Part I: Problems

Show working.

1. A 315 μF capacitor has dielectric (dielectric constant of 2.90) filling the 0.275 mm gap between its parallel plates. The charge on each plate has magnitude 0.167 μC. What is the magnitude of the surface charge density on each plate?

a) $17\,\text{pC/m}^2$

b) $49\,\text{pC/m}^2$

c) $94\,\text{pC/m}^2$

d) $140\,\text{pC/m}^2$

(2 points)

$$C = K\epsilon_0 \frac{A}{d} \implies A = \frac{C}{K\epsilon_0} d = 3.38 \times 10^3\,\text{m}^2 \implies \frac{|Q|}{A} = 4.9 \times 10^{-11}\,\text{C/m}^2$$

2. The three capacitors below are all fully charged. The charge on the 1.0 μF capacitor is 2.0 μC. What is the emf of the battery?

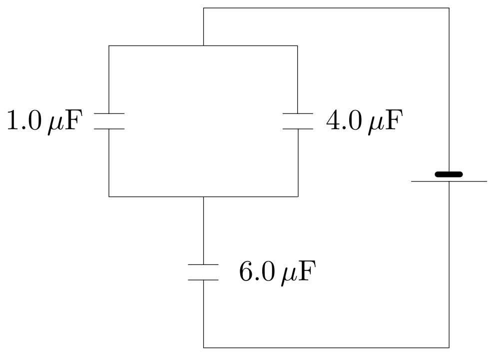
a) 1.7 V

b) 2.0 V

c) 2.3 V

d) 3.7 V

(4 points)



$$\Delta V_{1.0\,\mu\text{F}} = \frac{2.0\,\mu\text{C}}{1.0\,\mu\text{F}} = 2.0\,\text{V} = \Delta V_{4.0\,\mu\text{F}} \implies Q_{4.0\,\mu\text{F}} = (4.0\,\mu\text{F})(2.0\,\text{V}) = 8.0\,\mu\text{C}$$

$$\implies Q_{6.0\,\mu\text{F}} = Q_{1.0\,\mu\text{F}} + Q_{4.0\,\mu\text{F}} = 10.0\,\mu\text{F}$$

$$\mathcal{E} = \Delta V_{4.0\,\mu\text{F}} + \Delta V_{6.0\,\mu\text{F}} = 2.0 + \frac{10.0\,\mu\text{C}}{6.0\,\mu\text{F}} = 3.7\,\text{V}$$

3. A wire (diameter of 1.95 mm) carries a current of 5.15 A; there are 8.5×10^{28} free electrons per cubic meter. How much time does a free electron take to travel a 1.25 m length of this wire?

a) 1.4×10^3 s

b) 5.4×10^3 s

c) 9.8×10^3 s

d) 11×10^3 s

(2 points)

$$v_d = \frac{1}{nq}\frac{I}{A} = 1.27 \times 10^{-4}\,\mathrm{m/s} \implies \text{Time} = \frac{\text{Length}}{v_d} = 9.8 \times 10^3\,\mathrm{s}$$

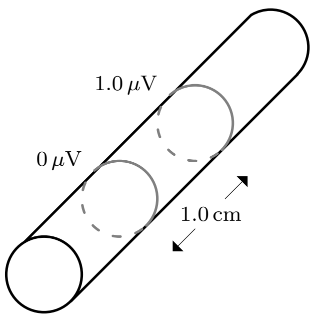
4. At 20 °C, the cylindrical wire below carries a current of 2.65 A. What is the current at 250 °C if the temperature coefficient of resistivity $\alpha = 4.5 \times 10^{-3}\,(^\circ\mathrm{C})^{-1}$?

