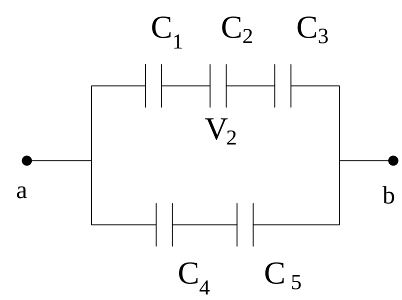
Problems - Show your solution in detail.

1. Find the potential difference between points a and b shown in the figure.
   Given $C_1 = 1\ \mu F$, $C_2 = 2\ \mu F$, $C_3 = 3\ \mu F$, $C_4 = 4\ \mu F$, $C_5 = 5\ \mu F$, and $V_2 = 12\ V$.

   3 points



$$Q_1 = Q_2 = Q_3 = C_2 V_2 = 24\mu C;$$

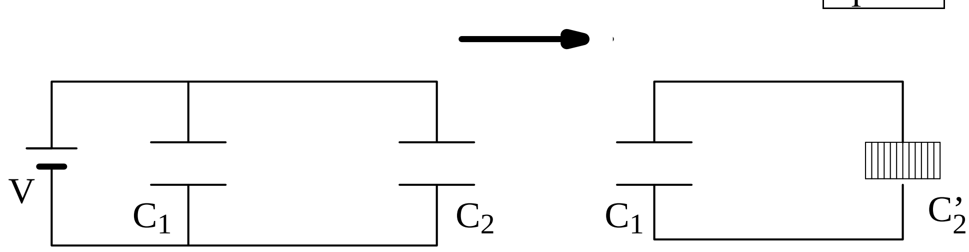
$$\Rightarrow V_1 = \frac{Q_1}{C_1} = \frac{24\mu C}{1\mu F} = 24\ V$$

$$\Rightarrow V_3 = \frac{Q_{31}}{C_3} = \frac{24\mu C}{3\mu F} = 8\ V$$

$$V = V_1 + V_2 + V_3 = 44\ V$$

2. Capacitors C_1 and C_2 are connected in parallel with a battery. The magnitude of plate-charge in C_2 (= $10\ \mu F$) is $Q_2 = 10\ \mu C$. The capacitance C_1 and the charge Q_1 are unknown. Then the battery is disconnected. After that a slab of dielectric material ($K = 4$) is inserted between the plates of C_2 and the charge in C_2' becomes $Q_2' = 30\ \mu C$. Find the capacitance C_1.

   4 points



$$Q_2 = \frac{C_2 Q_{total}}{C_1 + C_2};$$

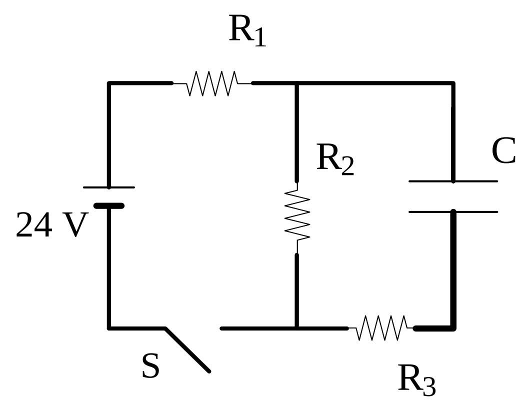
$$Q_2' = \frac{C'_2 Q_{total}}{C_1 + C'_2}$$

$$3Q_2 = Q_2' \Rightarrow \frac{3C_2}{C_1 + C_2} = \frac{C'_2}{C_1 + C'_2} \Rightarrow \quad \frac{3}{C_1 + C_2} = \frac{K}{C_1 + KC_2}$$

$$so \quad 3(C_1 + 4C_2) = 4(C_1 + C_2) \quad \Rightarrow \quad C_1 = 80\ \mu F$$

3. In the circuit shown, the switch S was closed for a very long time. Then at $t = 0$ the switch is opened. Find the energy stored in the capacitor $10\ ms$ after the switch is opened. Given $R_1 = R_2 = R_3 = 2\ k\Omega$, and $C = 5\ \mu F$.

3 points



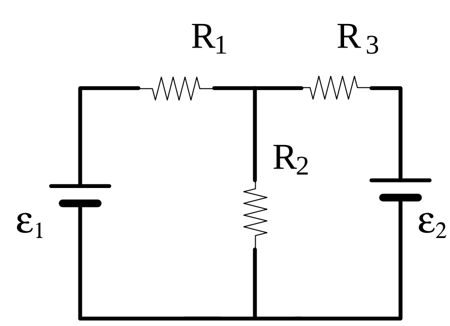
$$V_{C0} = 12\ V \Rightarrow Q_0 = 5\mu F 12V = 60\mu C$$

$$\tau = 2RC = 20\ \ ms \qquad Q(10ms) = Q_0 e^{-\frac{10ms}{20ms}} = 36\mu C$$

$$U(10ms) = \frac{Q^2(10ms)}{2C} = 132\mu J$$

4. In the circuit shown, $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 3\ \Omega$, $\mathcal{E}_1 = 4\ V$, and $\mathcal{E}_2 = 6V$. What is the rate of energy dissipation in the resistor R_1?

