(1) A bullet of mass m_1 and initial speed v_1, hits a block of wood of mass m_2 at rest. The bullet is stopped in the block. If $m_1 = 20$ gm, $m_2 = 980$ gm and $v_1 = 200$ m/s, find the final energy of the system.

Solution

We can't use the Energy conservation law because we do not know if surface is frictionless or rough.

– Therefore we can use the conservation of Linear Momentum

$$\vec{P}_i = \vec{P}_f$$

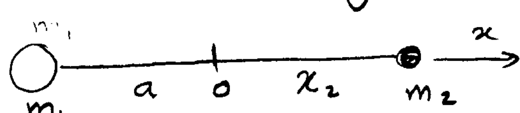
$$m_1 v_1 = (m_1 + m_2) v_f$$

$$\Rightarrow \quad v_f = \frac{m_1 v_1}{m_1 + m_2} = \frac{4}{.980 + .02} = \frac{4}{1} = 4 \text{ m/s}$$

Once we calculated the velocity we can calculate the energy

$$K = \frac{1}{2}(m_1 + m_2) v^2 = \frac{1}{2}(1)(4)^2 = \boxed{8\text{ J}} \ \#$$

(2) Two masses m_1 and m_2 are located at the points $x_1 = -a$ and x_2 respectively as shown. Suppose that $m_1 = 3m_2$. Find the position x_2 such that the CM (Center of mass) is at the origin is. Take $a = 2$ meters.



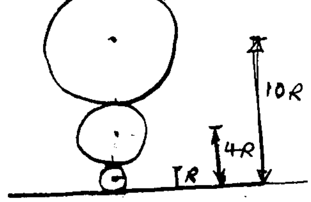
Solution.

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \text{zero}$$

$$\Rightarrow \quad m_1 x_1 = -m_2 x_2$$

$$\Rightarrow \quad x_2 = -\frac{m_1 x_1}{m_2} = -\frac{3m_2(-2)}{m_2} = \boxed{+6\text{ m}} \ \#$$

(3) Three ~~copper~~ discs of radii R, 2R, 4R and of the ~~same~~ same thickness are arranged as shown in the figure. Find the center of mass of the system with respect to the origin at (O) (in m). Take R = .2 m.



$$y_{com} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} =$$

we do not know their masses but we know their volumes:

(1) $\pi R^2 \times$ thickness (2) $\pi (2R)^2$ thick. (3) $\pi (4R)^2$ thick.

(1) πR^2 th. (2) $\pi 4R^2$ th. (3) $\pi 16R^2$ th.

$$\Rightarrow \frac{(\pi R^2 th) y_1 + (4\pi R^2 th) y_2 + (16\pi R^2 th) y_3}{\pi R^2 th + 4\pi R^2 th + 16 \pi R^2 th}$$

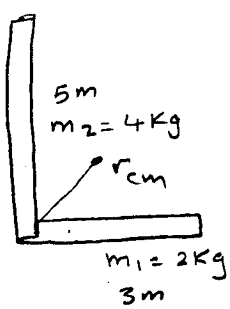
common factor $\pi R^2 th$

$$\frac{(\pi R^2 th)}{(\pi R^2 th)} \frac{(1)y_1 + (4)y_2 + (16)y_3}{1+4+16} =$$

But $y_1 = R$ $y_2 = 4R$ $y_3 = 10R$

$$\therefore \frac{(1)R + 16R + 160R}{21} = \frac{177R}{21} = \frac{(177)(.2)}{21} = \boxed{1.69\ m}$$ #

(4) Determine the distance from O of the center of mass of the L-shaped object shown, which consists of two uniform rods. The horizontal rod is 3m long and has a mass of 2 Kg and the vertical rod is 5m long and has a mass of 4 kg. (in meters)



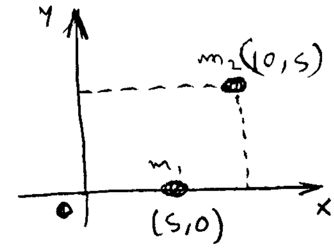
Solution

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(2)(1.5) + 0}{6} = .5m$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{0 + (4)(2.5)}{6} = \frac{10}{6} = 1.67m$$

$$\therefore r_{cm} = \sqrt{(.5)^2 + (1.67)^2} = \sqrt{3.028} = \boxed{1.74\ m}$$ #

(5) Two particles of masses $m_1 = 2\,Kg$ and $m_2 = 3\,Kg$ are located at points of coordinates (5,0) and (10,5) respectively, as shown in the figure. Determine the coordinates of the position of a third particle of mass $m_3 = 5\,Kg$ such that the center of mass of the three particles is located at the origin. All coordinates are in meters.



