Kuwait University **Physics Department**

Physics 101
Final Examination
Thursday, May 30, 2002
11 AM -1 PM

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For use by instructors only

Problem	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points													

Notes

1. Answer all questions
2. The solution should be given explicitly for each problem
3. No solution = no points
4. Check the correct answer for each question
5. Take g = 10 m/s^2

(P1) Three vectors are given by $\vec{a} = 2.0\hat{i} + 3.0\hat{j} - 6.0\hat{k}$, $\vec{b} = 2.0\hat{k}$ and $\vec{c} = 2.0\hat{i} + 2.0\hat{j}$. The angle between $\vec{a}$ and $(\vec{b} \times \vec{c})$ is

$\boxed{\text{(a) } 84.2^\circ}$ (b) 0° (c) 90° (d) 276° (e) 30°

Solution:

$$(\vec{b}\times\vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 2 \\ 2 & 2 & 0 \end{vmatrix} = (0-4)\hat{i} + (4-0)\hat{j} + (0-0)\hat{k} = -4\hat{i} + 4\hat{j}$$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = (2\hat{i}+3\hat{j}-6\hat{k})\cdot(-4\hat{i}+4\hat{j}) = -8+12+0 = 4$$

$$|\vec{a}| = \sqrt{2^2+3^2+6^2} = \sqrt{49} = 7$$

$$|\vec{b}\times\vec{c}| = \sqrt{4^2+4^2} = \sqrt{32} = 5.66$$

$$\theta = \cos^{-1}\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{|\vec{a}|\cdot|(\vec{b}\times\vec{c})|} =$$

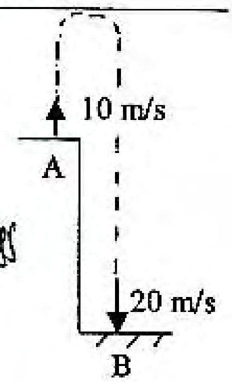
$$\theta = \cos^{-1}\frac{4}{(7)(5.66)} = \cos^{-1}\frac{4}{39.62}$$

$$= \cos^{-1}(.10096)$$

$$\boxed{\theta = 84.2^\circ}$$

(P2) An object is thrown upward from the edge of a cliff at point A with a speed of 10 m/s. The object returns to the ground at point B with a speed of 20 m/s as shown in the figure. The height of the cliff is

$\boxed{\text{(a) 15 m}}$ (b) 10 m (c) 8.33 m (d) 5 m (e) 20 m



Solution:

method 1

$$v^2 = v_0^2 - 2g\,\Delta y$$

$$400 = 100 - 20(\Delta y)$$

$$\Delta y = \frac{100-400}{20} = -15 \text{ m}$$

$\therefore$ height of cliff $= 15$ m #

method 2

Δy for maximum height =

$$v^2 = v_0^2 - 2g(\Delta y)$$

$$0 = 100 - 20(\Delta y)$$

$$= 5 \text{ m}$$

from height_{max} to bottom of cliff

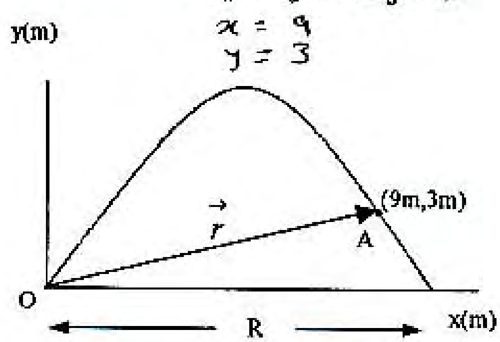
$$(-20)^2 = 0 - 2g(\Delta y)$$

$$\Delta y = -20 \text{ m}$$

$\therefore h = -20 + 5 = 15$ m #

(P3) The path of a projectile is shown in the figure. The position vector of the projectile at point A is $\vec{r} = 9.0\hat{i} + 3.0\hat{j}$. If the initial velocity component in the horizontal direction is $v_{ox} = 3$ m/s, then the range R of the projectile is

(a) 4.8 m (b) 9 m $\boxed{\text{(c) 9.6 m}}$ (d) 10 m (e) 14.4 m



Solution: Range equation R is

$$R = \frac{v_0^2 \sin 2\theta_0}{g}$$

$$v_{ox} = v_0\cos\theta_0 = 3$$

$$x = 9$$

$$y = 3$$

So we need to calculate θ_0 and v_0.

