Kuwait University **Department of Physics**

Physics 102

First Midterm Examination (Summer Semester, 2007-2008)

July 12, 2007
Time: 11:00 AM to 12:30 PM

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Name: ....Solution.............. **Student No.:** ....

Fundamental Constants

$k = \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N·m^2/C^2	Coulomb's constant
$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/(N·m^2)	Permitivity of free space
$\mu_0 = 4\pi \times 10^{-7}$ T·m/A	Permeability of free space
$e = 1.6 \times 10^{-19}$ C	Elementary unit of charge
$m_e = 9.11 \times 10^{-31}$ kg	Mass of an electron
$m_p = 1.67 \times 10^{-27}$ kg	Mass of a proton
1 eV = 1.6×10^{-19} J	Conversion from eV to J
$N_A = 6.022 \times 10^{23}$ /mol	Avogadro's number
$g = 9.8$ m/s^2	Acceleration due to gravity

Prefixes of units

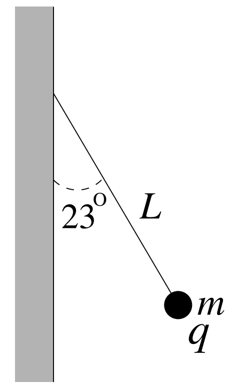
m = 10^{-3}	$\mu = 10^{-6}$	n = 10^{-9}	p = 10^{-12}
k = 10^3	M = 10^6	G = 10^9	T = 10^{12}

For use by instructors only

Problems	1	2	3	4	5	6	7	8	Questions	Total
Marks										

Part I: Solve the following problems

1. A tiny charged particle ($q = -20.0$ nC, $m = 0.2$ gram) hangs from an insulating thread of length $L = 50.0$ cm that makes an angle of 23^o with a vertical large uniformly charged sheet of surface charge density σ. Determine the surface charge density σ. **4 points**

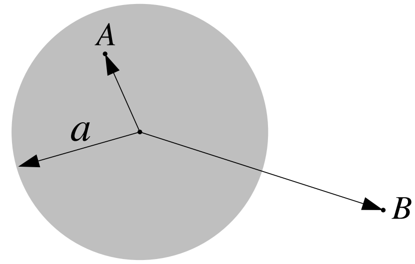


Solution: It is clear that $\sigma < 0$. At the equilibrium,

$$T\cos\theta = mg \qquad T\sin\theta = |qE| = \frac{|q\sigma|}{2\varepsilon_0} \implies \frac{|q\sigma|}{2\varepsilon_0(mg)} = \tan\theta$$

$$\implies |\sigma| = \frac{2\varepsilon_0 mg\ \tan\theta}{|q|} = 7.4\times10^{-7} \implies \sigma = -7.4\times10^{-7}\ \mathrm{C/m^2}$$

2. A spherical region of radius $a = 5.0$ cm has a uniformly distributed charge Q. The points A and B respectively are $r_A = 2.0$ cm and $r_B = 7.0$ cm away from the centre of the sphere. If $(V_A - V_B) = -1.8$ kV, determine the magnitude and sign of the charge Q. **2 points**



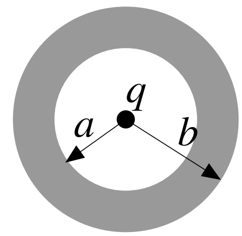
Solution: The potentials are A and B are given by

$$V_A = \frac{kQ}{2a}\left(3 - \frac{r_A^2}{a^2}\right) \qquad V_B = \frac{kQ}{r_B}$$

So

$$Q\left[\frac{3k}{2a} - \frac{kr_A^2}{2a^3} - \frac{k}{r_B}\right] = -1800 \implies Q = -1.4\times10^{-8}\ \mathrm{C}$$

3. A **conducting** shell of inner radius $a = 4.0$ cm and outer radius $b = 7.0$ cm has a net charge of $Q = 6.0$ nC. When a point charge q is placed at its centre, the electric potential at a distance of $r = 9.0$ cm from the centre is -300.0 V (taking $V = 0$ at $r \to \infty$). Determine the charge density on the **inner** surface of the shell. **3 points**



