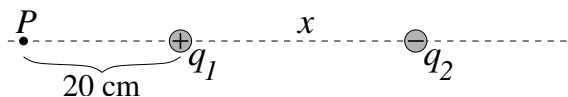


Part I: Solve the following problems

1. In the figure shown below, $q_1 = 8.0 \mu\text{C}$ and $q_2 = -16.0 \mu\text{C}$. The electric field at point P is zero. Find the distance between q_1 and q_2 .

2 points

- (a) 28 cm
 (b) 20 cm
 (c) 10 cm
 (d) 8 cm



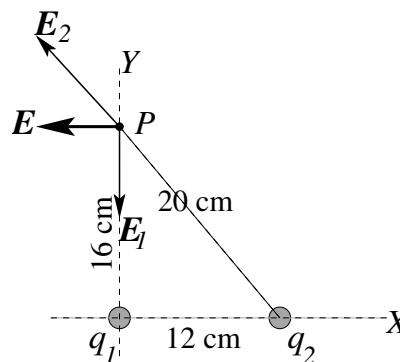
Solution: Let x be the distance between q_1 and q_2 . Then

$$\frac{k|q_1|}{(0.2)^2} = \frac{k|q_2|}{(0.2+x)^2} \implies \left(\frac{0.2+x}{0.2}\right)^2 = \frac{|q_2|}{|q_1|} \implies \frac{0.2+x}{0.2} = 1.414 \implies \boxed{x = 0.08 \text{ m}}$$

2. Two point charges, q_1 and q_2 , are placed on the X -axis as shown. The net electric field at point P on the Y -axis is $\vec{E} = -5.40 \hat{i} \text{ N/C}$. Find the charge q_1 .

4 points

- (a) 15.4 pC
 (b) 20.5 pC
 (c) -20.5 pC
 (d) -15.4 pC



Solution: The X -component of \vec{E} at P is due to q_2 only. So $\boxed{q_2 > 0}$. And

$$E = E_{2x} = \frac{kq_2}{(0.2)^2} \frac{0.12}{0.2} = 5.4 \implies \boxed{q_2 = 4.0 \times 10^{-11} \text{ C}}$$

The Y -components cancel, so $\boxed{q_1 < 0}$. And

$$\frac{kq_2}{(0.2)^2} \frac{0.16}{0.2} = \frac{k|q_1|}{(0.16)^2} \implies |q_1| = 2.048 \times 10^{-11} \implies \boxed{q_1 = -2.05 \times 10^{-11} \text{ C}}$$

3. An alpha particle (charge $q = +2e$, mass $m = 6.64 \times 10^{-27}$ kg) enters a uniform electric field with velocity $\vec{v}_0 = 2.50 \hat{i}$ km/s. Find the magnitude of the electric field if the velocity of the alpha particle is $\vec{v} = -1.50 \hat{i}$ km/s after $1.95 \mu\text{s}$.

2 points

- (a) 11 N/C
 (b) 43 N/C
 (c) 60 N/C
 (d) 85 N/C

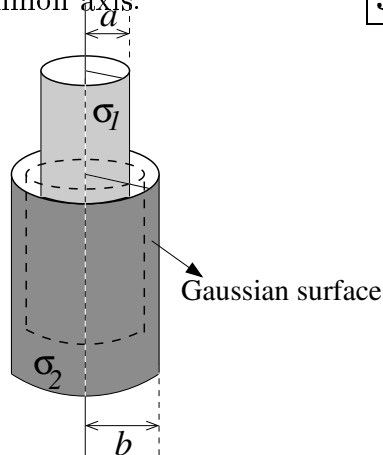
Solution:

$$\vec{v} = \vec{v}_0 + \vec{a}t \implies \vec{a} = \frac{\vec{v} - \vec{v}_0}{t} = -2.05 \times 10^9 \hat{i} \text{ m/s}^2 \implies \vec{E} = \frac{m\vec{a}}{q} = -42.6 \hat{i} \text{ N/C}$$

4. A very long hollow cylinder of radius $a = 1.0$ cm is coaxial with another very long hollow cylinder of radius $b = 3.0$ cm. The inner cylinder has a surface charge density $\sigma_1 = -6$ nC/m² and the outer cylinder has a surface charge density $\sigma_2 = 6$ nC/m². Find the electric field at a distance of 2.0 cm from the common axis.