we know $y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2}$

$$3 = \tan\theta_0(9) - \frac{10(81)}{2(3)^2}$$

$$\frac{48}{9} = \tan\theta_0 \Rightarrow \theta_0 = 79.38^\circ$$

$\because v_{ox} = v_0\cos\theta_0 \quad \therefore v_0 = \frac{3}{\cos\theta_0} = 16.28$ m/s

$$\therefore R = \frac{(16.28)^2 \sin(158.76)}{10} = 9.6 \text{ m} \quad \#$$

Another method

$\because x = v_{ox}t \Rightarrow 9 = 3t \Rightarrow t = 3$ sec

also $y = v_{oy}t - \frac{1}{2}gt^2 \Rightarrow 3 = v_{oy}(3) - 5(9) \Rightarrow v_{oy} = 16$ m/s

$\therefore v_0 = \sqrt{v_{ox}^2 + v_{oy}^2} = 16.28$ m/s $\Rightarrow \theta_1 = \tan^{-1}\left(\frac{16}{3}\right) \Rightarrow R = \frac{v_0^2 \sin 2\theta}{g} = 9.6$ m

(P4) A block starts descending the 30° slope from rest at point A and comes to rest at point B. If the coefficient of kinetic friction between the block and the incline is $\mu_{k1} = 0.1$, then the coefficient of kinetic friction μ_{k2} between the block and the horizontal surface is

$\boxed{\text{(a) } 0.2}$ (b) 0.3 (c) 0.4 (d) 0.6 (e) 0.1

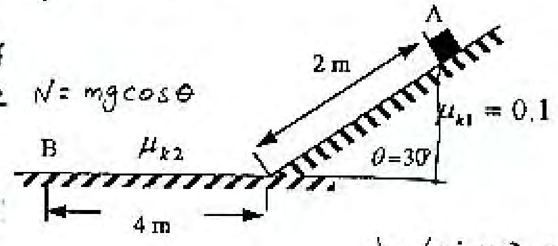
Solution: let us first calculate v at the end of the incline using Energy theorems

$\Delta E = W_f \Rightarrow \Delta K + \Delta U = W_f$

$(\frac{1}{2}mv^2 - 0) + (0 - mgh) = -\mu_{k_1} N d$ where $N = mg\cos\theta$

$\frac{1}{2}\cancel{m}v^2 - \cancel{m}gh = -\mu_{k_1}\cancel{m}g\cos\theta$

$\therefore v^2 = 2gh - 2(.1)(10)(2)(.866) = \sqrt{16.54}$



$h = (\sin 30)\, 2$
$= 1\text{ m}$

Second part for the horizontal again $\Delta E = W_f$

$0 - \frac{1}{2}\cancel{m}v^2 = -\mu_{k_2}\cancel{m}g\, d$

$\frac{1}{2}(16.54) = \mu_{k_2}(10)(4) \Rightarrow \mu_{k_2} = \frac{8}{40} = .2067 \simeq .21$ #

(P5) An object of mass m = 5 kg is attached to a light cord of length R = 2 m. What is the minimum speed that the object should have at point A in order to keep its vertical circular path? [Hint: The object is on the verge of falling down at point B].

(a) 8 m/s (b) 6.3 m/s $\boxed{\text{(c) 10 m/s}}$ (d) 8.9 m/s (e) 12.6 m/s

Solution:

for the object at point not to fall $\therefore T = 0$

$\therefore -\cancel{m}g = -\cancel{m}\frac{v^2}{R} \Rightarrow v_B = \sqrt{gR} = \sqrt{10(2)} = \sqrt{20}$

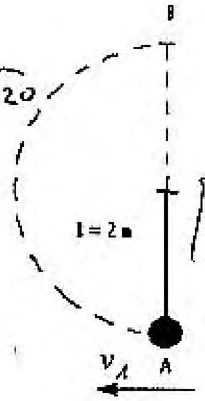
now using Energy theorems to find v_A

$\Delta E = 0$

$\Delta K + \Delta U = 0 \Rightarrow (\frac{1}{2}\cancel{m}v_A^2 - \frac{1}{2}\cancel{m}v_B^2) + (0 - \cancel{m}gh)$

$\therefore \frac{1}{2}v_A^2 - \frac{1}{2}(20) - (10)(4) = 0$ where $h = 2R = 4\text{ m}$

$\Rightarrow v_A = \sqrt{100} = 10\text{ m/s}$ #



$h = 2R$
$h = 4\text{ m}$

(P6) Two equal masses m = 0.5 kg, are initially situated as shown in the figure with the center of mass of one held a vertical distance h = 1 m above the center of mass of the other. The left mass is released and strikes the other. Assume that the collision is head-on elastic collision and neglect the mass of the strings and any frictional effects. The maximum compression of the spring whose spring constant k = 1000 N/m is

