Kuwait University **Physics Department**

Physics 102

Final Examination

January 18, 2009

Time: 11.00 a.m to 13.00 p.m.

Name……………………………………Student No…………………

Instructors: Drs. Abdel-Karim, Afrousheh, Davis, Kokay, Lajko, Marafi, Rakhshani, Razee, Sabah & Sharma

Fundamental constants

$k = \frac{1}{4\pi\varepsilon_o} = 9.0 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2$	***(Coulomb constant)***
$\varepsilon_o = 8.85 \times 10^{-12}\ C^2/(N\cdot m^2)$	***(Permittivity of free space)***
$\mu_o = 4\pi \times 10^{-7} = 1.26 \times 10^{-6}\ H/m$	***(Permeability of free space)***
$e = +1.60 \times 10^{-19}\ C$	***(Elementary unit of charge)***
$N_A = 6.022 \times 10^{23}$	***(Avogadro's number)***
$g = 9.8\ m/s^2$	***(Acceleration due to gravity)***
$1\ eV = 1.6 \times 10^{-19}\ J$	***(Conversion from eV to J)***
$m_e = 9.11 \times 10^{-31}\ kg$	***(Electron mass)***
$m_p = 1.67 \times 10^{-27}\ kg$	***(Proton mass)***

Prefixes of units

$m = 10^{-3}$	$\mu = 10^{-6}$	$n = 10^{-9}$	$p = 10^{-12}$
$k = 10^{3}$	$M = 10^{6}$	$G = 10^{9}$	$T = 10^{12}$

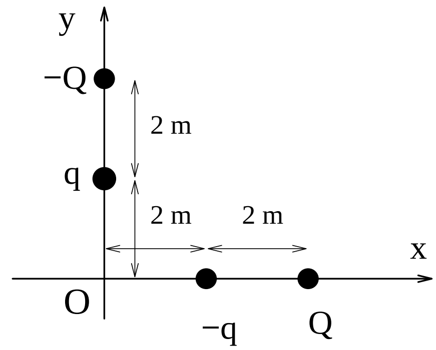
For use by Instructors only

	1	2	3	4	5	6	7	8	9	10	Total
Prob.											
Ques.											

The credit points for each problem are indicated in parentheses. The conceptual questions carry 1 point each.

PART I. Solve the following problems. Show your solution in detail.

1. Four point charges are placed in the xy-plane as shown. Q = 8 μC and q = 1 μC. Find the direction of the net electric field at O relative to the positive x-axis. **[3 points]**

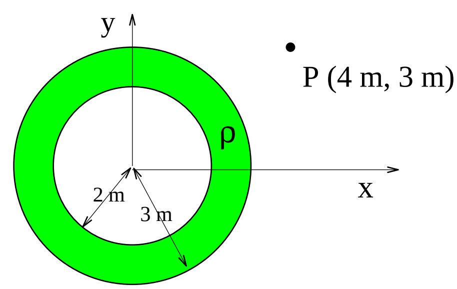


$$E_x = +\frac{kq}{(2m)^2} - \frac{kQ}{(4m)^2}$$

$$E_y = -\frac{kq}{(2m)^2} + \frac{kQ}{(4m)^2}$$

$$\Theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{-\frac{1}{4}+\frac{1}{2}}{+\frac{1}{4}-\frac{1}{2}}\right) = 135^o$$

2. A plastic spherical shell (R_{in} = 2 m, R_{out} = 3 m) centered at the origin has uniform volume charge density ρ. The magnitude of electric field at point P(4 m, 3 m) is E = 16 N/C. What is the value of ρ? **[3 points]**



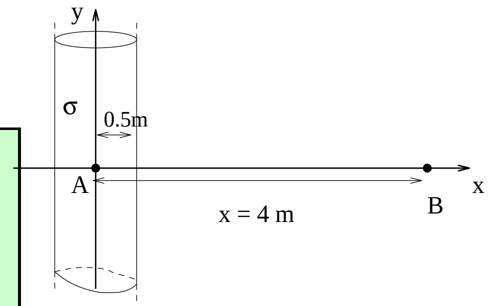
$$\overbrace{\Phi_E = \oint \overline{E}\cdot d\overline{A} = E4\pi r^2}^{\text{due to the symmetry}} = \frac{Q_{enc}}{\varepsilon_0}$$

$$with \quad r = \sqrt{(3m)^2 + (4m)^2} = 5m$$

$$Q_{enc} = \rho\frac{4}{3}\pi\left[(3m)^3 - (2m)^3\right]$$

$$so \qquad \rho = \frac{\varepsilon_0 E 4\pi (5m)^2}{\frac{4}{3}\pi[(3m)^3 - (2m)^3]} = 5.6\times 10^{-10}\, C/m^3$$

