B is a polarisable insulator.

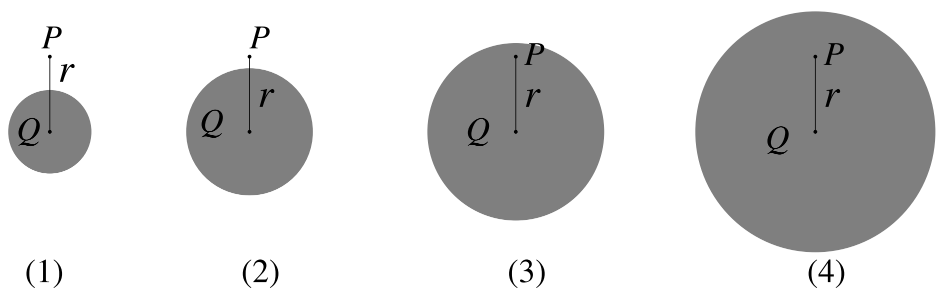
3. The spheres shown below each have the same charge Q distributed uniformly over the volume. If E_1, E_2, E_3 and E_4 are the magnitudes of the electric fields at P (at a distance of r from the centres),

   (a) $E_1 = E_2 = E_3 = E_4$.

   (b) $\boxed{E_1 = E_2 > E_3 > E_4.}$

   (c) $E_1 > E_2 > E_3 > E_4$.

   (d) $E_1 < E_2 < E_3 < E_4$.



4. Gauss' law can be applied to a closed surface S

   (a) only if all the charges are stationary.

   (b) only if all the charges inside the surface S are stationary.

   (c) only if no charges inside the surface S are accelerating.

   (d) $\boxed{\text{even if all the charges are moving.}}$

5. A metal object is given a positive charge. What can one conclude about the mass of the object?

   (a) It increases.

   (b) It decreases.

   (c) It does not change.

   (d) No conclusions are possible.

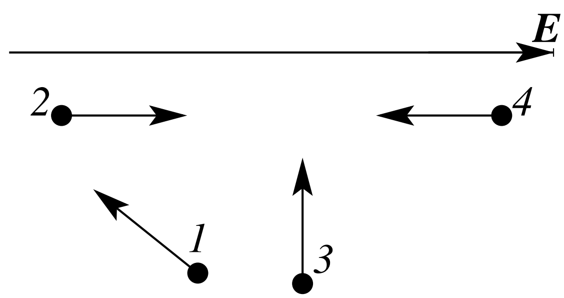
6. The velocities of four protons (of identical speed) in a region of uniform electric field $\vec{E}$ are shown below. For which proton is the rate at which the electric field is doing work greatest?

   (a) Proton 1.

   (b) Proton 2.

   (c) proton 3.

   (d) proton 4.



7. The electric potential **can not** be used to find

   (a) electric field in a region by partial differentiation.

   (b) the work done by electrostatic fields.

   (c) where conservative electric fields are zero.

   (d) the work done by non-electric forces in some processes.

8. An infinite sheet has uniform surface charge density σ. The equipotential surfaces are

   (a) cubes.

   (b) spheres.

   (c) cylinders.

   (d) planes.