a) 1.2 A

b) 1.3 A

c) 3.3 A

d) 3.4 A



(2 points)

$$I_{250\,^\circ\mathrm{C}} = \frac{\Delta V}{R_{250\,^\circ\mathrm{C}}} = \frac{R_{20\,^\circ\mathrm{C}}}{R_{250\,^\circ\mathrm{C}}} I_{20\,^\circ\mathrm{C}} \implies I_{250\,^\circ\mathrm{C}} = \frac{\rho_{20\,^\circ\mathrm{C}}}{\rho_{250\,^\circ\mathrm{C}}} I_{20\,^\circ\mathrm{C}} = \frac{1}{1 + (230\,^\circ\mathrm{C})\alpha} I_{20\,^\circ\mathrm{C}} = 1.3\,\mathrm{A}$$

5. What is the potential difference $V_a - V_b$ in the 2-loop circuit below?

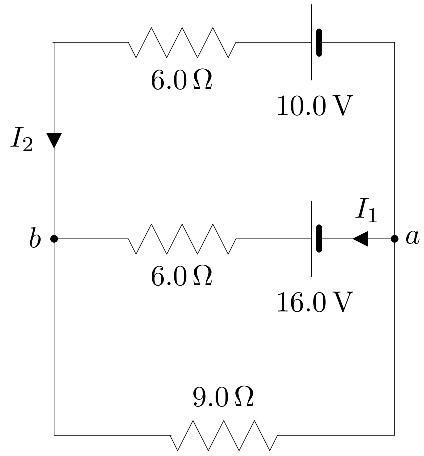
a) $-6.3\,\mathrm{V}$

b) $-9.8\,\mathrm{V}$

c) $-14\,\mathrm{V}$

d) $-23\,\mathrm{V}$

(4 points)



2 loop equations:

$$16\,\mathrm{V} - 6I_1 + 6I_2 - 10\,\mathrm{V} = 0 \quad\Longrightarrow\quad I_2 = I_1 - 1$$

$$16\,\mathrm{V} - 6I_1 - 9(I_1 + I_2) = 0 \quad\Longrightarrow\quad 15I_1 + 9I_2 = 16\,\mathrm{V}$$

$$\text{Solution for } I_1: \; I_1 = \frac{25}{24} = 1.04\,\mathrm{A} \quad\Longrightarrow\quad V_a - V_b = +6I_1 - 16\,\mathrm{V} = -9.8\,\mathrm{A}$$

6. First, switch S_1 below is closed for 5.0 ms and then opened; thereafter, switch S_2 is closed. The capacitor was initially uncharged. What is the plate charge of the capacitor 5.0 ms after S_2 is closed?

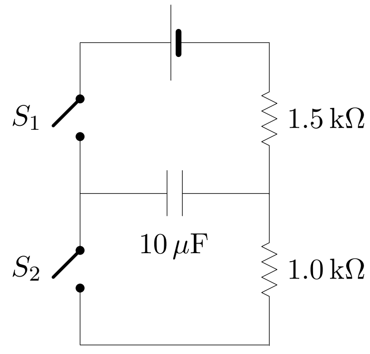
a) $21\,\mu\mathrm{C}$

b) $34\,\mu\mathrm{C}$

c) $73\,\mu\mathrm{C}$

d) $120\,\mu\mathrm{C}$

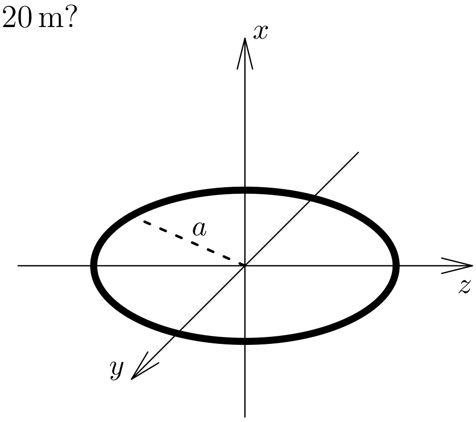
(3 points)



$$\text{Plate charge after charging:} \quad Q_0 = (120\,\mu\mathrm{C})\left[1 - \exp\left(-\frac{5\,\mathrm{ms}}{15\,\mathrm{ms}}\right)\right] = 34\,\mu\mathrm{C}$$

$$\text{Plate charge after discharging:} \quad Q = 34\,\mu\mathrm{C})\exp\left(-\frac{5\,\mathrm{ms}}{10\,\mathrm{ms}}\right) = 20.6\,\mu\mathrm{C}$$