(a) 0.001 m (b) 0.2 m (c) 0.01 m $\boxed{\text{(d) 0.1 m}}$ (e) 0.02 m

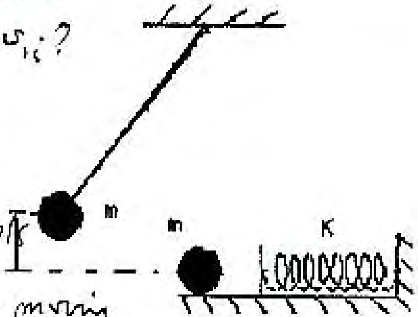
Solution:

Because collision is elastic and masses are equal then $v_{2f} = v_{1i}$ and $v_{1f} = 0$ so what is v_{1i}?

$\Delta E = 0$ for the object on the left

$\Delta K + \Delta U = 0 \Rightarrow (\frac{1}{2}mv_{1i}^2 - 0) + (0 - mgh) = 0$

$\Rightarrow \frac{1}{2}\cancel{m}v_{1i}^2 = \cancel{m}gh \quad \therefore v_{1i} = \sqrt{20} = 4.47\text{ m/s}$



now we have the second object after collision is moving with 4.47 m/s $\Rightarrow \Delta E = 0$

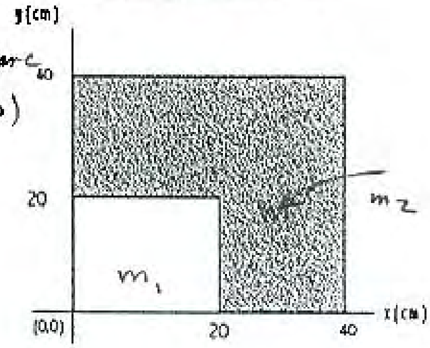
$\Delta K + \Delta U = 0 \Rightarrow (0 - \frac{1}{2}mv_2^2) + (\frac{1}{2}kx^2 - 0) = 0$

$-\frac{1}{2}mv_2^2 = -\frac{1}{2}kx^2 \Rightarrow x^2 = \frac{mv_2^2}{k}$

$\therefore x = \sqrt{.01} = .1\text{ m}$ #

(P7) A small squared area has been removed from a uniform square shaped plate as shown in the figure. The <u>distance</u> of the center of mass of the remaining plate relative to the origin (0,0) is

(a) 20 cm (b) 28 cm (c) 23 cm $\boxed{\text{(d) 33 cm}}$ (e) 25 cm



Solution:

if we assume that we put back the missing square
∴ the center of mass of the two plates is (20,20)

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$y_{com} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

We do not have masses but we have areas which is derived originally from volumes

$$\therefore x_{com} = 20 = \frac{(400) x_1 + (1600 - 400) x_2}{1600} = \frac{(400)(10) + (1200)(x_2)}{1600}$$

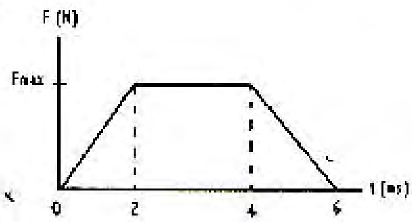
$$\Rightarrow x_2 = \frac{28000}{1200} = 23.3$$ and the same for $y_2 = 23.33$ because it is square

$$\therefore r = \sqrt{(23.3)^2 + (23.3)^2} = 32.998 \simeq 33 \ \#$$

(P8) The plot shows the force <u>**magnitude versus time**</u> during the collision of a 0.1 kg ball with a wall. The initial velocity of the ball is 30 m/s perpendicular to wall. If the ball rebounds directly back perpendicular to the wall with the same speed, then the maximum magnitude of the force on the ball from the wall is

[<u>Note</u>: the time scale on the plot is in milliseconds (ms)].

(a) 10 N (b) 500 N (c) 800 N $\boxed{\text{(d) 1500 N}}$ (e) 400 N



Solution:

We know that $J \text{ (impulse)} = \Delta P = P_f - P_i$

also $J = \int F(x)\, dt$

$$\therefore \Delta p = \int f(x)\, dt = \text{area under curve}$$

$$mv_2 - (-mv_1) = 2\left(\tfrac{1}{2}(2\times10^{-3}) f_{max}\right) + (2\times10^{-3}) f_{max}$$

$$(.1)(30) + (.1)(30) = 4\times10^{-3} f_{max}$$

$$\therefore f_{max} = \frac{6}{4\times10^{-3}} = 1500 \text{ N} \ \#$$

(P9) A 60 kg runner runs with 3 m/s and jumps on a 40 kg stationary cart. How far will they move together until stop, if the coefficient of kinetic friction between the cart and the ground is 0.2 ?

