CONCEPTUAL EXAMPLE 4.3

As a projectile moves in its parabolic path, is there any point along its path where the velocity and acceleration vectors are (a) perpendicular to each other? (b) parallel to each other?

Reasoning (a) At the top of its flight, v is horizontal and a is vertical. This is the only point where the velocity and acceleration vectors are perpendicular. (b) If the object is thrown straight up or down, then v and a will be parallel throughout the downward motion. Otherwise, the velocity and acceleration vectors are never parallel.

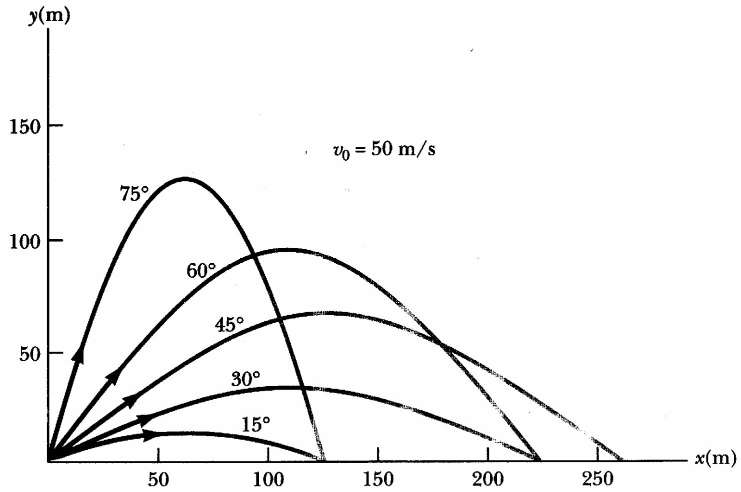

FIGURE 4.8 A projectile fired from the origin with an initial speed of 50 m/s at various angles of projection. Note that complementary values of θ_0 will result in the same value of x (range of the projectile).

EXAMPLE 4.4 The Long-Jump

A long-jumper leaves the ground at an angle of 20.0° to the horizontal and at a speed of 11.0 m/s. (a) How far does he jump? (Assume that his motion is equivalent to that of a particle.)

Solution The horizontal motion is described by Equation 4.12:

$$x = (v_0 \cos \theta_0)t = (11.0 \text{ m/s})(\cos 20.0°)t$$

The value of x can be found if t, the total time of the jump, is known. We are able to find t using Equation 4.11, $v_y = v_0 \sin \theta_0 - gt$, and by noting that at the top of the jump the vertical component of velocity goes to zero:

$$v_y = v_0 \sin \theta_0 - gt$$

$$0 = (11.0 \text{ m/s}) \sin 20.0° - (9.80 \text{ m/s}^2)t_1$$

$$t_1 = 0.384 \text{ s}$$

Note that t_1 is the time interval to reach the *top* of the jump. Because of the symmetry of the vertical motion, an identical time interval passes before the jumper returns to the ground. Therefore, the *total* time in the air is $t = 2t_1 = 0.768$ s. Substituting this value into the above expression for x gives

$$x = (11.0 \text{ m/s})(\cos 20.0°)(0.768 \text{ s}) = \mathbf{7.94 \text{ m}}$$

(b) What is the maximum height reached?

Solution The maximum height reached is found using Equation 4.13 with $t = t_1 = 0.384$ s:

$$y_{\text{max}} = (v_0 \sin \theta_0)t_1 - \tfrac{1}{2}gt_1^2$$

$$= (11.0 \text{ m/s})(\sin 20.0°)(0.384 \text{ s}) - \tfrac{1}{2}(9.80 \text{ m/s}^2)(0.384 \text{ s})^2$$

$$= \mathbf{0.722 \text{ m}}$$

The assumption that the motion of the long-jumper is that of a particle is an oversimplification. Nevertheless, the values obtained are reasonable. Note that we also could have used Equations 4.17 and 4.18 to find the maximum height and horizontal range. However, the method used in our solution is more instructive.

(Example 4.4) In a long-jump event, Willie Banks can leap horizontal distances of at least 8 meters. *(© R. Mackson/FPG)*

EXAMPLE 4.5 It's a Bull's-Eye Every Time

In a very popular lecture demonstration, a projectile is fired at a target in such a way that the projectile leaves the gun at the same time the target is dropped from rest, as in Figure 4.9. Let us show that if the gun is initially aimed at the stationary target, the projectile hits the target.

