Kuwait University **Physics Department**

Physics 102

Final Exam - Semester II (2006-2007)

May 30, 2007 (11.00 a.m. - 1:00 p.m.)

Solutions

Name: ……………………………… Student No: ………………..

Instructors: Drs. M. Abel Karim, Bhatia, Davis, Farhan, Lajko, Manaa, Marafi, Razee, and Sharma

---

(Fundamental Constants)

$k = \dfrac{1}{4\pi\varepsilon_0} = 9\times10^{9}\ \text{N.m}^2/\text{C}^2$	(Coulomb's constant)
$\varepsilon_o = 8.85 \times 10^{-12}\ \text{C}^2 / (\text{N}\cdot\text{m}^2)$	(Permittivity of free space)
$\mu_0 = 4\pi \times 10^{-7}$ T.m/A	(Permeability constant)
$\lvert e\rvert = 1.60 \times 10^{-19}$ C	(Elementary unit of charge)
$N_A = 6.02 \times 10^{23}$	(Avogadro's number)
$g = 9.8\ \text{m/s}^2$	(Acceleration due to gravity)
$m_e = 9.11 \times 10^{-31}$ kg	(Electron mass)
$m_p = 1.67 \times 10^{-27}$ kg	(Proton mass)

Prefixes of units

$\text{m} = 10^{-3}$	$\mu = 10^{-6}$	$\text{n} = 10^{-9}$	$\text{p} = 10^{-12}$
$\text{k} = 10^{3}$	$\text{M} = 10^{6}$	$\text{G} = 10^{9}$	$\text{T} = 10^{12}$

For use by Instructors only

Prob.	1	2	3	4	5	6	7	8	9	10	Questions	Total
Marks												

1. A charge particle q = +10 nC (mass = 6.0×10^{-15} kg) enters perpendicular to a region of electric field E = 3.0×10^5 V/m with a velocity v = 2.0×10^5 m/s at the origin O. What are the coordinates (x, y) of the particle after 4.2 µs?

a. 2.2 m, 0.42 m
b. 4.4 m, 0.84 m (Ans)
c. 2.2 m, 0.84 m
d. 4.4 m, 0.42 m
e. 1.1 m, 0.21 m

(3 points)

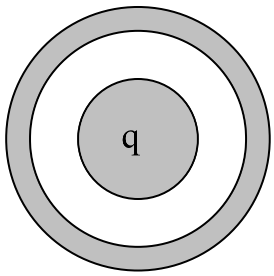
$$a_x = \frac{qE}{m} = \frac{10\times10^{-9}\times3.0\times10^5}{6.0\times10^{-15}} = 5\times10^{11}\ \text{m/s}^2$$

$$x = \text{v}_{x_0}t + \frac{1}{2}a_x t^2 = 0 + 0.5\times5\times10^{11}\times(4.2\times10^{-6})^2 = 4.4\ \text{m}$$

$$y = \text{v}_{y_0}t = 2.0\times10^5\times4.2\times10^{-6} = 0.84\ \text{m}$$

2. A sphere of radius 0.20 m with a charge q = +8 µC distributed uniformly throughout its volume is concentric with a *conducting* shell of radii 0.40 m and 0.50 m. The surface charge density on the outer surface of the shell is 3.2 µC/m². What is the net electric field at a distance of 0.60 m from the centre of the sphere?

a. 1.0×10^5 N/C
b. 1.5×10^5 N/C
c. 2.0×10^5 N/C
d. 2.5×10^5 N/C (Ans)
e. zero



(3 points)

Charge q' on the outer surface of the shell:

$$q' = \sigma.4\pi r^2 = 3.2\times10-6\times4\pi\times0.50^2 = 1.0\times10^{-5}\ \text{C} = 10\ \mu\text{C}$$

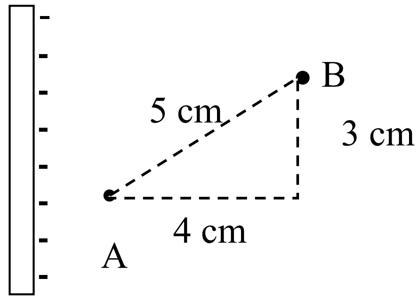
From Gauss Law: $\oint \mathbf{E}.d\mathbf{A} = \frac{q_{encl}}{\varepsilon_0} \Rightarrow E = k\frac{q_{encl}}{r^2}$

$$E = k\frac{(+q-q+q')}{r^2} = k\frac{q'}{r^2} = \frac{9\times10^9\times1.0\times10^{-5}}{(0.60)^2} = 2.5\times10^5\ \text{N/C}$$