3. An infinitely long metal cylinder (R = 0.5 m), with its axis along the y-axis, has uniform surface charge density σ = 4 μC/m². Calculate the potential difference V_A-V_B between points A (x = 0 m) and B (x = 4 m) along the x-axis. **[4 points]**

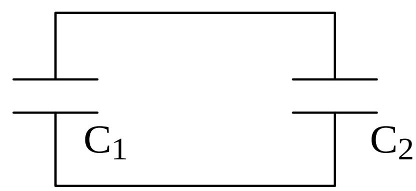


$$\overbrace{\Phi_E = \oint \overline{E}\cdot d\overline{A} = E2\pi rL}^{\text{due to the symmetry}} = \frac{Q_{enc}}{\varepsilon_0} = \frac{\sigma 2\pi(0.5m)L}{\varepsilon_0}$$

$$so \qquad E = \frac{\sigma 0.5m}{\varepsilon_0 r} \qquad and$$

$$\underbrace{V_A - V_B = \int Edr = \int_{0.5m}^{4m} \frac{\sigma 0.5m}{\varepsilon_0 r}dr}= \underbrace{\frac{\sigma 0.5m}{\varepsilon_0}\int_{0.5m}^{4m} \ln\left[\frac{4m}{0.5m}\right] = 4.7\times 10^5 V}$$

4. A capacitor C_1 = 4 μF is charged to a potential difference V_0 and then connected to an uncharged capacitor C_2 = 2 μF, as shown. The final energy stored in C_2 is 4 μJ. What was the original energy stored in C_1 before it was connected to C_2? **[3 points]**



$$U_2 = \frac{C_1 V^2}{2} \Rightarrow\Rightarrow V = 2\ V$$

The charge is conserved so

$$Q_0 = C_{eq}V = 6\mu F \cdot 2V = 12\mu C$$

$$U_0 = \frac{Q_0^2}{2C_1} = \frac{(12\mu C)^2}{2\cdot 4\mu F} = 18\mu J$$

5. Two wires A and B are made of the same copper material. Wire A has twice the diameter and half the length of wire B and both are connected to the same potential difference. The energy dissipated in wire A is 16 J per second. What is the rate of energy dissipation in wire B? **[2 points]**

$$R_A = \frac{\rho L_B / 2}{A_B 4} = \frac{R_B}{8} \Rightarrow\Rightarrow R_B = 8R_A$$

$$P_B = \frac{V^2}{R_B} = \frac{V^2}{8R_A} = \frac{P_A}{8} = 2\ W$$

6. A current I = 2 A flows in the right section of the circuit, as shown. What is the current supplied by the battery? **[4 points]**

From the loop rule in the right:

$$RI = 8\Omega I_{8\Omega} = 6\Omega 2A \Rightarrow\Rightarrow I_{8\Omega} = 1.5A$$

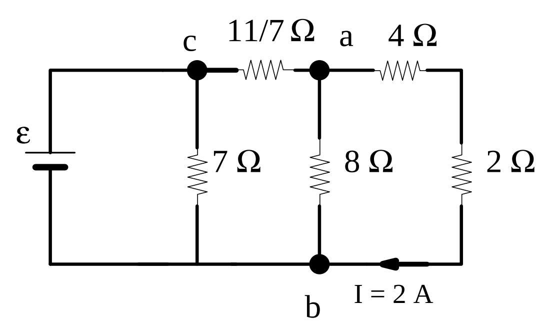
$$I_{ab} = 2A + 1.5A = 3.5A$$

$$R_{ab} = \frac{24}{7}\Omega; \qquad R_{ab} + \frac{11}{7}\Omega = 5\Omega = R_{cb-right}$$