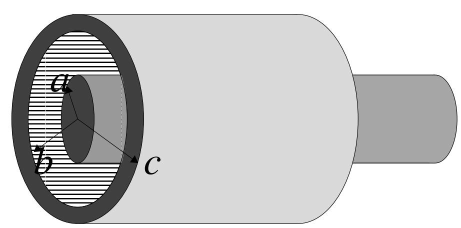
Solution: To satisfy Gauss' law, the charge on the inner surface of the shell must be $Q_{in} = -q$ and the charge on the outer surface of the shell is $Q_{out} = Q + q$. So

$$\frac{k(Q+q)}{0.09} = -300.0 \implies Q + q = -3.0 \times 10^{-9} \implies q = -9.0 \times 10^{-9}$$

So the charge density on the inner surface is

$$\sigma_{in} = \frac{-q}{4\pi a^2} = +4.5 \times 10^{-7}\ \mathrm{C/m^2}$$

4. A very long insulating cylinder of radius $a = 2.0$ cm is coaxial with another very long insulating hollow cylinder of inner radius $b = 4.0$ cm and outer radius $c = 7.0$ cm. The inner cylinder has a volume charge density $\rho_1 = -60.0\ \mathrm{nC/m^3}$ and the outer cylinder has a volume charge density $\rho_2 = 60.0\ \mathrm{nC/m^3}$. Find the magnitude of the electric field at a distance of 5.0 cm from the common axis. **3 points**



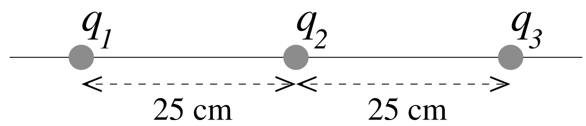
Solution: We have a cylindrical symmetry. So we choose a cylindrical Gaussian surface of radius r and length L. Then

$$\Phi_E = (2\pi r L)\, E \qquad Q_{enc} = \left[\pi a^2 L \rho_1 + \pi\left(r^2 - b^2\right) L \rho_2\right]$$

Using Gauss' law

$$E = \frac{\left|a^2\rho_1 + (r^2 - b^2)\,\rho_2\right|}{2\varepsilon_0 r} = 33.9\ \mathrm{N/C}$$

5. Three identical charged particles with charges, $q_1 = q_2 = q_3 = 20.0\ \mu$C, and masses $m = 0.5$ grams lie on a straight line as shown. They are released from rest at their positions. What will be the speed of q_1 when it is very far away from the other two particles? **3 points**



Solution: The net force on q_2 is zero and it will remain so all the time, so it is not going to move. Only q_1 and q_3 will move and they will have equal speeds. So

$$\Delta K = -\Delta U \implies 2 \times \left(\frac{1}{2}mv^2\right) - 0 = -\left[0 - \left(\frac{kq_1q_2}{0.25} + \frac{kq_2q_3}{0.25} + \frac{kq_1q_3}{0.5}\right)\right]$$

$$\implies mv^2 = 36.0 \implies v = 268.3 \text{ m/s}$$

6. Two isolated spherical conductors of radii $R_1 = 5.0$ cm and $R_2 = 7.0$ cm carry charges $Q_1 = 50.0$ nC and $Q_2 = -20.0$ nC respectively. If the two spheres are connected by a thin long conducting wire, how many electrons will be gained or lost by the sphere of radius R_1? **4 points**

Solution: Let the new charges be Q_1' and Q_2' respectively. The two spheres will have equal potential, so

$$\frac{kQ_1'}{R_1} = \frac{kQ_2'}{R_2} \implies Q_2' = \frac{Q_1'R_2}{R_1}$$

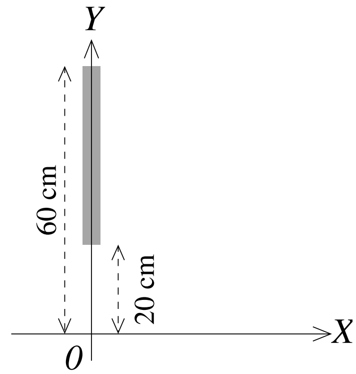
and the total charge

$$Q_1' + Q_2' = Q_1 + Q_2 \implies Q_1'\left(1 + \frac{R_2}{R_1}\right) = 30.0 \times 10^{-9} \implies Q_1' = 12.5 \times 10^{-9} \text{ C}$$

$$\implies \Delta Q_1 = -37.5 \times 10^{-9} \text{ C} \implies N_e = \frac{\Delta Q_1}{-e} = 2.3 \times 10^{11}$$

So the sphere 1 has **gained** 2.3×10^{11} electrons.