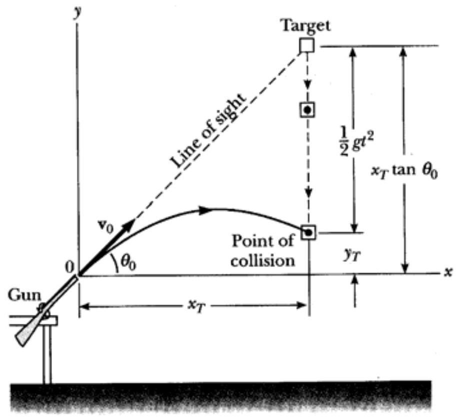

FIGURE 4.9 (Example 4.5) Schematic diagram of the projectile-and-target demonstration. If the gun is aimed directly at the stationary target and is fired at the same instant the target begins to fall, the projectile will hit the target. Both fall through the same vertical distance in a time t, because both experience the same acceleration, $a_y = -g$.

Reasoning and Solution We can argue that a collision will result under the conditions stated by noting that both the projectile and the target experience the same acceleration, $a_y = -g$, as soon as they are released. First, note from Figure 4.9 that the initial y coordinate of the target is $x_T \tan\theta_0$ and that it falls through a distance $\frac{1}{2}gt^2$ in a time t. Therefore, the y coordinate of the target as a function of time is, from Equation 4.14,

$$y_T = x_T \tan\theta_0 - \tfrac{1}{2}gt^2$$

Now if we write equations for x and y for the projectile path over time, using Equations 4.12 and 4.13 simultaneously, we get

$$y_p = x_p \tan\theta_0 - \tfrac{1}{2}gt^2$$

Thus, we see by comparing the two equations above that when $x_p = x_T$, $y_p = y_T$ and a collision results.

The result could also be arrived at with vector methods, using expressions for the position vectors for the projectile and target.

You should also note that a collision will *not* always take place. There is the further restriction that a collision results only when $v_0 \sin\theta_0 \geq \sqrt{gd/2}$, where d is the initial elevation of the target above the *floor*. If $v_0 \sin\theta_0$ is less than this value, the projectile will strike the floor before reaching the target.

EXAMPLE 4.6 That's Quite an Arm

A stone is thrown from the top of a building upward at an angle of 30.0° to the horizontal and with an initial speed of 20.0 m/s, as in Figure 4.10. If the height of the building is 45.0 m, (a) how long is the stone "in flight"?

Solution The initial x and y components of the velocity are

$$v_{x0} = v_0 \cos\theta_0 = (20.0\text{ m/s})(\cos 30.0°) = 17.3\text{ m/s}$$

$$v_{y0} = v_0 \sin\theta_0 = (20.0\text{ m/s})(\sin 30.0°) = 10.0\text{ m/s}$$

To find t, we can use $y = v_{y0}t - \frac{1}{2}gt^2$ (Eq. 4.13) with $y = -45.0$ m and $v_{y0} = 10.0$ m/s (we have chosen the top of the building as the origin, as in Figure 4.10):

$$-45.0\text{ m} = (10.0\text{ m/s})t - \tfrac{1}{2}(9.80\text{ m/s}^2)t^2$$

Solving the quadratic equation for t gives, for the positive root, $t = 4.22$ s. Does the negative root have any physical meaning? (Can you think of another way of finding t from the information given?)

(b) What is the speed of the stone just before it strikes the ground?

Solution The y component of the velocity just before the stone strikes the ground can be obtained using the equation $v_y = v_{y0} - gt$ (Eq. 4.11) with $t = 4.22$ s:

$$v_y = 10.0\text{ m/s} - (9.80\text{ m/s}^2)(4.22\text{ s}) = -31.4\text{ m/s}$$

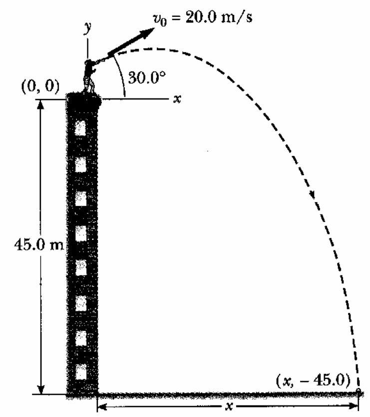

FIGURE 4.10 (Example 4.6)

Since $v_x = v_{x0} = 17.3$ m/s, the required speed is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(17.3)^2 + (-31.4)^2}\text{ m/s} = 35.9\text{ m/s}$$

Exercise Where does the stone strike the ground?

Answer 73.0 m from the base of the building.