3. The electric field due to a large sheet as shown in the figure is 4.5×10^6 V/m. What is the work required to move the charge –2 μC from the point A to the point B?

a. –0.30 J
b. +0.42 J
c. −0.36 J (Ans)
d. +0.36 J
e. zero

(3 points)



$$\Delta V = -\vec{E}.\vec{d}$$
$$V_B - V_A = -(-Ed\cos\theta) = E \times 0.05\cos\theta$$
$$V_B - V_A = E \times 0.04 = 1.8 \times 10^5 \text{ V}$$
$$W = \Delta U = q\Delta V = -2.0 \times 10^{-6} \times 1.8 \times 10^5 \text{ J} = -0.36 \text{ J}$$

4. A capacitor C_1 = 2 μF has 100 μJ of energy stored on it. It is then connected to an uncharged capacitor C_2 = 8 μF. What is the charge on C_2 now?

a. 16 μC (Ans)
b. 8 μC
c. 6 μC
d. 4 μC
e. 2 μC

(3 points)

$$U = \frac{Q^2}{2C_1} \Rightarrow Q = \sqrt{2UC_1} = \sqrt{2 \times 100 \times 10^{-6} \times 2 \times 10^{-6}} = 20 \times 10^{-6} \text{ C} = 20\,\mu\text{C}$$

After C_1 and C_2 are connected together:

Common voltage across the capacitors $V = \dfrac{Q_{\text{total}}}{C_{\text{eq}}} = \dfrac{20\,\mu\text{C}}{(2+8)\,\mu\text{F}} = 2\text{ V}$

$$q_2 = C_2 V = 8\,\mu\text{F} \times 2\text{ V} = 16\,\mu\text{C}$$

5. A potential difference of 6 V is applied across a conducting wire of length 5 m with a resistivity of 2.4×10^{-8} Ω.m. What is the current density in the wire?

a. 2.0×10^7 A/m^2
b. 4.0×10^7 A/m^2
c. 5.0×10^7 A/m^2 (Ans)
d. 6.0×10^7 A/m^2
e. 7.0×10^7 A/m^2

(3 points)

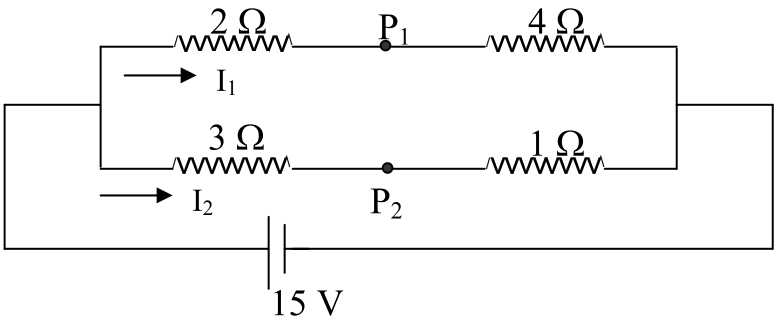
Current density $J = \sigma E = \frac{E}{\rho} = \frac{V/L}{\rho} = \frac{V}{\rho L}$

or

$$J = \frac{I}{A} = \frac{V}{RA} = \frac{V}{\rho \frac{L}{A} A} = \frac{V}{\rho L}$$

$$J = \frac{6}{2.4 \times 10^{-8} \times 5} = 5 \times 10^7 \text{ A/m}^2$$

6. In the circuit below, what is the magnitude of the electric potential difference between the points P_1 and P_2?



a. 0 V
b. 3.50 V
c. 6.25 V (Ans)
d. 9.75 V
e. 15.0 V

(4 points)

Two branches with the resistors are connected in parallel to the battery. Hence currents I_1 and I_2 can be calculated as:

$$I_1 = \frac{15}{(2+4)} = 2.5 \text{ A}$$

$$I_2 = \frac{15}{(3+1)} = 3.75 \text{ A}$$

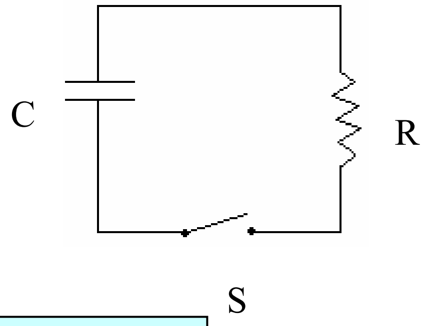
$$V_{P_1} + 2I_1 - 3I_2 = V_{P_2}$$

$$\Rightarrow V_{P_1} - V_{P_2} = 3I_2 - 2I_1 = 3 \times 3.75 - 2 \times 2.5 = 6.25 \text{ V}$$