7. A −50.0 nC charge is distributed uniformly along the y-axis, from $y = 20.0$ cm to $y = 60.0$ cm. Calculate the electric field vector at the origin. **3 points**



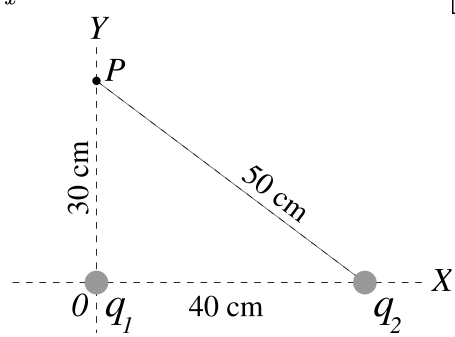
Solution: We choose a charge element $dq = (\lambda\, dy)$ at a distance of y from the origin, where λ is the linear charge density. The electric field due to this charge element is

$$d\vec{\boldsymbol{E}} = \frac{k|\lambda|\, dy}{y^2}\,\hat{\boldsymbol{j}}$$

The electric field due to the whole of the charge is then

$$\begin{aligned}\vec{\boldsymbol{E}} &= k|\lambda| \int_{0.2}^{0.6} \frac{dy}{y^2}\,\hat{\boldsymbol{j}} = k\left|\frac{-50\times 10^{-9}}{0.4}\right| \left[\frac{-1}{y}\right]_{0.2}^{0.6} \hat{\boldsymbol{j}} \\ &= k\left(\frac{50\times 10^{-9}}{0.4}\right)\left(\frac{1}{0.2} - \frac{1}{0.6}\right)\hat{\boldsymbol{j}} = 3.75\times 10^{3}\,\hat{\boldsymbol{j}}\ \mathrm{N/C}\end{aligned}$$

8. Two point charges, q_1 and q_2, are placed on the x-axis as shown below. The charge $q_1 = -2.0$ nC but q_2 is not known. The y-component of the net electric field at the point P is $E_y = 340.0$ N/C. Find its x-component E_x at P. **4 points**



Solution: The net electric field at P is

$$\vec{\boldsymbol{E}} = \frac{kq_1}{0.3^2}\,\hat{\boldsymbol{j}} + \frac{kq_2}{0.5^2}\left(-\frac{0.4}{0.5}\,\hat{\boldsymbol{\imath}} + \frac{0.3}{0.5}\,\hat{\boldsymbol{j}}\right) = -\frac{kq_2}{0.5^2}\,\frac{0.4}{0.5}\,\hat{\boldsymbol{\imath}} + \left(\frac{kq_1}{0.3^2} + \frac{kq_2}{0.5^2}\,\frac{0.3}{0.5}\right)\hat{\boldsymbol{j}}$$

The y-component is

$$\frac{kq_1}{0.3^2} + \frac{kq_2}{0.5^2}\,\frac{0.3}{0.5} = 340.0 \implies \frac{kq_2}{0.5^2}\,\frac{0.3}{0.5} = 340.0 - \frac{kq_1}{0.3^2} = 540.0 \implies q_2 = 2.5\times 10^{-8}\ \mathrm{C}$$

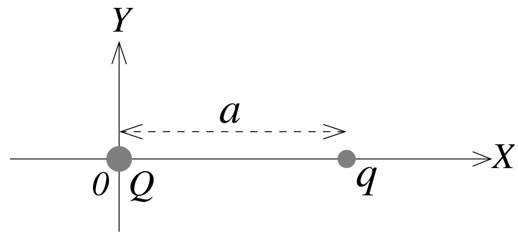
The x-component is

$$E_x = -\frac{kq_2}{0.5^2}\,\frac{0.4}{0.5} = -720.0\ \mathrm{N/C}$$

Part II: Conceptual Questions

In the following, tick ($\surd$) the best answer. Each question carries 1 point.