EXAMPLE 4.7 The Stranded Explorers

An Alaskan rescue plane drops a package of emergency rations to a stranded party of explorers, as shown in Figure 4.11. If the plane is traveling horizontally at 40.0 m/s at a height of 100 m above the ground, where does the package strike the ground relative to the point at which it is released?

Reasoning The coordinate system for this problem is selected as shown in Figure 4.11, with the positive x direction to the right and the positive y direction upward.

Consider first the horizontal motion of the package. The only equation available to us is $x = v_{x0}t$ (Eq. 4.12).

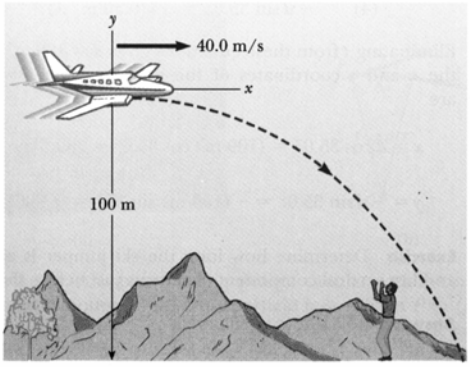

FIGURE 4.11 (Example 4.7) According to a ground observer, a package released from the rescue plane travels along the path shown. How does the path followed by the package appear to an observer on the plane (assumed to be moving at constant speed)?

The initial x component of the package velocity is the same as that of the plane when the package is released, 40.0 m/s. Thus, we have

$$x = (40.0\text{ m/s})t$$

If we know t, the length of time the package is in the air, we can determine x, the distance traveled by the package in the horizontal direction. To find t, we move to the equations for the vertical motion of the package. We know that at the instant the package hits the ground, its y coordinate is -100 m. We also know that the initial component of velocity of the package in the vertical direction, v_{y0}, is zero because the package was released with only a horizontal component of velocity.

Solution From Equation 4.13, we have

$$y = -\tfrac{1}{2}gt^2$$

$$-100\text{ m} = -\tfrac{1}{2}(9.80\text{ m/s}^2)t^2$$

$$t^2 = 20.4\text{ s}^2$$

$$t = 4.51\text{ s}$$

The value for the time of flight substituted into the equation for the x coordinate gives

$$x = (40.0\text{ m/s})(4.51\text{ s}) = 180\text{ m}$$

The package hits the ground not directly under the drop point but 180 m to the right of that point.

Exercise What are the horizontal and vertical components of the velocity of the package just before it hits the ground?

Answer $v_x = 40.0$ m/s; $v_y = -44.1$ m/s.

EXAMPLE 4.8 The End of the Ski Jump

A ski jumper travels down a slope and leaves the ski track moving in the horizontal direction with a speed of 25.0 m/s as in Figure 4.12. The landing incline below her falls off with a slope of 35.0°. (a) Where does she land on the incline?

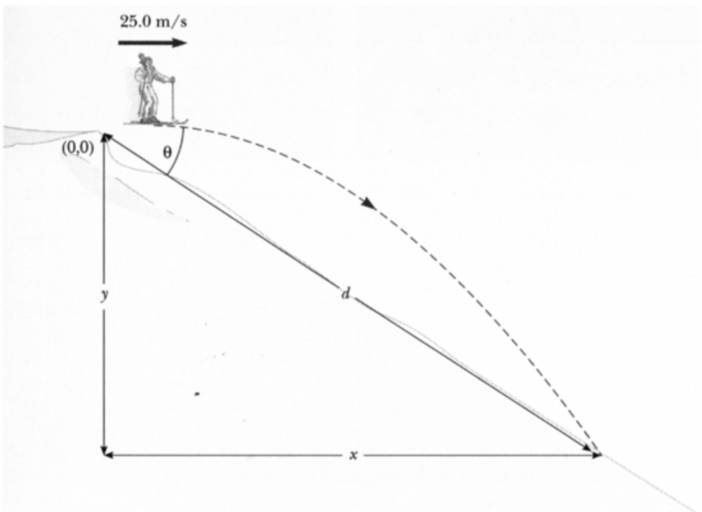

FIGURE 4.12

Solution It is convenient to select the origin ($x = y = 0$) at the beginning of the jump. Since $v_{x0} = 25.0$ m/s, and $v_{y0} = 0$ in this case, Equations 4.12 and 4.13 give

$$(1) \qquad x = v_{x0}t = (25.0 \text{ m/s})t$$

$$(2) \qquad y = v_{y0}t - \tfrac{1}{2}gt^2 = -\tfrac{1}{2}(9.80 \text{ m/s}^2)t^2$$