7. The capacitor C = 50 μF is charged to 6×10^3 V. The switch S is closed for 10^{-3} s. If R = 245 Ω, how much energy is dissipated through the resistor during this time?

a. 91 J
b. 106 J
c. 121 J
d. 136 J (Ans)
e. 151 J

(3 points)



Initial charge on the capacitor:

$$q_0 = CV = 50\times10^{-6}\times6\times10^{3} = 0.3\,\mathrm{C}$$

Charge q after time t:

$$q = q_0 e^{-t/RC} = 0.3\times e^{-10^{-3}/(245\times50\times10^{-6})}\ \mathrm{C} = 0.276\ \mathrm{C}$$

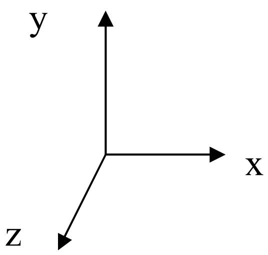
Energy dissipated in the resistor during the time t (energy lost from the capacitor) :

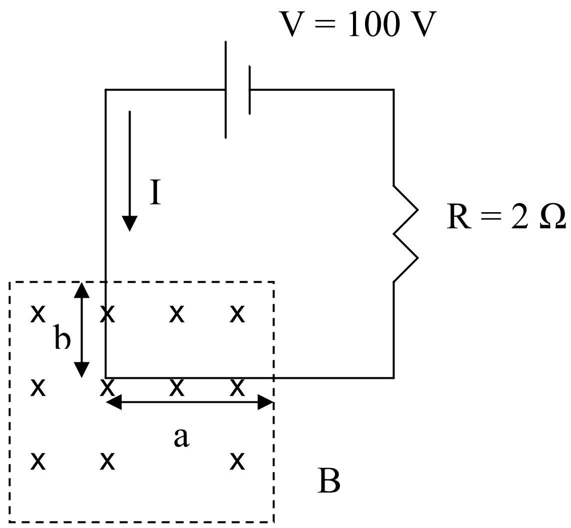
$$\Delta U = U_i - U_f = \frac{q_0^2}{2C} - \frac{q^2}{2C} = \frac{(q_0^2 - q^2)}{2C} = \frac{(0.3^2 - 0.276^2)}{2\times50\times10^{-6}} = 136\ \mathrm{J}$$

8. The circuit shown is placed partly in a region of the uniform magnetic field B = 2 T. Given, a = 4 cm and b = 3 cm, calculate the magnitude of the net force on the wire.

a. 3 N
b. 4 N
c. 7 N
d. 5 N (Ans)
e. 0 N

(3 points)





Current I = V/R = 100/2 = 50 A.

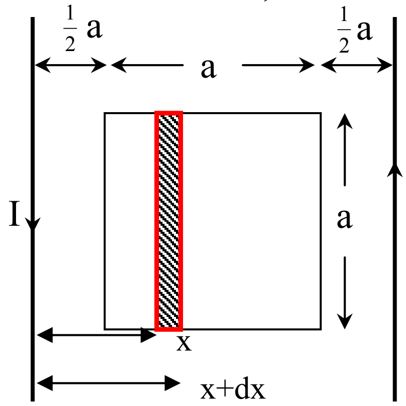
Considering the right-handed coordinate system as above: $\mathrm{B} = -2\ \hat{\mathrm{k}}\ \mathrm{T}$

$$\vec{F}_1 = I\vec{L}_1\times\vec{B} = Ia\hat{\mathrm{i}}\times(-2\hat{\mathrm{k}}) = 2Ia\,\hat{\mathrm{j}} = 4\hat{\mathrm{j}}\ \mathrm{N}$$

$$\vec{F}_2 = I\vec{L}_2\times\vec{B} = Ib(-\hat{\mathrm{j}})\times(-2\hat{\mathrm{k}}) = 2Ib\,\hat{\mathrm{i}} = 3\,\hat{\mathrm{i}}\ \mathrm{N}$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = (4\hat{\mathrm{j}} + 3\hat{\mathrm{i}})\ \mathrm{N} \quad\Rightarrow\quad F = \sqrt{F_1^2 + F_2^2} = \sqrt{4^2 + 3^2} = 5\ \mathrm{N}$$

9. A square loop of wire is placed midway between two long straight wires carrying equal steady currents, as shown in the figure. If a = 10 cm and I = 5 A, what is the magnitude of the magnetic flux through the loop?



a. 0
b. 0.22 μWb (Ans)
c. 0.18 μWb
d. 0.11 μWb
e. 0.09 μWb

(4 points)

Let $\hat{n}$ be a unit vector pointing out of page. Let the distance x of a rectangular slab be measured from wire 1 (left wire).