(a) 3.24 m (b) 4.05 m (c) 8.1 m $\boxed{\text{(d) 0.81 m}}$ (e) 4.2 m

Solution

$$P_i = P_f$$

$$mv_1 = (m_1 + m_2) V \Rightarrow V = \frac{(60)(3)}{100} = 1.8 \text{ m/s}$$

now $\Delta E = W_f$

$$\Delta K + \Delta U = 0 \Rightarrow \left(0 - \tfrac{1}{2} M V^2\right) + 0 = -\mu M g d$$

where $M = m_1 + m_2$

$$\therefore d = \frac{162}{200} = .81 \text{ m}$$

(P10) A thin uniform rod has mass 2 kg, length L= 0.3 m and moment of intertia about its center of mass ($I = \frac{ML^2}{12} kg.m^2$) is pivoted about O. The rod is held in a horizontal position, then released from rest. Neglecting the friction at the pivot, the angular velocity of the rod in the vertical position is

(a) 0 rad/s (b) 20 rad/s $\boxed{\text{(c) 10 rad/s}}$ (d) 3.25 rad/s (e) 30 rad/s

Solution

$I = I_{cm} + Mh^2 = \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{3} = \frac{(2)(.3)^2}{3} = .06\ kg.m^2$

now $\Delta E = 0$

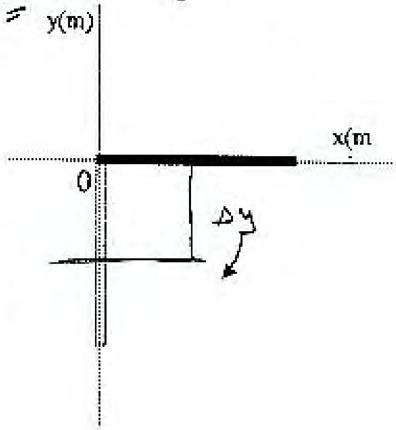
$\Rightarrow \Delta K + \Delta U = 0$

$\left(\frac{1}{2}I\omega^2 - 0\right) + (mg\,\Delta y) = 0$

$\frac{1}{2}(.06)\omega^2 + (2)(10)\left(-\frac{L}{2}\right)$

$\Rightarrow \omega^2 = \frac{3}{.03} = 100$

$\omega = 10$ rad/s



$h = \Delta y$

$= -\frac{L}{2}$

(P11) A disk of radius R = 10 cm that is rotating with 60 rev / min is brought to rest after 2 seconds. What is the angular displacement of a point on the rim of the disk at this time.

(a) π rad $\boxed{\text{(b) } 2\pi \text{ rad}}$ (c) 3π rad (d) $\pi/2$ rad (e) $\pi/4$ rad

Solution

$\omega = \omega_0 + \alpha t$

$0 = \left(\frac{60 \times 2\pi}{60}\right) + 2\alpha$

$\alpha = -\pi$

$\therefore \Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 2\pi(2) - \frac{\pi}{2}(2)^2 = 2\pi$ rad #

(P12) A wheel **1m in diameter** rotates on a fixed, frictionless, horizontal axle. Its moment of inertia about this axis is 5 kg.m^2. A constant tension of 20 N is maintained on a rope wrapped around the rim of the wheel, so as to cause the wheel to accelerate. If the wheel starts from rest at t = 0, find the angular velocity of the wheel at t = 3 s.

(a) 12 rad/s $\boxed{\text{(b) 6 rad/s}}$ (c) 3 rad/s (d) 9 rad /s (e) 15 rad/s

Solution

$\tau = I\alpha$ also $\tau = TR$

counter clockwise τ is positive

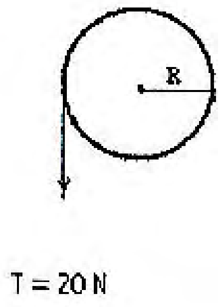
$TR = I\alpha$

$(20)(.5) = 5\alpha$

$\alpha = 2$ rad/s^2

$\omega = \omega_0 + \alpha t$

$= 0 - 2(3) = 6$ (rad/s) #



Solution of the Final Exam. (May 30, 2002)