Taking d to be the distance she travels along the incline before landing, then from the right triangle in Figure 4.12, we see that her x and y coordinates at the point of landing are $x = d\cos 35.0°$ and $y = -d\sin 35.0°$. Substituting these relationships into (1) and (2) gives

$$(3) \qquad d\cos 35.0° = (25.0 \text{ m/s})t$$

$$(4) \qquad -d\sin 35.0° = -\tfrac{1}{2}(9.80 \text{ m/s}^2)t^2$$

Eliminating t from these equations gives $d = 109$ m. Hence, the x and y coordinates of the point at which she lands are

$$x = d\cos 35.0° = (109 \text{ m})\cos 35.0° = 89.3 \text{ m}$$

$$y = -d\sin 35.0° = -(109 \text{ m})\sin 35° = -62.5 \text{ m}$$

Exercise Determine how long the ski jumper is airborne and her vertical component of velocity just before she lands.

Answer 3.57 s; $v_y = -35.0$ m/s.

EXAMPLE 4.11 A Boat Crossing a River

A boat heading due north crosses a wide river with a speed of 10.0 km/h relative to the water. The river has a uniform speed of 5.00 km/h due east relative to Earth. Determine the velocity of the boat relative to a stationary ground observer.

Solution We know

$\mathbf{v}_{br}$ = the velocity of the boat, b, relative to the river, r

$\mathbf{v}_{re}$ = the velocity of the river, r, relative to Earth, e

and we want $\mathbf{v}_{be}$, the velocity of the *boat* relative to *Earth*. The relationship between these three quantities is

$$\mathbf{v}_{be} = \mathbf{v}_{br} + \mathbf{v}_{re}$$

The terms in the equation must be manipulated as vector quantities; the vectors are shown in Figure 4.19. The quantity $\mathbf{v}_{br}$ is due north, $\mathbf{v}_{re}$ is due east, and the vector sum of the two, $\mathbf{v}_{be}$, is at an angle θ, as defined in Figure 4.19. Thus, the speed of the boat relative to Earth can be found from the Pythagorean theorem:

$$v_{be} = \sqrt{v_{br}^2 + v_{re}^2} = \sqrt{(10.0)^2 + (5.00)^2} \text{ km/h} = 11.2 \text{ km/h}$$

The direction of $\mathbf{v}_{be}$ is

$$\theta = \tan^{-1}\left(\frac{v_{re}}{v_{br}}\right) = \tan^{-1}\left(\frac{5.00}{10.00}\right) = 26.6°$$

Therefore, the boat will be traveling at a speed of 11.2 km/h in the direction 63.4° north of east relative to Earth.

Exercise If the width of the river is 3.0 km, find the time it takes the boat to cross the river.

Answer 18 min.

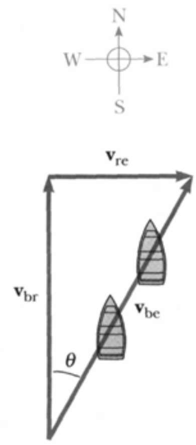

FIGURE 4.19 (Example 4.11)

EXAMPLE 4.12 Which Way Should We Head?

If the boat of the preceding example travels with the same speed of 10.0 km/h relative to the water and is to travel due north, as in Figure 4.20, what should be its heading?

Solution As in the previous example, we know $\mathbf{v}_{br}$ and $\mathbf{v}_{re}$, and we want to find $\mathbf{v}_{be}$. The relationship between these three quantities, $\mathbf{v}_{be} = \mathbf{v}_{br} + \mathbf{v}_{re}$, is shown in Figure 4.20. That is, the boat must head upstream in order to be pushed directly northward across the river. The speed v_{be} can be found from the Pythagorean theorem:

$$v_{be} = \sqrt{v_{br}^2 - v_{re}^2} = \sqrt{(10.0)^2 - (5.00)^2} \text{ km/h} = 8.66 \text{ km/h}$$

The direction of $\mathbf{v}_{be}$ is

$$\theta = \tan^{-1}\left(\frac{v_{re}}{v_{be}}\right) = \tan^{-1}\left(\frac{5.00}{8.66}\right) = 30.0°$$

The boat must steer a course 30.0° west of north.

Exercise If the width of the river is 3.0 km, how long does it take the boat to cross the river?

Answer 21 min.

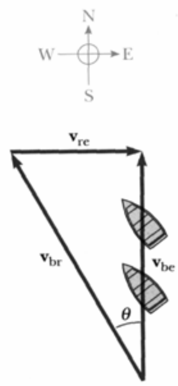

FIGURE 4.20 (Example 4.12)