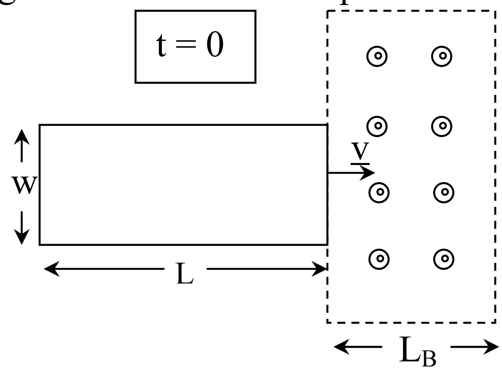
Magnetic field B_1 through the slab of infinitesimal thickness at a distance x:

$$B_1 = \frac{\mu_0 I}{2\pi x}\hat{n} \quad \Rightarrow \quad \text{Flux through the slab:} \quad d\Phi_1 = B_1 dA = \frac{\mu_0 I\, a dx}{2\pi x}$$

$$\Phi_1 = \int d\Phi_1 = \frac{\mu_0 I\, a}{2\pi}\int_{a/2}^{3a/2}\frac{dx}{x} = \frac{\mu_0 I\, a}{2\pi}\ln x \Big|_{a/2}^{3a/2} = \frac{\mu_0 I\, a}{2\pi}\ln 3$$

$$\text{Total flux} = 2\,\Phi_1 = \frac{\mu_0 I\, a}{\pi}\ln 3 = \frac{4\pi\times 10^{-7}\times 5\times 0.10\times \ln 3}{\pi} = 2.2\times 10^{-7}\ \text{Wb} = 0.22\ \mu\text{Wb}$$

10. At time t = 0, a rectangular single-loop coil of resistance R = 5 Ω and dimensions W = 2 cm and L = 10 cm enters a region of constant magnetic field directed out of the page with B = 2.5 T. The length of the region containing the magnetic field is L_B = 5 cm as shown. The coil moves to the right with the constant speed of v = 2 cm/s. The induced current at t = 2 s is:



a. 200 μA counterclockwise.
b. 200 μA clockwise. (Ans)
c. 1 mA clockwise.
d. 1 mA counterclockwise.
e. 0.

(3 points)

Let x be the length of the loop inside the field at any time t.

$$\text{Flux } \Phi_B = BA = BWx \quad \Rightarrow \quad \varepsilon = -\frac{d\Phi_B}{dt} = -BW\frac{dx}{dt} = -BW\,\text{v}$$

At t =2, the loop has traveled x = vt = 4 cm and is inside the field.

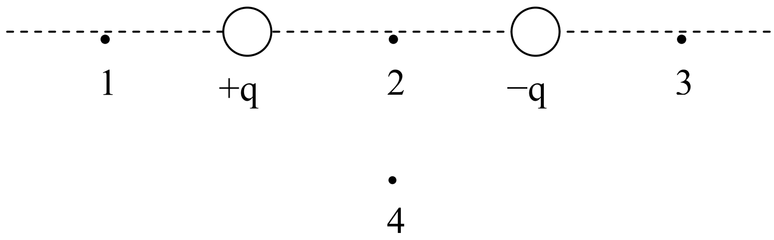
$$|\varepsilon| = 2.5\times 0.02\times .02 = 10^{-3}\ \text{V}$$

$$I = \varepsilon / R = 10^{-3}/5 = 2\times 10^{-4}\ \text{A} = 200\times 10^{-6}\ \text{A} = 200\ \mu\text{A}$$

Since the flux is increasing out of the page, the current induced will be clockwise to produce a magnetic field into the page.

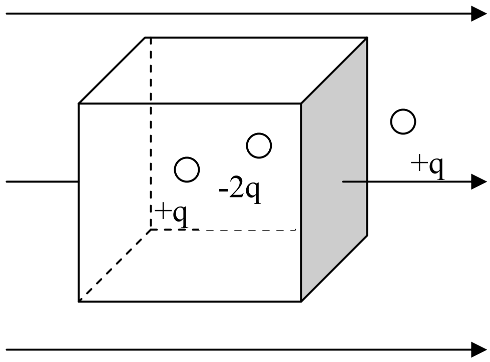
Conceptual Questions: Tick a correct answer.