7. An electron moves along the x-axis shown away from a uniform ring of charge $-7.5\,\mathrm{pC}$ (radius $a = 0.25\,\mathrm{m}$) in the yz-plane. What is the speed of the electron at $0.40\,\mathrm{m}$ from the origin if the electron was released from rest at $x = 0.20\,\mathrm{m}$?



a) $43\,\mathrm{km/s}$

b) $120\,\mathrm{km/s}$

c) $150\,\mathrm{km/s}$

d) $220\,\mathrm{km/s}$

(3 points)

$$\Delta KE = -(-e)\Delta V = e\,\Delta\left(\frac{kQ}{\sqrt{a^2+x^2}}\right) = 1.08\times10^{-20}\,\mathrm{J}$$

$$\Longrightarrow \quad v_{0.4\,\mathrm{m}}^2 = \frac{2}{m_\mathrm{e}}\Delta KE = 2.37\times10^{10}\,\mathrm{m/s} \quad \Longrightarrow \quad v_{0.4\,\mathrm{m}} = 154\,\mathrm{km/s}$$

8. A charge ($Q = +250\,\mathrm{nC}$) is spread uniformly throughout the volume of a sphere (radius $R = 35.0\,\mathrm{cm}$). What is the potential at the center of the sphere if the potential is zero on the outer surface of the sphere?

a) $2.12\,\mathrm{kV}$

b) $3.21\,\mathrm{kV}$

c) $6.43\,\mathrm{kV}$

d) $9.64\,\mathrm{kV}$

(2 points)

$$V_{\text{Outer surface}} - V_{\text{Center}} = -\int_0^R E_r dr \Longrightarrow V_{\text{Center}} = \int_0^R E_r dr = \int_0^R \frac{kQ}{R^3} r dr$$

$$= \frac{kQ}{2R} = 3\,210\,\mathrm{V}$$

Part II: Conceptual Questions

Tick (✓) the best answer.

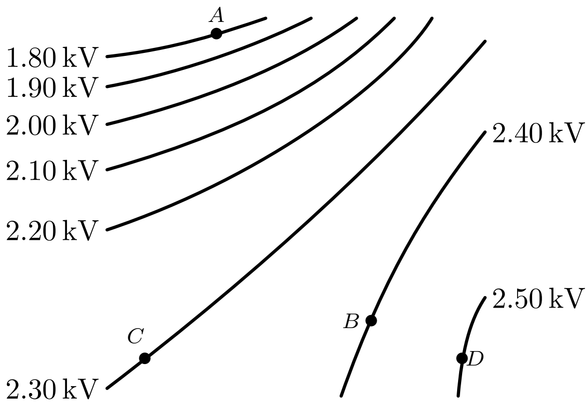
1. For points A, B, C and D on the equipotentials below, the magnitude of the electric field is biggest at:

✓ a) A

b) B

c) C

d) D



2. The electric potential (due to a charge distribution) is zero at some point P. Then,

a) the electric field at P *must* be zero and the total charge in the region *must* be zero.

b) the electric field at P *must* be zero and the total charge in the region *may* be zero.

c) the electric field at P *may* be zero and the total charge in the region *must* be zero.

✓ d) the electric field at P *may* be zero and the total charge in the region *may* be zero.