(P1)

$$\vec{d} = \vec{b} \times \vec{c} = (2\hat{k} \times 2\hat{i}) + (2\hat{k} \times 2\hat{j}) = -4\hat{i} + 4\hat{j}$$

$$\vec{a} \cdot \vec{d} = -8 + 12 = 4$$

$$|a| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{49} = 7$$

$$|d| = \sqrt{4^2 + 4^2} = \sqrt{32}$$

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{d}}{|a||d|}\right) = 84.2°$$

(P2)

$$v^2 = v_0^2 - 2g(\Delta y)$$

$$400 = 100 - 20(\Delta y)$$

$$\Delta y = \frac{100 - 400}{20} = -15$$

$$height = 15m$$

(P3)

$$x = v_{ox} t$$

$$9 = 3t$$

$$t = 3s$$

$$y = v_{oy} t - \frac{1}{2} g t^2$$

$$3 = v_{oy}(3) - 5(9)$$

$$v_{oy} = 16 m/s$$

$$v_o = \sqrt{3^2 + 16^2} = 16.27$$

$$\theta = \tan^{-1}(16/3)$$

$$R = \frac{v_o^2 \sin 2\theta}{g} = 9.6m$$

(P4)

$$mgh = W_{f_1} + W_{f_2}$$

$$mg(2 \sin 30) = \mu_{k_1} mgd_1 \cos 30 + \mu_{k_2} mgd_2$$

$$10(2)(0.5) = 0.1(10)(2) + \mu_{k_2}(10)(4)$$

$$10 = 2 + 40\mu_{k_2}$$

$$\mu_{k_2} = 8/40 = 0.2$$

(P5)

$$\frac{1}{2} m v_A^2 = 2mgR + \frac{1}{2} m v_B^2$$

$$mg + T = \frac{m v_B^2}{R}$$

$$v_B^2 = mgR$$

$$\frac{1}{2} m v_A^2 = 2mgR + \frac{1}{2} m(mgR)$$

$$v_A = \sqrt{5gR} = \sqrt{5(10)(2)} = 10 m/s$$

(P6)

Left

$$mgh = \frac{1}{2} m v_i^2$$

$$v_i = \sqrt{2gh} = \sqrt{2(10)(1)} = \sqrt{20} = 4.47 m/s$$

$$v_{if} = v_{li}$$

$$\frac{1}{2} m v_{if}^2 = \frac{1}{2} k x_m^2$$

$$x_{max} = \sqrt{\frac{m v_{if}^2}{k}} = \sqrt{\frac{(20)(0.5)}{1000}} = 0.1m$$

(P7)

$$x_{com} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2} = \frac{1600(20) - 400(10)}{1600 - 400} = 23.33$$

$$y_{com} = x_{com}$$

$$d = \sqrt{x_{com}^2 + y_{com}^2} = \sqrt{2(23.33)^2} = 32.99 \approx 33cm$$

(P8)

$$J = \Delta P = m(v_f - v_i) = area$$

$$(2[\frac{1}{2}(2)(F_{max})] + 2F_{max}) \times 10^{-3} = m(-30 - 30)$$

$$4F_{max} \times 10^{-3} = -60(0.1)$$

$$|F_{max}| = 1500\ N$$

(P9)

$$P_i = P_f$$

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)V$$

$$V = \frac{60 \times 3}{(60 + 40)} = 1.8 m/s$$

$$\frac{1}{2} M V^2 = \mu_k M g d$$

$$\frac{1}{2}(100)(1.8)^2 = (0.2)(100)(10)d$$

$$d = 0.81\ m$$

(P10)

$$mgh = \frac{1}{2} I_o \omega^2$$

$$I_o = \frac{ML^2}{12} + M\frac{L^2}{4} = \frac{ML^2}{3}$$

$$mg\frac{L}{2} = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$$

$$\omega = \sqrt{\frac{3g}{L}} = \sqrt{\frac{3(10)}{0.3}} \sqrt{100} = 10\ rad/s$$

(P11)

$$\omega = \omega_0 + \alpha t$$

$$0 = (60 \times 2\pi/60) + 2\alpha$$

$$\alpha = -\pi$$

$$\Delta\theta = \omega_o t + 0.5\alpha t^2 = 2\pi(2) - 0.5(\pi)(2^2) = 2\pi(rad)$$

(P12)

$$\tau = I\alpha$$

$$-TR = 5\alpha$$

$$-20 \times 0.5 = 5\alpha$$

$$\alpha = -2(rad/s^2)$$

$$\omega = \omega_o + \alpha t = 0 - 2(3) = -6(rad/s)$$

$$|\omega| = 6(rad/s)$$