10.5 Elastic Collisions in One Dimention

Stationary Target (at rest):

We can use the conservation of momentum Law and Kinetic Energy Law: if the second particle was at rest.

- **momentum:** $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$ ..... (1)

$$\Rightarrow m_1 v_{1i} - m_1 v_{1f} = m_2 v_{2f}$$

$$\Rightarrow m_2 v_{2f} = m_1 (v_{1i} - v_{1f}) \quad \text{------- (2)}$$

- **Kinetic Energy:** $\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$ divide by $\frac{1}{2}$

$$m_1 v_{1i}^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2$$

$$m_2 v_{2f}^2 = m_1 v_{1i}^2 - m_1 v_{1f}^2$$

$$m_2 v_{2f}^2 = m_1 (v_{1i}^2 - v_{1f}^2) \quad \text{(فرق بين مربعين)}$$

$$m_2 v_{2f}^2 = m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f}) \quad \text{--- (3)}$$

divide equ. (3) by (2)

$$\frac{m_2 v_{2f}^2}{m_2 v_{2f}} = \frac{m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f})}{m_1 (v_{1i} - v_{1f})}$$

$$v_{2f} = v_{1i} + v_{1f} \quad \text{------- (4)}$$

also $\Rightarrow$

$$v_{1f} = v_{2f} - v_{1i} \quad \text{------- (5)}$$

Substitute equ. (4) into (1)

$$m_1 v_{1i} = m_1 v_{1f} + m_2 (v_{1i} + v_{1f})$$

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{1i} + m_2 v_{1f}$$

$$m_1 v_{1i} = v_{1f} (m_1 + m_2) + m_2 v_{1i}$$

$$v_{1i} (m_1 - m_2) = v_{1f} (m_1 + m_2)$$

$$\boxed{v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}} \quad \#_1$$

Substitute equ (5) into (1)

$$m_1 v_{1i} = m_1 (v_{2f} - v_{1i}) + m_2 v_{2f}$$

$$m_1 v_{1i} = m_1 v_{2f} - m_1 v_{1i} + m_2 v_{2f}$$

$$m_1 v_{1i} + m_1 v_{1i} = v_{2f} (m_1 + m_2)$$

$$2 m_1 v_{1i} = v_{2f} (m_1 + m_2)$$

$$\boxed{v_{2f} = \frac{2 m_1}{m_1 + m_2} v_{1i}} \quad \#_2$$

For a moving target :

momentum :

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{-------- (1)}$$

$$\Rightarrow \quad m_1 v_{1i} - m_1 v_{1f} = -m_2 v_{2i} + m_2 v_{2f}$$

$$\Rightarrow \quad m_1 (v_{1i} - v_{1f}) = -m_2 (v_{2i} - v_{2f}) \quad \text{-------- (2)}$$

Kinetic :

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

divide by $\frac{1}{2}$ $\quad m_1 v_{1i}^2 + m_2 v_{2i}^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2$

$$\Rightarrow \quad m_1 (v_{1i}^2 - v_{1f}^2) = -m_2 (v_{2i}^2 - v_{2f}^2) \quad \text{(الفرق بين مربعين)}$$

$$m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f}) = -m_2 (v_{2i} - v_{2f})(v_{2i} + v_{2f}) \quad \text{---- (3)}$$

divide equation (3) by (2)

$$\frac{m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f})}{m_1 (v_{1i} - v_{1f})} = \frac{-m_2 (v_{2i} - v_{2f})(v_{2i} + v_{2f})}{-m_2 (v_{2i} + v_{2f})}$$

$$v_{1i} + v_{1f} = v_{2i} + v_{2f}$$

$$\Rightarrow \quad v_{2f} = v_{1i} + v_{1f} - v_{2i} \quad \text{-------- (4)}$$

also $\Rightarrow \quad v_{1f} = v_{2i} + v_{2f} - v_{1i} \quad \text{-------- (5)}$

Substitute equation (4) into (1)

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 (v_{1i} + v_{1f} - v_{2i})$$

$$\Rightarrow \quad m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{1i} + m_2 v_{1f} - m_2 v_{2i}$$

$$m_1 v_{1i} + m_2 v_{2i} = v_{1f} (m_1 + m_2) + m_2 v_{1i} - m_2 v_{2i}$$

$$v_{1f} (m_1 + m_2) = m_1 v_{1i} + m_2 v_{2i} - m_2 v_{1i} + m_2 v_{2i}$$

$$= v_{1i} (m_1 - m_2) + v_{2i} (m_2 + m_2)$$

$$= v_{1i} (m_1 - m_2) + v_{2i}\, 2m_2$$

$$\boxed{v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}}$$

Substitute equation (5) into (1)

$$m_1 v_{1i} + m_2 v_{2i} = m_1 (v_{2i} + v_{2f} - v_{1i}) + m_2 v_{2f}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{2i} + m_1 v_{2f} - m_1 v_{1i} + m_2 v_{2f}$$

$$= v_{2f} (m_1 + m_2) + m_1 v_{2i} - m_1 v_{1i}$$

$$v_{2f} (m_1 + m_2) = m_1 v_{1i} + m_2 v_{2i} - m_1 v_{2i} + m_1 v_{1i}$$

$$= v_{1i} (m_1 + m_1) + v_{2i} (m_2 - m_1)$$

$$\Rightarrow \quad \boxed{v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}}$$