5 points



$$junction\ \ rule: \quad i_1 + i_3 = i_2$$

$$loop\ \ left: \quad 4V - i_1 R_1 - i_2 R_2 = 0$$

$$loop\ \ right: \quad i_2 R_2 + i_3 R_3 - 6V = 0$$

$$For\ \ finding\ \ i_1 = 0.73\ \ A$$

$$p = i^2 R = 0.53W$$

5. A copper wire of radius *2 mm* carries a current of *12 A*. The density of charge carriers is $8.5\times10^{28}\ m^{-3}$. Find the drift speed of the electrons in the wire.

2 points

$$J = \frac{I}{A} = \frac{12A}{\pi r^2} = 9.5\times10^5\ A/m^2$$

$$v_d = \frac{J}{nq} = 7\times10^{-5}\ m/s$$

6. At $45\ ^oC$ the resistance of a segment of a gold wire is *85* Ω. When the wire is placed in a liquid bath the resistance decreases to *80* Ω. What is the temperature of the liquid bath? Given $\alpha = 3.4\times10^{-3}\ (C^o)^{-1}$.

2 points

$$\frac{R}{R_0} = \frac{80}{85} = 1 + \alpha(T - T_0)$$

$$\Rightarrow T - T_0 = -17.3^o C \rightarrow\rightarrow T = 27.7^o C$$

7. A proton moves in a uniform magnetic field $\vec{B} = (\vec{i} + 2\vec{j} - \vec{k})\,T$. At the instant when its velocity is $\vec{v} = (2\vec{i} - 4\vec{j} + \vec{k})\,m/s$, what is the magnitude of the magnetic force on this proton?

3 points

By writing out the vector product:

$$v \times B = (2\hat{i} + 3\hat{j} + 8\hat{k})\ \ mT/s$$

$$|v \times B| = \sqrt{77}mT/s = 8.8\ \ mT/s$$

$$F = q\,|v \times B| = 1.4 \times 10^{-18}\ \ N$$

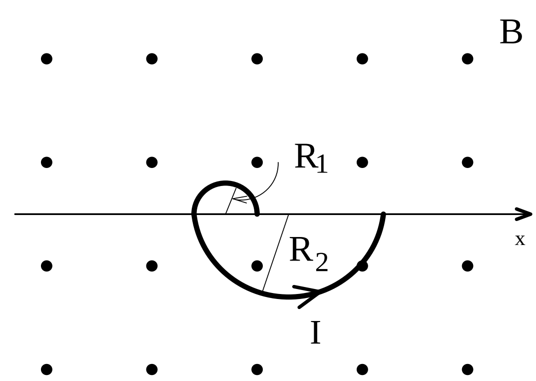
8. A wire consisting of two semicircular segments (with radii $R_1 = 0.3$ *and* $R_2 = 0.9\ m$), as shown in the figure, carries a current of I = 3A. The wire is in a uniform magnetic field $B = 2\ T$, out of the page. Find the magnitude of the net magnetic force acting on the wire.

3 points

$$F_1 = I2R_1B = 3.6\ \ N$$

$$F_2 = I2R_2B = 10.8\ \ N$$

$$F = F_2 - F_1 = 7.2\ \ N$$



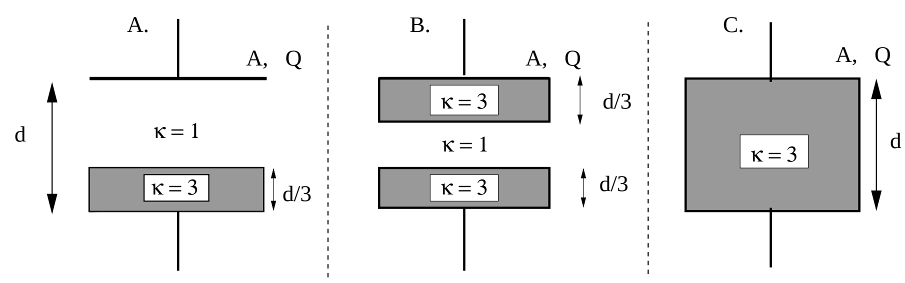
Conceptual Question – Tick the correct answer (One point each)

1. The electric field in a resistor connected to a battery is

a. inversely proportional to the resistance of the resistor.
b. proportional to the potential difference across the resistor.
c. inversely proportional to the current density.
d. none of the above.