3 points

- (a) 678 N/C, outward
 (b) 339 N/C, inward
 (c) 339 N/C, outward
 (d) zero



Solution: We have a cylindrical symmetry. Choose a cylindrical Gaussian surface of radius r and length L . Then

$$\Phi_E = (2\pi rL) E \quad Q_{enc} = (2\pi aL) \sigma_1 \quad (\sigma_2 \text{ is outside the Gaussian surface})$$

Using Gauss' law

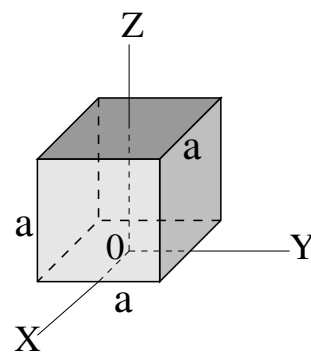
$$E = \frac{\sigma_1 a}{\epsilon_0 r} = -339 \text{ N/C}$$

Since the flux is negative, \mathbf{E} is inward.

5. Electric field in a region is given by $\vec{E} = 8.4x \hat{i}$ kN/C when x is expressed in metres. What is the electric flux through the rear face of the cubical surface shown in the figure? The cube has side $a = 15$ cm. The origin of the coordinate system is at the centre of the base of the cube.

3 points

- (a) $-14.2 \text{ N}\cdot\text{m}^2/\text{C}$
 (b) $14.2 \text{ N}\cdot\text{m}^2/\text{C}$
 (c) $28.4 \text{ N}\cdot\text{m}^2/\text{C}$
 (d) zero



Solution: For the rear face of the cube, $x = -a/2$ and the area vector $\vec{A} = -a^2 \hat{i}$.

$$\Phi_{rear} = [\vec{E} \cdot (-a^2 \hat{i})]_{x=-a/2} = [- (8.4 \times 10^3 x) a^2]_{x=-a/2} = 4.2 \times 10^3 a^3 = 14.2 \text{ N}\cdot\text{m}^2/\text{C}$$

6. The potential energy of a two-charge system when they are separated by a distance of 60 cm is -0.54 J. The charges are released at those positions. What will be the potential energy of the system after each charge has moved a distance of 10 cm?

2 points

- (a) -0.41 J
 (b) -0.81 J
 (c) -0.54 J
 (d) 0.54 J

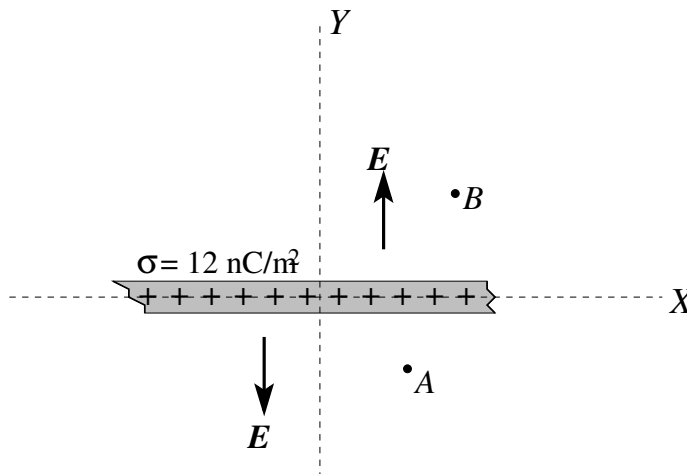
Solution: The potential energy is negative, so the charges are of **opposite** signs. So when released they will move **towards** each other. So,

$$U_i = \frac{kq_1q_2}{0.6} = -0.54 \implies q_1q_2 = -3.6 \times 10^{-11} \quad \text{So} \quad U_f = \frac{kq_1q_2}{0.4} = -0.81 \text{ J}$$

7. A large sheet of charge in the XZ -plane shown below has a uniform surface charge density $\sigma = 12.0 \text{ nC/m}^2$. (The Z -axis is perpendicular to the plane of paper.) Find $V_A - V_B$ for points A and B in the XY -plane with coordinates $x_A = 20 \text{ cm}$, $y_A = -10 \text{ cm}$ and $x_B = 30 \text{ cm}$, $y_B = 20 \text{ cm}$.

