Problems - **Detailed solution is required.** ***(Solutions)***

1. Four point charges are located on the x-y-plane as shown in the figure. Given q = 1 nC, the x-component of the electric field at the origin is:

3 points

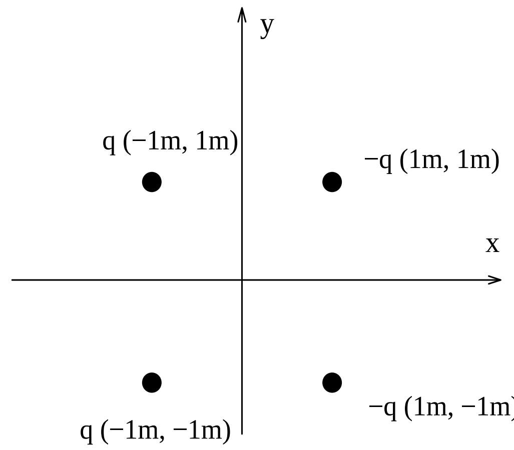
a) 18.0 N/C

b) 12.7 N/C

c) 6.4 N/C

d) 9.0 N/C

e) zero



$$E = \frac{kq}{2}$$

$$E_x = 4E\cos\theta = \frac{4kq}{2}\frac{\sqrt{2}}{2} = \mathbf{12{,}72\ N/C}$$

2. Two spheres of radii a and 2a (a < 1m) are centered on the x-axis at x = 0 m and at x = 6 m, as shown. The spheres have equal uniform surface charge densities. Where on the x-axis can a point charge Q be placed so that the net force on it is zero?

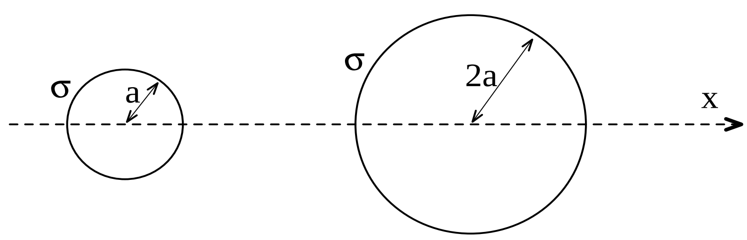
3 points

a) x = 3 m

b) x = –4 m

c) x = 8 m

d) x = 2 m



e) x = – 2m

$q_1 = \sigma 4\pi a^2$ $\qquad$ $q_2 = \sigma 4\pi (2a)^2 = 4q_1$

$$|E_1| = |E_2|$$

$$\frac{kq_1}{x^2} = \frac{4kq_1}{(6-x)^2}$$

$$\frac{1}{x^2} = \frac{4}{(6-x)^2} \qquad \Rightarrow x = 2m$$

3. Two rods with non-uniform charge densities are located along the x and y-axis as shown in the figure. The linear charge densities are $\lambda_1 = 3x^2$ nC/m and $\lambda_2 = -4y^2$ nC/m. The magnitude of the electric field at the origin is:

4 points

a) 90 N/C

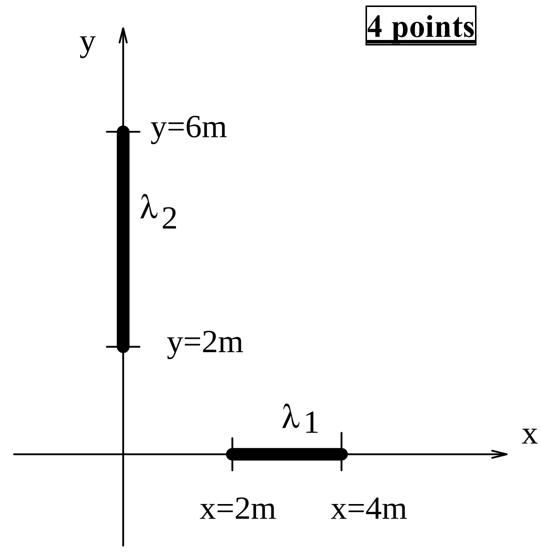
b) 132 N/C

c) 198 N/C

d) 154 N/C

e) 148 N/C

$$dE = \frac{k\lambda dr}{r^2}$$



$$dE_1 = \frac{k\lambda_1 dx}{x^2} = k3n\frac{C}{m}dx = 27 \ \ N/C \qquad E_1 = 27\int_2^4 dx = 27*2N/C = 54 \ \ N/C$$

$$dE_2 = \frac{k\lambda_2 dx}{y^2} = k4n\frac{C}{m}dx = 36 \ \ N/C \qquad E_2 = 36\int_2^6 dy = 36*4N/C = 144 \ \ N/C$$

$$E_{net} = \sqrt{E_1^2 + E_2^2} = 153.8 \ \ N/C$$

4. A sphere of radius 1m and surface charge density σ = 4 μC/m^2 is centered at the origin. The work done by the electric field on a point charge q = 1 μC that travels from point A to B is:

4 points

a) -77×10^{-3} J

$$r_A = \sqrt{2^2 + 3^2} = 3.6 \ \ m$$

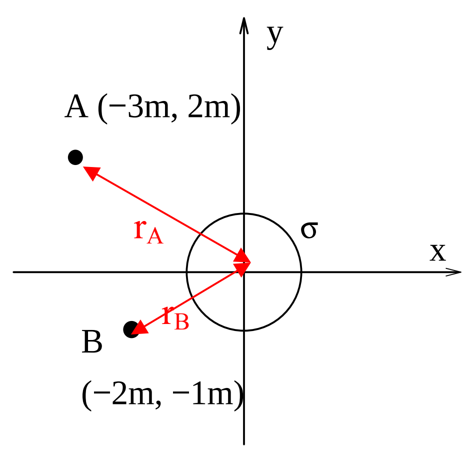
b) 77×10^{-3} J

$$r_B = \sqrt{2^2 + 1^2} = 2.24 \ \ m$$

c) 328×10^{-3} J

d) -328×10^{-3} J

e) zero

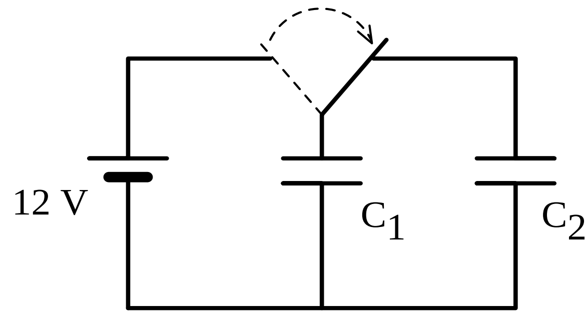


$$Q = \sigma 4\pi r^2 = 16\pi\times10^{-6}C$$

$$V_A = \frac{kQ}{r_A}; \qquad V_B = \frac{kQ}{r_B}$$

$$W_E = -q(V_B - V_A) = qkQ(\frac{1}{r_A} - \frac{1}{r_B}) = -77\times10^{-3}J$$

5. Capacitor C_1 is charged to a potential difference 12 V so that the energy stored in the capacitor is 432 μJ. Then the charging battery is disconnected and an uncharged capacitor of capacitance C_2 is connected to C_1. The energy stored in C_1 decreases to 48 μJ. What is the energy stored in the C_2 capacitor? **4 points**



a) 384 μJ

b) 64 μJ

c) 216 μJ

d) 126 μJ

e) 96 μJ

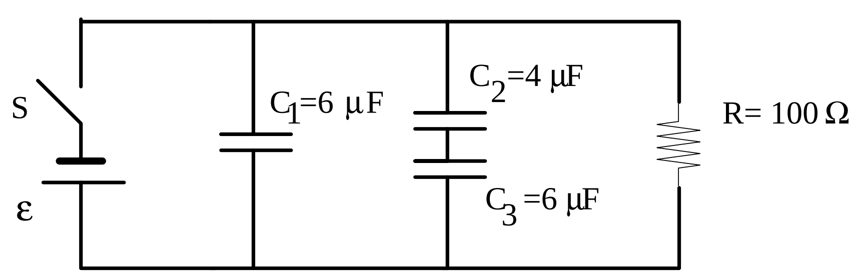
$$U_0 = \frac{1}{2} Q_0 V \Rightarrow\Rightarrow Q_0 = 72 \mu C \text{ and } C_1 = \frac{Q}{V} = 6 \ \mu C$$

$$\text{and } U_1 = \frac{1}{2C_1} q_1^2 \Rightarrow\Rightarrow q_1 = 24 \ \mu C \Rightarrow\Rightarrow V_1 = \frac{q_1}{C_1} = 4 \ V$$

$$q_2 = (72 - 24) \ \mu C$$

$$U_2 = \frac{1}{2} q_2 V_2 = \frac{1}{2} \times 48 \times 10^{-6} \times 4 = 96 \ \mu J$$