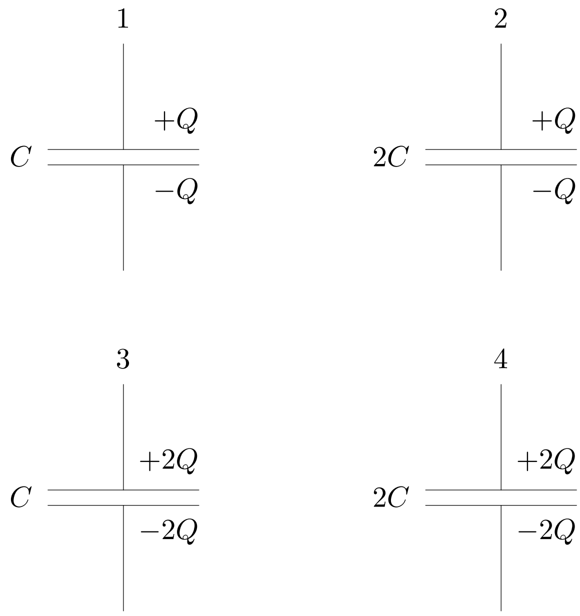
3. The potential differences V_1, V_2, V_3 and V_4 across the four capacitors below are such that:

a) $V_3 = V_4 > V_1 = V_2$

b) $V_1 = V_3 > V_2 = V_4$

✓ c) $V_3 > V_1 = V_4 > V_2$

d) $V_4 > V_2 = V_3 > V_1$



4. If a dielectric is inserted between the plates of an isolated charged capacitor (not connected to a battery), the energy stored in the capacitor

a) always increases.

✓ b) always decreases.

c) will increase for solids and decrease for fluids.

d) will decrease for solids and increase for fluids.

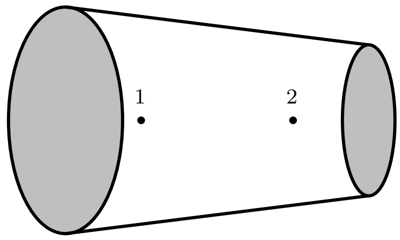
5. For points 1 and 2 in the current-carrying wire (of non-uniform cross-section) below, the correct statement is:

a) $(v_d)_1 > (v_d)_2$

b) $\vec{E}_1 > \vec{E}_2$

✓ c) $\vec{J}_1 < \vec{J}_2$

d) $I_1 < I_2$



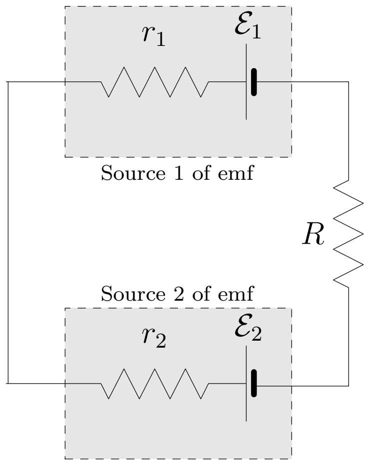
6. In the circuit shown, the terminal voltage of the source 1 of emf is larger than $\mathcal{E}_1$. If $r_1 = r_2$, then:

a) $\mathcal{E}_1 > \mathcal{E}_2$ and the current is clockwise.

b) $\mathcal{E}_1 > \mathcal{E}_2$ and the current is anticlockwise.

✓ c) $\mathcal{E}_1 < \mathcal{E}_2$ and the current is clockwise.

d) $\mathcal{E}_1 < \mathcal{E}_2$ and the current is anticlockwise.



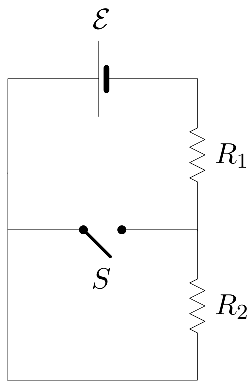
7. If the switch S is closed in the circuit shown, the power dissipated in R_1 will:

✓ a) increase.

b) decrease.

c) remain the same.

d) become zero.



8. The graph shows the currents when 4 different capacitors (each with the same capacitance) are discharged separately through 4 different resistors. The largest resistance is:

a) R_1

✓ b) R_2

c) R_3

d) R_4