1. Two charges +q and −q are along the x-axis as shown. At which point is the electric field highest?



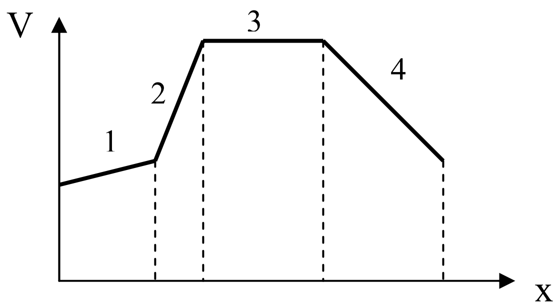
a. 1
b. 2 (Ans)
c. 3
d. 4

2. Charges +q and −2q are inside a cubic box and a charge +q is near the surface as shown in the figure. The box is placed in an external electric field E. The electric flux through the cube is:



a. $-\frac{q}{\varepsilon_0}+q$.
b. $-\frac{q}{\varepsilon_0}+\varepsilon_0 E$.
c. 0.
d. $-\frac{q}{\varepsilon_0}$. (Ans)

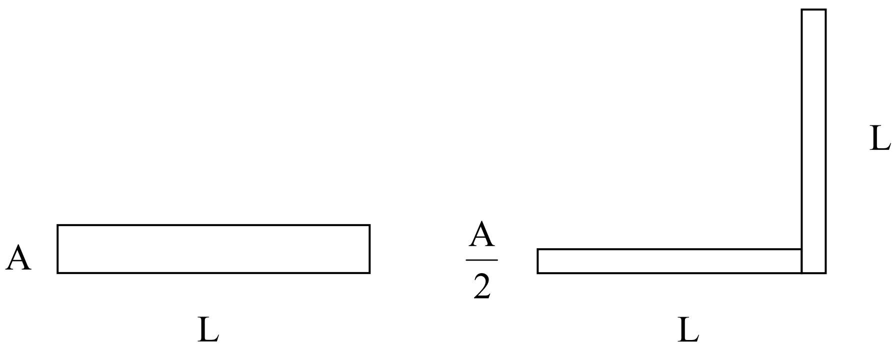
3. The electric potential along the x-axis is shown in the figure. Rank the regions according to the magnitude of x-component of the electric field, from the highest to the lowest.



a. 1, 2, 3, 4
b. 2, 4, 1, 3 (Ans)
c. 1, 3, 2, 4
d. 3, 1, 4, 2

4. A conductor with a length L and a cross-sectional area A has a resistance R (the figure on the left). What is the resistance R' of the conductor of the same material, as shown in the figure on the right?

a. R' = R
b. R' = 2R
c. R' = √2R
d. R' = 4R (Ans)



5. Which of the following does not produce a magnetic field?

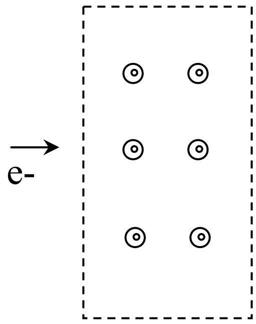
   a. A varying current-carrying wire
   b. A stationary electric charge (Ans)
   c. A moving electric charge
   d. An electromagnet

6. Voltage can be induced in a loop of wire

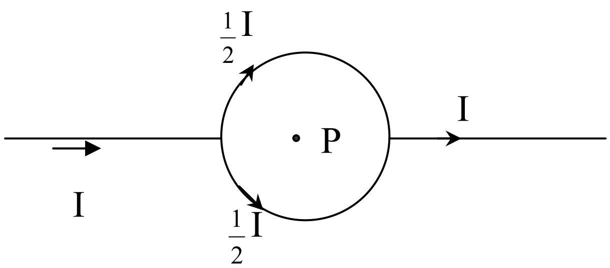
   a. only by moving a magnet near the loop.
   b. only by moving the loop near the magnet.
   c. only by changing the current in a nearby loop.
   d. all of the above. (Ans)

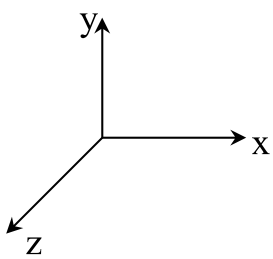
7. An electron moving to the right enters a region of uniform magnetic field that points out of the paper. Upon entering the magnetic field the electron will initially be

   a. deflected out of the plane of the paper.
   b. deflected into the plane of the paper.
   c. deflected upward. (Ans)
   d. deflected downward.
   e. undeflected in its motion.



8. A long wire splits into two identical semicircular segments as shown in the figure below. What is the direction of the magnetic field at point P, the centre of the circle?





   a. +x   b. -x
   c. +y   d. -y
   e. +z   f. -z
   g. none of these - there is no field at P. (Ans)