3 points

- (a) 203 V
 (b) 96 V
 (c) 68 V
 (d) -214 V



Solution: The electric field is along the negative y -direction below the sheet and in the positive y -direction above the sheet, and is uniform. Let V_0 be the potential on the surface of the sheet, then

$$V_A - V_0 = -\vec{E} \cdot [(x_A - 0)\hat{i} + (y_A - 0)\hat{j}] = \left(\frac{\sigma}{2\epsilon_0}\right) y_A$$

$$V_B - V_0 = -\vec{E} \cdot [(x_B - 0)\hat{i} + (y_B - 0)\hat{j}] = -\left(\frac{\sigma}{2\epsilon_0}\right) y_B$$

So

$$V_A - V_B = \left(\frac{\sigma}{2\epsilon_0}\right) (y_A + y_B) = 67.8 \text{ V}$$

8. The electric field at a distance of $r = 1.0 \text{ cm}$ from the centre of a uniformly charged non-conducting sphere of radius $a = 2.0 \text{ cm}$ is **inward** with a magnitude of 135 N/C . Find the electric potential at that point (assuming the potential to be zero at infinity).

3 points

- (a) -7.4 V
 (b) 7.4 V
 (c) 13.5 V
 (d) -13.5 V

Solution: The electric field is inward, so $Q < 0$. The magnitude is (since $r < a$)

$$E = \frac{k|Q|r}{a^3} = 135 \implies |Q| = 1.2 \times 10^{-11} \text{ C} \implies \boxed{Q = -1.2 \times 10^{-11} \text{ C}}$$

Electric potential inside a uniformly charged sphere is

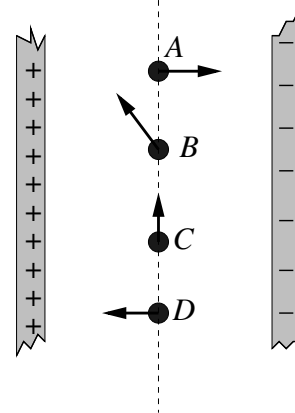
$$V = \frac{kQ}{2a} \left(3 - \frac{r^2}{a^2}\right) = -7.43 \text{ V}$$

Part II: Conceptual Questions

In the following questions tick () the best answer.

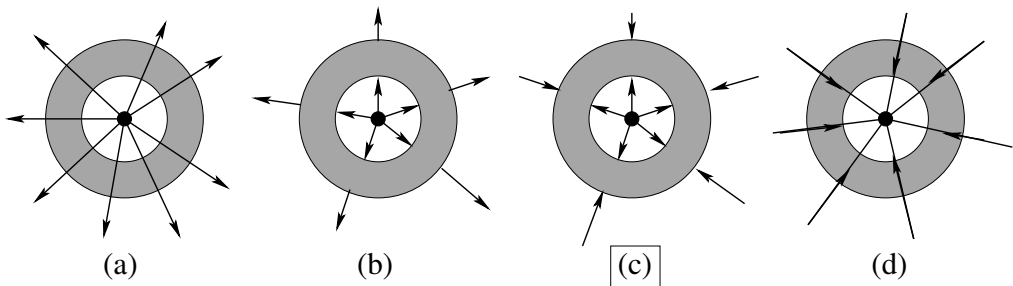
1. The two large parallel plates below are parallel to each other. Four **identical** positive charges are each launched with a speed v_0 from points half-way between the plates in different directions as shown. Assume that no charge touches the positive plate. Which charge will hit the negative plate with highest **speed**?

- (a) *A*
 (b) *C*
 (c) *D*
 (d) Speed will be the same for all the charges.



$$\Delta K = -\Delta U = qE \left(\frac{d}{2}\right) \text{ for all the charges.}$$

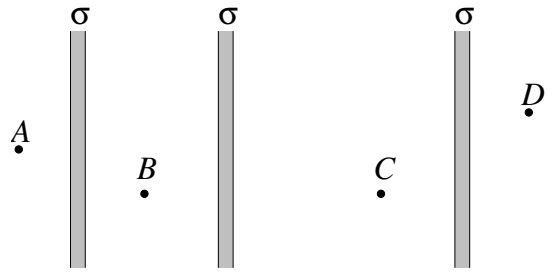
2. A point charge q ($q > 0$) is placed at the centre of a **conducting** spherical shell having a net charge $-2q$. The electric field lines will be as



The inner and outer surfaces of the shell have charges $-q$ each and electric field inside a conductor must be zero.