1. Two point charges, Q (> 0) and q (< 0) with $Q > |q|$ are placed on the x-axis as shown. The point(s) at which the net electric field $\vec{E}$ is zero



   (a) must be on the x-axis at $x > a$.
   (b) must be on the x-axis at $x < 0$.
   (c) must be on the x-axis at $0 < x < a$.
   (d) could be anywhere in the xy-plane.

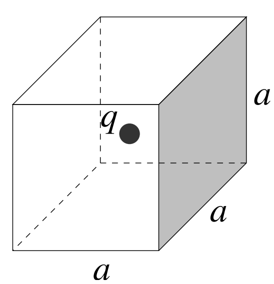
2. When a negatively charged rod is brought near tiny pieces of paper, the paper pieces are initially attracted towards the rod, then are repelled away after touching the rod. We may conclude that, before touching the rod the paper pieces were

   (a) positively charged.
   (b) neutral.
   (c) either positively charged or neutral.
   (d) either negatively charged or neutral.

3. The total electric flux through a closed surface is zero. Then we conclude that,

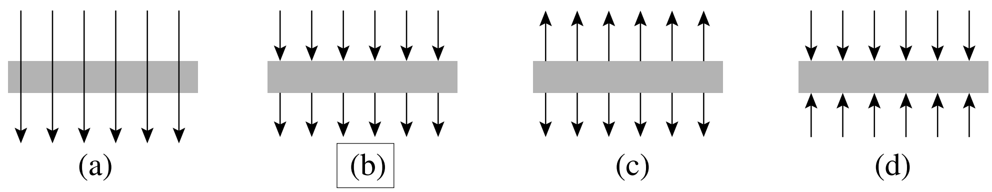
   (a) the electric field at all points on the surface must be zero.
   (b) the electric field at all points inside the surface must be zero.
   (c) the net charge in the volume bounded by the surface must be zero.
   (d) there must be no charge present inside the surface.

4. A point charge q is placed at the centre of a cube of side a. If the electric flux through the shaded face of the cube is $-\alpha$, the charge q is then



   (a) $\frac{-\alpha\varepsilon_0}{6}$.
   (b) $-6\alpha\varepsilon_0$.
   (c) $-\alpha\varepsilon_0$.
   (d) zero.

5. An electrically neutral large conducting slab is placed in a region of uniform electric field with its face perpendicular to the electric field. Which figure best represents the resulting electric field?



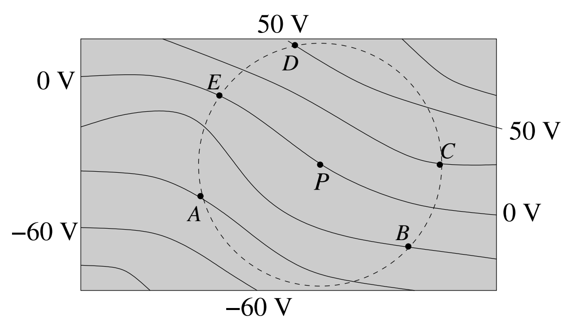
6. The solid lines in the figure represent equipotential lines in the shaded region. The dashed line is a circle centred at point P. A point charge q ($q > 0$) is taken from P to the points A, B, C, D and E in turn by an external agent. Which task among these five requires the external agent to do maximum work?

(a) Moving the charge from P to A.

(b) Moving the charge from P to B.

(c) Moving the charge from P to C.

(d) Moving the charge from P to D.



7. Two large plastic sheets with uniform charge densities σ_1 and σ_2 ($\sigma_1 \neq \sigma_2$) are parallel to each other. If the separation between the sheets is increased,

(a) the magnitude of the electric field between the plates will decrease.

(b) the magnitude of the electric field between the plates will increase.

(c) the potential difference between the plates will increase.

(d) the potential difference between the plates will decrease.

8. In a certain region of space the electric potential is given by

$$V = A + Bx + Cy^2 + Dxy$$

where A, B, C and D are positive constants. Then we conclude that

(a) $\vec{E}$ in this region has no z-component.

(b) $\vec{E} = 0$ at the origin.

(c) increasing the value of A will increase the magnitude of $\vec{E}$ in this region.

(d) increasing the value of A will decrease the magnitude of $\vec{E}$ in this region.