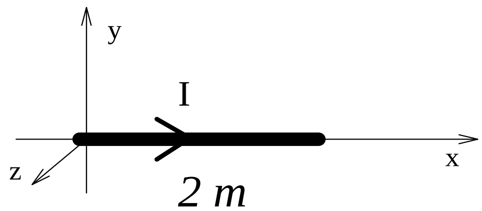
From the middle loop with $R_{cb-right}$ in the right

$$7\Omega I_{7\Omega} = 5\Omega 3.5A \Rightarrow I_{7\Omega} = (5/7)3.5A = 2.5A$$

$$I_{battery} = 2.5A + 3.5A = 6A$$

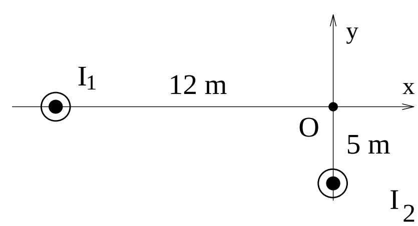


7. A 2m-long wire carrying current I = 3 A is placed along the x-axis in a magnetic field given by $\overline{B} = (2.6\hat{i} + 4.4\hat{j} + 5.3\hat{k})T$. Calculate the force acting on the wire in terms of the unit vectors. **[3 points]**



$$\overline{F} = I\overline{L} \times \overline{B}$$
$$\overline{F} = I2m\hat{i} \times (2.6\hat{i} + 4.4\hat{j} + 5.3\hat{k})T$$
$$\Downarrow$$
$$\overline{F} = \underbrace{-31.8\hat{j}N}\ \underbrace{+26.4\hat{k}N}$$

8. Two infinitely long wires carrying currents $I_1 = 2$ A, $I_2 = 3$ A are perpendicular to the page and pointing outward, as shown. Find the magnitude of the net magnetic field at point O.
[2 points]



$$B_{1x} = 0$$
$$B_1 y = \frac{\mu_0 I_1}{2\pi 12m}$$
$$B_{2x} = -\frac{\mu_0 I_2}{2\pi 5m}$$
$$B_{2y} =$$
so
$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi}\sqrt{\left(\frac{2}{12}\right)^2 + \left(\frac{3}{5}\right)^2} = 1.24 \times 10^{-7}\,T$$

9. An infinitely long hollow cylinder (R_{in} = 0.3 m, R_{out} = 0.7 m), placed along the y-axis, carries a current that has uniform density J = 2.4 A/m^2. A circular loop (R = 0.2 m) lying in the xy-plane with its center at point P (2 m, 0 m) has current I flowing as shown. The magnetic field at the center of the circular loop is 0. Find the value of **I**. **[4 points]**

From Amperes law due to cylindrical symmetry:

$$\oint B_{cyl} dl = B_{cyl} 2\pi r = \mu_0 I_{enc} = \mu_0 J\pi[0.7^2 - 0.3^2]$$

so $$B_{cyl} = \frac{\mu_0 J\pi[0.7^2 - 0.3^2]}{2\pi 2m}$$

$$B_{circle} = \frac{\mu_0 I}{2 \cdot 0.2m}$$

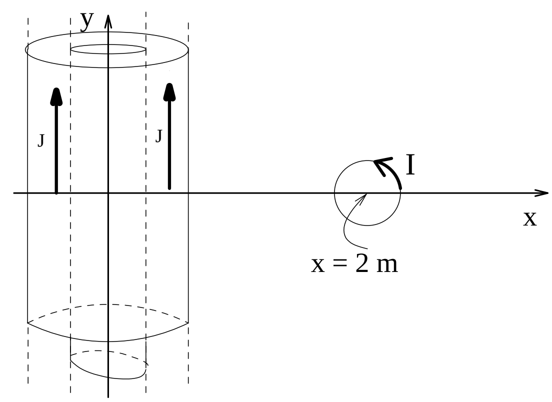
At the center of the circle

$$|B_{cylinder}| = |B_{circle}|$$

$$\Downarrow$$

$$\frac{\mu_0 \pi[0.7^2 - 0.3^2]J}{2\pi 2m} = \frac{\mu_0 I_{circle}}{2a} \Rightarrow\Rightarrow$$

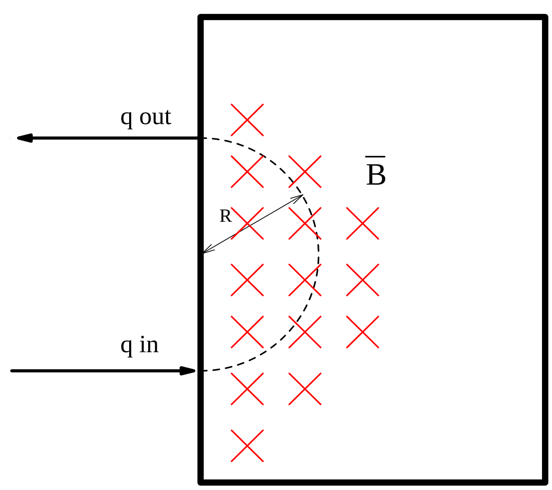
$$I_{circle} = \frac{a[0.7^2 - 0.3^2]J}{2} = 9.6\times 10^{-2}\,A$$