6. In the circuit shown below, the switch S is closed for a very long time. Energy stored in capacitor C_2 is 200 μJ. What is the rate of energy dissipation in the resistor R? **3 points**



a) 0.44 W

b) 1.66 W

c) 0.66 W

d) 2.77 W

e) 16.6 W

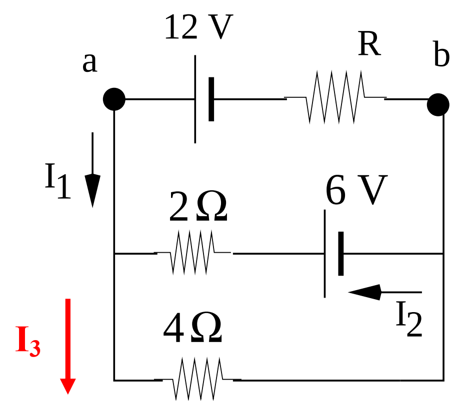
$$U_2 = \frac{1}{2C} q_{23}^2 \Rightarrow\Rightarrow q_{23} = 40 \ \mu C$$

$$\frac{1}{C_{23}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{6+4}{24} \Rightarrow C_{23} = 2.4 \ \mu F$$

$$V_{23} = \frac{Q_{23}}{C_{23}} = \frac{40}{2.4} \ V = 16.67 \ V$$

$$P = IV = \frac{V^2}{R} = 2.77 \ W$$

7. In the circuit shown $I_1 = I_2$. Calculate the potential difference V_a-V_b. **3 points**



a) 13.2 V

b) –4.8 V

c) 11.8 V

d) 4.8 V

e) 13.2 V

Junction rule $I_1 + I_2 = I_3 = 2I_1$

From the lower loop
$$\left.\begin{aligned} &6V - 2\Omega I_2 - 4\Omega(2I_2) = 0 \\ &6V - 10I_2 = 0 \Rightarrow\Rightarrow I_2 = 0.6\ A \end{aligned}\right\}$$

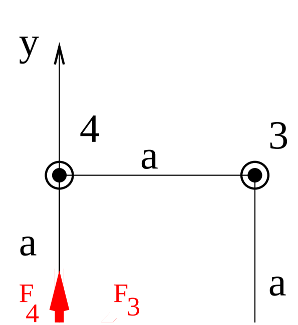
$$V_a - 2I_2 4 = V_b \Rightarrow\Rightarrow V_a - V_b = 4.8V$$

8. Four very long parallel wires are placed at the corners of a square with currents flowing either in or out as shown in the figure. If the magnitude of current in each wire is the same, the direction of the net magnetic force on wire 1 relative to the x-axis is **3 points**

a) 108°

b) 45°

c) 135°



d) 90°

e) 72°

$$|F_2| = |F_4| = \frac{\mu_0 I^2 L}{2\pi a}; \qquad |F_3| = \frac{\mu_0 I^2 L}{2\pi a\sqrt{2}};$$

$$\left.\begin{aligned} F_{net,x} &= F_3 \cos\theta - F_2 = F_3 \frac{1}{\sqrt{2}} - F_2 = -\frac{\mu_0 I^2 L}{4\pi a} \\ F_{net,y} &= F_3 \sin\theta + F_2 = F_3 \frac{1}{\sqrt{2}} + F_2 = \frac{3\mu_0 I^2 L}{4\pi a} \end{aligned}\right\}$$

$$\alpha = 180^0 - \tan^{-1}\left(\frac{F_{net,y}}{F_{net,x}}\right) = 180^o - 72^o = 108^o$$

9. A straight wire of length 10 m carries a current of 20 A along the z-axis in a region where the magnetic field is B = $(2\hat{i} - 8\hat{j} + \hat{k}) \times 10^{-4}$ T. What is the magnitude of magnetic force on the wire?

3 points

a) 2.6×10^{-1} N

b) 5.8×10^{-2} N

c) 1.7×10^{-1} N

d) 7.2×10^{-2} N

e) 9.4×10^{-1} N

$\overline{L} = 10\ \hat{k}$;

I= 20 A

$$\overline{F} = I\overline{L} \times \overline{B} = 0.02\overline{k} \times (2\hat{i} - 8\hat{j} + \hat{k})N = 0.02(-2\hat{j} + 8\hat{i})$$

$$F = 0.04\sqrt{1+4^2}\,N = 1.7\times10^{-1}\,N;$$

10. An electron of velocity **v** $= 2\times10^6\,\hat{i}\,\frac{m}{s}$ passes undeflected through a velocity selector with electric field E = – 200 $\hat{j}$ N/C. If the electric field is then switched off, the radius of the resulting circular path is: **2 points**

a) 0.1 cm

b) 21.4 cm

c) 0.07 cm

d) 11.4 cm

e) 8.5 cm

$$v = \frac{E}{B} \Rightarrow\Rightarrow B = \frac{200}{2\times10^6}T = 10^{-4}T;$$

$$R = \frac{mv}{qB} = 11.4\times10^{-2}\,m.$$

Conceptual questions (Tick the best answer) 1 point each

1. An inflatable rubber sphere of radius r has uniform surface charge density σ. The electric field due to the sphere just outside the sphere is E. Then the rubber sphere is inflated to a radius of 2r, while the total charge on the sphere remains the same. The electric field just outside the sphere is now

(a) 4E.