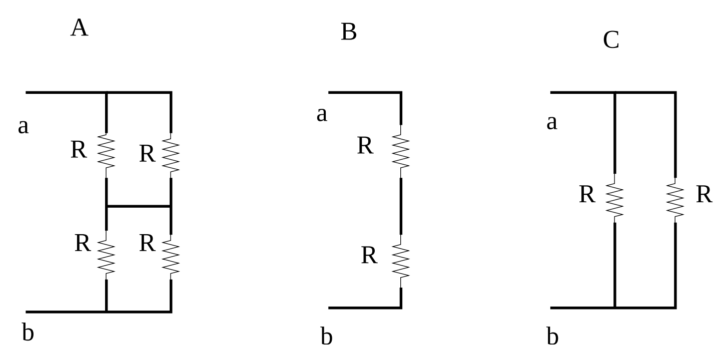
2. Three parallel-plate capacitors A, B, and C have the same geometrical parameters *A, d* and have equal plate-charge *Q*. Different fractions of the space between the plates are filled with a dielectric material (K=3) as shown in the figure. Rank the capacitors according to their potential differences. Smallest first.

a. A, B, C
b. C, B, A
c. B, C, A
d. A, C, B



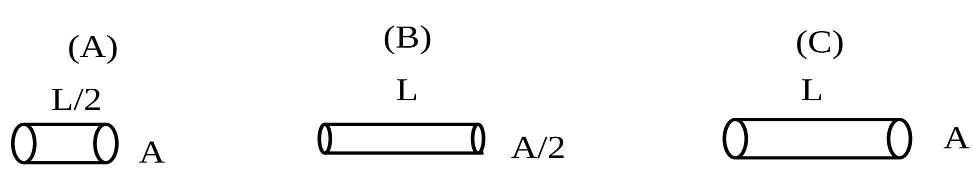
3. Rank the arrangement of resistors according to their equivalent resistances between the points a and b. Greatest first.

a. B. C. A.
b. A. C. B.
c. B. A. C.
d. A. B. C.



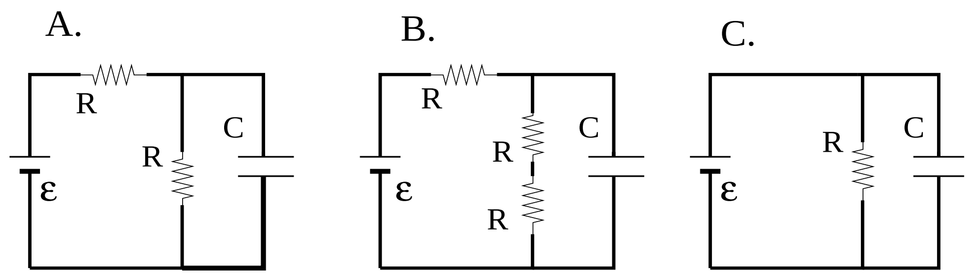
4. Three cylindrical copper wires with different lengths and cross-sectional areas are connected to the same potential differences. Rank the wires according to the power dissipated in them. Greatest first.

a. A, B, C,
b. B, C, A,
c. A, C, B,
d. C, B, A,



5. In the circuits shown, the resistances *R*, the capacitances *C* and the emfs **ε** have the same values. Rank the circuits according to the charges stored in the capacitor. Greatest first.

a. B. A. C.
b. B. C. A.
c. C. A. B.
d. C. B. A.

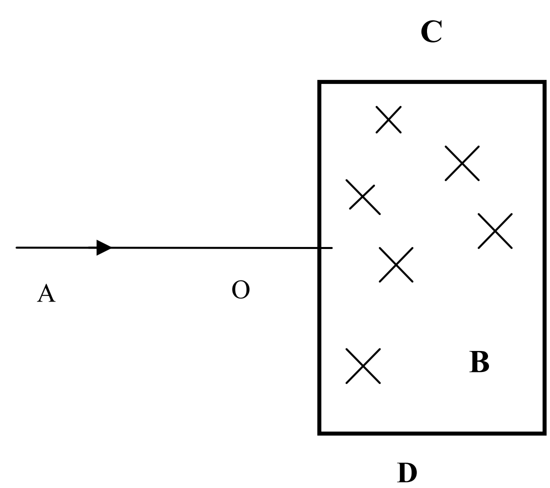


6. The temperature coefficient of resistivity of carbon is negative. Which statement is true for a resistor made of carbon at temperatures T_1 and T_2 ($T_1 < T_2$)?

a. $R(T_1) < R(T_2)$
b. $R(T_1) = R(T_2)$
c. $R(T_1) > R(T_2)$
d. There is not enough information to decide.

7. A positively charged particle projected into a very large a region of magnetic field **B**, as shown in the figure will be. The particle will

a) move straight without deflection.
b) be deflected perpendicular to the direction AO and will reach the point C.
c) be deflected perpendicular to the direction AO and will reach the point D.
d) make a U turn.



8. A wire having a mass M carries a current I in gravitational field, as shown in the figure. What is the direction of the magnetic field that can lift the wire up?

a. Along the +y axis.
b. Along the –y-axis.
c. Along the +z-axis.
d. Along the –z-axis.