10. A particle of mass m= 5×10^{-5} kg and charge q = 5 mC enters a region of uniform magnetic field (B = 2 T) and leaves it with a speed of 10 m/s after completing a half-circle path in the magnetic field. What is the time spent by the particle in the magnetic field? **[2 points]**

$$t = T/2$$

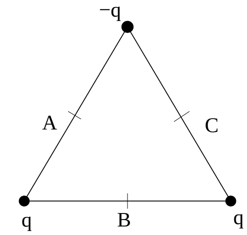
$$t = \frac{2\pi}{2\omega} = \frac{\pi m}{qB} = 1.57\times 10^{-2}\,s$$



Part II. Conceptual questions. Tick the best answer.

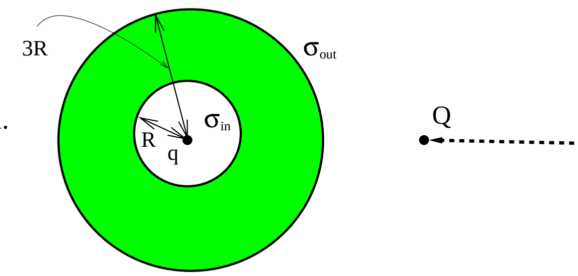
1. Three point charges are fixed at the vertices of an equilateral triangle, as shown. Points A, B, and C are midpoints of the sides. At which midpoint is the net electric field the smallest?

a. A
b. B
c. C
d. All have the same electric field.



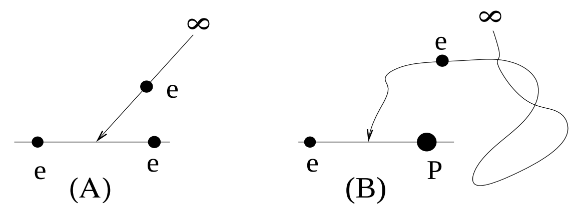
2. A metal spherical shell with inner radius R outer radius 3R has a point charge q at its center as shown. The inner and outer surface charge densities (σ_{in} and σ_{out}) are uniform. Then a point charge Q (= 2q) is placed at a distance 5R from the center of the shell.

a. σ_{in} will change to become non-uniform.
b. σ_{out} will change to become non-uniform.
c. Both σ_{in} and σ_{out} will change to become non-uniform.
d. Not enough information to decide.



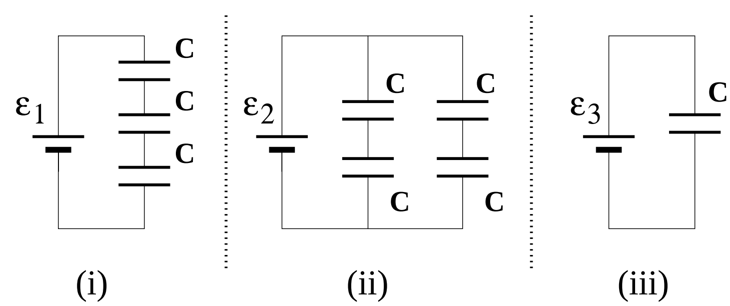
3. In two separate situations we move an electron from an infinitely far point to a point midway between two charges (either proton or electron). What is the sign of the work done by the electric field on the electron?

a. A negative, B positive.
b. A positive, B zero.
c. A negative, B zero.
d. A zero, B negative.