3. Three large plastic sheets with equal surface charge densities are parallel to each other. The magnitude of the net electric field is maximum at the point(s)

- (a) *B* only.
 (b) *A* and *D*.
 (c) *B* and *C*.
 (d) *A* only.



$$E_A = E_D = \frac{3\sigma}{2\epsilon_0}, E_B = E_C = \frac{\sigma}{2\epsilon_0}$$

4. Two conducting spheres with radii R_1 and R_2 (with $R_1 > R_2$) have charges Q_1 and Q_2 . When they are connected together by a thin conducting wire, the charges will flow between them until

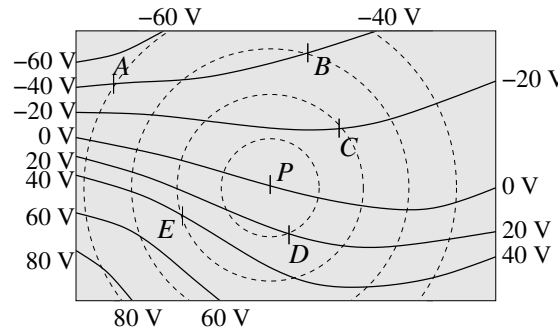
- (a) the charges on the two spheres are equal.
 (b) the charge on the smaller sphere becomes zero.
 (c) the electric potentials of the two spheres become equal.
 (d) the charge on the bigger sphere becomes zero.

Quite obvious.

5. The bold lines in the figure are equipotentials (labeled with values of the potentials). The dashed circles represent different distances from the centre P . A positive point charge is moved by an external force from point P to various points: P to A , P to B , P to C , P to D , and P to E . For which point(s) is the work done maximum?

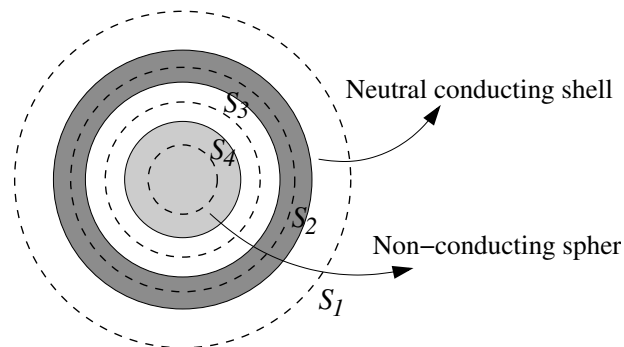
- (a) B
 (b) E
 (c) E and C
 (d) A

$$W_{ext} = \Delta U = q\Delta V$$



6. A positively charged non-conducting sphere with uniform charge density is concentric with a neutral conducting shell. Four spherical Gaussian surfaces (dashed circles) are shown. If Φ_1 , Φ_2 , Φ_3 and Φ_4 are the electric fluxes through these surfaces, then

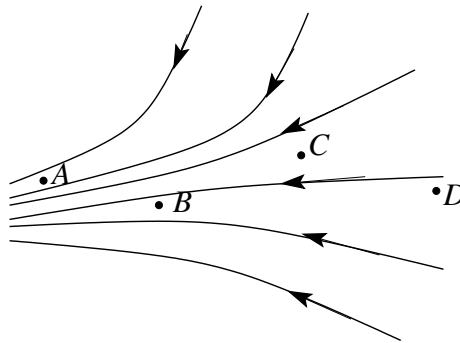
- (a) $\Phi_1 > \Phi_2 > \Phi_3 > \Phi_4$
 (b) $\Phi_1 = \Phi_2 = \Phi_3 > \Phi_4$
 (c) $\Phi_3 > \Phi_4 > \Phi_2 > \Phi_1$
 (d) $\Phi_1 = \Phi_3 > \Phi_4 > \Phi_2$



The inner and outer surfaces of the shell have charges $-q$ and q respectively and \vec{E} inside a conductor must be zero. So $\Phi_1 = \Phi_3 = q/\epsilon_0$, $\Phi_2 = 0$, $\Phi_4 = q'/\epsilon_0$ with $q' < q$

7. The electric field lines in a region are shown. At which of the point(s) among A , B , C and D is the electric potential highest?

- (a) A
 (b) B
 (c) C
 (d) D



Electric field lines are always from a higher potential region to a lower potential region.

8. Two point charges q_1 and q_2 , both positive, are located on the X -axis. If the net electric field at a point is zero, then the point

- (a) must be on the X -axis and between the two charges.
 (b) may lie anywhere in the XY -plane.
 (c) must be on the X -axis but not between the two charges.
 (d) must be on the X -axis but could be anywhere.

If the point is not on the X -axis then only one of the components of \vec{E} may become zero.