Chapter 2

Equations for Motion with **Constant Acceleration**

No.	Equation	missing
(1)	$v = v_o + at$	$x - x_o$
(2)	$x - x_o = v_o t + \frac{1}{2}at^2$	v
(3)	$v^2 = v_o^2 + 2a(x - x_o)$	t
(4)	$x - x_o = \frac{1}{2}(v_o + v)t$	a
(5)	$x - x_o = vt - \frac{1}{2}at^2$	v_o

The first two equations (1) and (2) are the essential equations because the other three equations can be derived from their combinations.

Let us first derive the first two essential equations:

Equation 1:

If the acceleration is constant then the average acceleration and the instantaneous acceleration are equal.

$$a_{in} = a_{avg} = a = \frac{v - v_o}{t - 0} = \frac{v - v_o}{t}$$

where v_o is the initial velocity at t = 0

rewriting the equation

$$at = v - v_0 \quad \Rightarrow \quad v = v_o + at \qquad (1)$$

Equation 2:

To Derive equation (2) we need three equations, one of them is equation (1), and the other two equations are (6) and (8) as follows:

We know that the average of two values can be obtained by adding them and dividing the result by two, therefore for a linear velocity we can calculate the average velocity as:

$$v_{avg} = \frac{v_o + v}{2} = \frac{1}{2}(v_o + v) \qquad (6)$$

For the third equation we can use the standard v_{avg} equation which we learned previously:

$$v_{avg} = \frac{x - x_o}{t} \qquad (7)$$

Now combining both equations (1) and (6) by substituting v in equation (6)

$$v_{avg} = \frac{1}{2}(v_o + (v_o + at)) \qquad (8)$$

Then replacing v_{avg} by its value from equation (7) into equation (8)

$$\frac{x - x_o}{t} = \frac{1}{2}v_o + \frac{1}{2}v_o + \frac{1}{2}at$$

Multiplying both sides by t then

$$x - x_o = v_o t + \frac{1}{2}at^2 \qquad (2)$$

Equation 3:

Starting from equation (1) : $v = v_o + at$ and then rewriting it again as : $t = \frac{v - v_o}{a}$

substitute this into equation (2) :

$$x - x_o = v_o\left(\frac{v - v_o}{a}\right) + \frac{1}{2}a\left(\frac{v - v_o}{a}\right)^2$$

$$x - x_o = \frac{v_o v - v_o^2}{a} + \frac{1}{2}a\frac{(v - v_o)^2}{a^2}$$

$$x - x_o = \frac{v_o v - v_o^2}{a} + \frac{(v - v_o)^2}{2a}$$

$$x - x_o = \frac{v_o v - v_o^2}{a} + \frac{(v^2 - 2v_o v + v_o^2)}{2a}$$

$$x - x_o = \frac{2v_o v - 2v_o^2 + v^2 - 2v_o v + v_o^2}{2a}$$

$$x - x_o = \frac{v^2 - v_o^2}{2a}$$

$$2a(x - x_o) = v^2 - v_o^2$$

$$v^2 = v_o^2 + 2a(x - x_o) \qquad (3)$$

Equation 4:

Starting from equation (1) : $v = v_o + at$ then rewriting it again as : $a = \frac{v - v_o}{t}$

substitute this into equation (2) :

$$x - x_o = v_o t + \frac{1}{2}\left(\frac{v - v_o}{t}\right)t^2$$

$$x - x_o = v_o t + \tfrac{1}{2}\left(v - v_o\right)t$$

$$x - x_o = v_o t + \tfrac{1}{2}vt - \tfrac{1}{2}v_o t$$

$$x - x_o = \tfrac{1}{2}(v_o + v)t \qquad (4)$$

Equation 5:

Starting from equation (1) : $v = v_o + at$ then rewriting it again as : $v_o = v - at$

substitute this into equation (2) :

$$x - x_o = (v - at)t + \tfrac{1}{2}at^2$$

$$x - x_o = vt - at^2 + \tfrac{1}{2}at^2$$

$$x - x_o = vt - \tfrac{1}{2}at^2 \qquad (5)$$

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