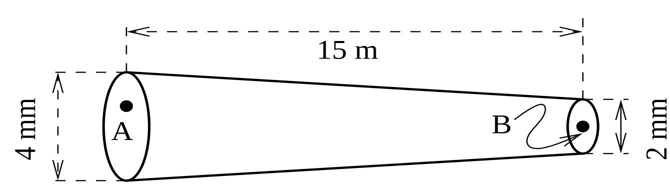
4. In three separate circuits identical capacitors C are fully charged with different batteries and have identical charges Q on each of the capacitors. Rank the batteries according their emfs.

a. $\mathcal{E}_1 < \mathcal{E}_2 < \mathcal{E}_3$
b. $\mathcal{E}_3 < \mathcal{E}_2 < \mathcal{E}_1$
c. $\mathcal{E}_2 < \mathcal{E}_1 < \mathcal{E}_3$
d. $\mathcal{E}_1 < \mathcal{E}_3 < \mathcal{E}_2$

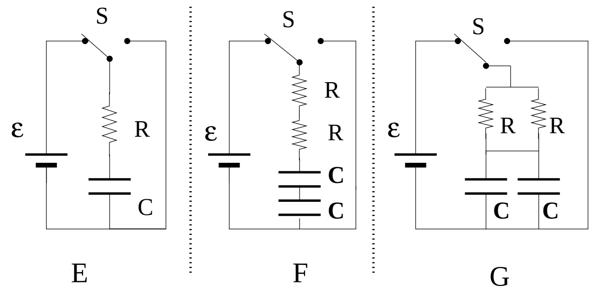


5. The diameter of a 15m-long copper wire smoothly and linearly changes from 4 mm (point A) to 2mm (point B), as shown. The wire carries 2 A current. The electric field at the point A is E_A and at point B is E_2. The relation between E_1 and E_2 is the following

a. $E_A = E_B/4$
b. $E_A = E_B/2$
c. $E_A = 2E_B$
d. $E_A = 4E_B$

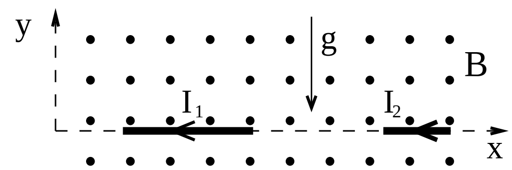


6. Identical resistors, R = 1 kΩ, capacitors, C = 4 μF, and batteries, ε = 10 V, are connected as shown in three circuits. Each of the switches are thrown to the left for a long time, then at t = 0 the switches are thrown to the right. In which circuit is the current the smallest at t = 4ms?



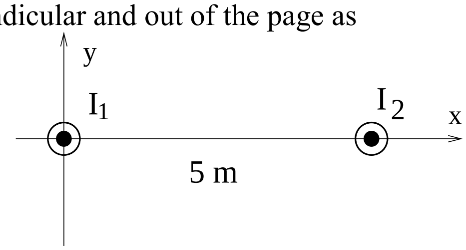
a. E
b. F
c. G
d. All have the same current.

7. In uniform magnetic and gravitational field, a wire that is carrying a current I_1, has a length L_1, mass m_1 is in equilibrium, as shown. A second wire in the same fields with $L_2 = L_1/2$ and with mass $m_2 = 2m_1$ is also in equilibrium. What is the current I_2?



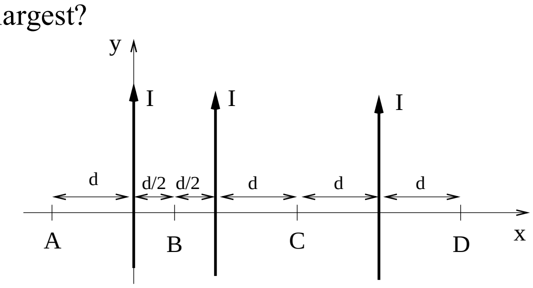
a. $I_2 = I_1$
b. $I_2 = 2I_1$
c. $I_2 = 4I_1$
d. $I_2 = 8I_1$

8. Two identical currents $I_1 = I_2$ are flowing in wires perpendicular and out of the page as shown. The magnetic force acting on wire 2 is



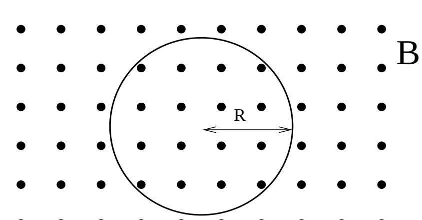
a. along the positive x-axis.
b. along the positive y-axis.
c. along the negative x-axis.
d. along the negative y-axis.

9. Three identical currents are flowing in wires parallel with the y-axis, as shown. At which of the points A, B, C, D is the net magnetic field the largest?



a. A
b. B
c. C
d. D

10. A sphere of radius R is placed in a uniform magnetic field, as shown. What is the flux of the magnetic field on the closed surface of the sphere?



a. $B\pi R^2$
b. $B2\pi R$
c. $B4\pi R^2$
d. 0