(b) E/2.

(c) E/4.

(d) E.

2. A copper wire of length L and cross-sectional area A is connected to a potential difference V. The drift velocity is doubled if

(a) the length of the wire is doubled.

(b) the diameter of the wire is halved.

(c) another identical wire is connected parallel with the first one.

(d) the potential is doubled.

3. One eV is a unit of

a) electric potential difference

b) electric field

c) energy

d) electric power

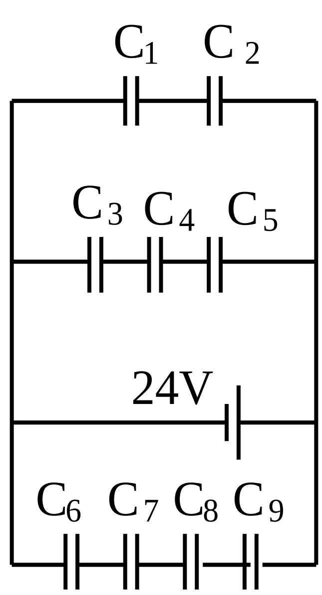
4. Capacitors C_1, C_2, C_3,...C_9 with identical capacitance are arranged in a network and battery is connected to the them, as shown. Which statement is correct about the charges on the capacitors?

(a) $Q_1 = Q_3$

(b) $Q_2 = 2Q_4$

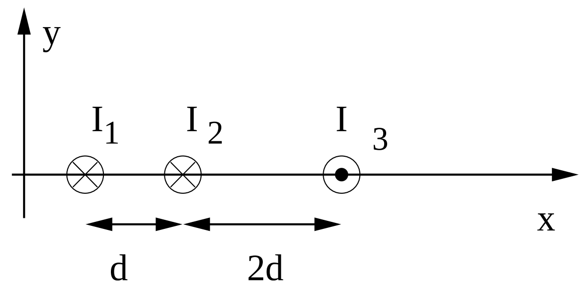
(c) $Q_1 = 2Q_6$

(d) $4Q_1 = Q_4$



5. The figure shows a cross-sectional view of three long straight wires, which are perpendicular to the page and carry currents in the directions as shown. The force acting on wire 1 is zero. The absolute value of the ratio I_2/I_3 is

(a) 1/2
(b) 1/3
(c) 2/3
(d) 1/4

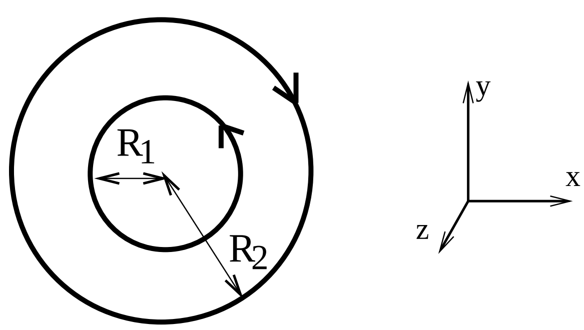


6. The Hall effect is useful to measure

a) magnetic fields.
b) charge carrier concentration in materials.
c) the drift velocity of charge carriers.
d) all of the above.

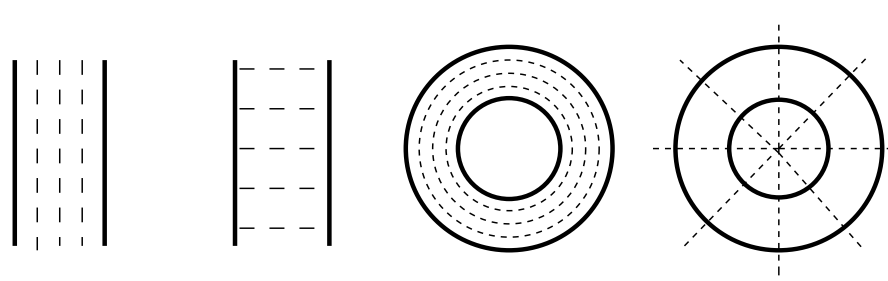
7. Two concentric rings in the plane of page and of radii R_1 and R_2 carry equal currents but in opposite direction as shown. If $R_1 < R_2$, the net magnetic field at the center is along:

a) $\hat{k}$
b) $-\hat{k}$
c) $-\hat{j}$
d) $+\hat{j}$



8. In the figures shown, A and B are parallel-plate capacitors C and D are spherical capacitors. The equipotential surfaces are shown by dashed lines. The correct figures are

a) A and D
b) B and D
c) A and C
d) C and B



A B C D