Solution

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$0 = \frac{2(5) + 3(10) + 5(x_3)}{2+3+5}$$

$$2(5) + 3(10) + 5(x_3) = 0$$

$$5(x_3) = -40$$

$$x_3 = -\frac{40}{5} = \boxed{-8\,m}$$

$$Y_{cm} = \frac{m_1 Y_1 + m_2 Y_2 + m_3 Y_3}{m_1 + m_2 + m_3} = 0$$

$$m_3 Y_3 = -m_1 Y_1 + m_2 Y_2 =$$

$$= \frac{-0 + 3(5)}{5} = \boxed{-3\,m}$$

$$\therefore \quad r_{cm} = \boxed{(-8, -3)\,m} \quad \#$$

(6) Three particles of equal masses $m_1 = m_2 = m_3 = 1\,Kg$. have the following position vectors:

$$\vec{r}_1 = 2\hat{i} \quad , \quad \vec{r}_2 = 4\hat{i} + 3\hat{j} \quad , \quad \vec{r}_3 = 3\hat{i} + 6\hat{j}$$

What is the position vectors of their center of mass

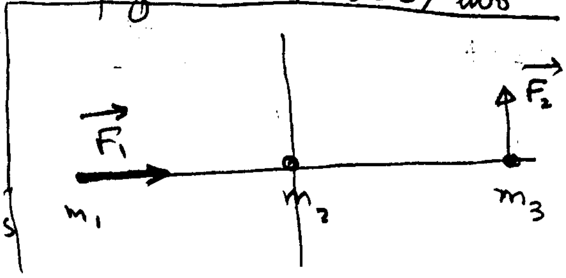
Solution

$$\vec{r}_{cm} = \frac{m}{M}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3)$$

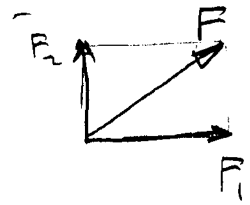
$$\vec{r}_{cm} = \frac{1}{3}\left[(2\hat{i}) + (4\hat{i} + 3\hat{j}) + (3\hat{i} + 6\hat{j})\right]$$

$$= \frac{1}{3}(9\hat{i} + 9\hat{j}) = \frac{9}{3}(\hat{i} + \hat{j}) = \boxed{3(\hat{i} + \hat{j})} \quad \#$$

(7) Three particles are arranged such that m_2 is located at the center of mass of the system as shown in the figure. At $t=0$, two forces of magnitude 20 N each are applied to m_1 and m_3 as shown. If $m_1 = 2$ Kg, $m_2 = 3$ Kg and $m_3 = 5$ Kg, at what distance from m_2 will the center of mass to be at $t = 3$ s.



Solution



$$F = \sqrt{F_1^2 + F_2^2} = 20\sqrt{2}$$

$$\Sigma F_{net} = M a_{cm}$$

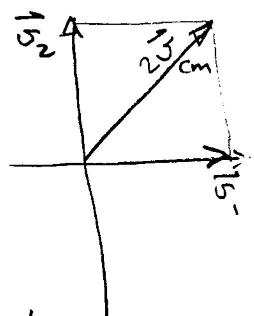
$$20\sqrt{2} = (2+3+5)\, a_{cm}$$

$$a_{cm} = 2\sqrt{2} \ \ m/s^2$$

$$x = \tfrac{1}{2} a t^2 = \tfrac{1}{2}(2\sqrt{2})(3)^2 = 12.7\,m \simeq \boxed{13\ m} \quad \#$$

(8) An object explodes into two fragments of equal masses; one is projected towards the east at 4 m/s, the other is projected towards north at 6 m/s. What was the magnitude and direction (relative to the positive x-axis) of the object's velocity before the explosion?

Solution



$$M\vec{v}_{cm} = m_1\vec{v_1} + m_2\vec{v_2}$$

$$2m\,\vec{v}_{cm} = m\vec{v_1} + m\vec{v_2} \qquad \text{divide by } m$$

$$2\vec{v}_{cm} = \vec{v_1} + \vec{v_2}$$

$$2|v_{cm}| = \sqrt{v_1^2 + v_2^2} = \sqrt{(4)^2 + (6)^2} = 7.2\ m/s$$

$$\therefore\ v_{cm} = 3.6\ \ m/s$$

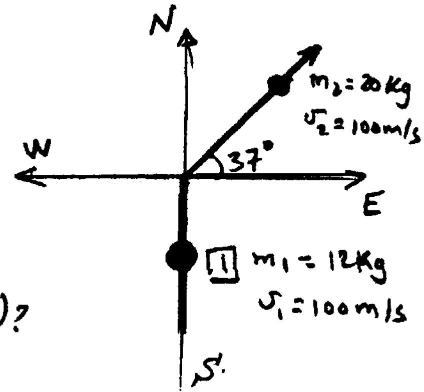
$$\theta = \tan^{-1}\frac{6}{4} = 56^0$$

Since $\Sigma F_{net} = \text{zero}$, momentum is conserved

$$(M v_{cm})_{before} = (M v_{cm})_{after}$$

This means that v_{cm} has the same magnitude and direction before explosion as after explosion

(9) A bomb of 40 Kg initially at rest explodes into three parts. The first one has mass 12 kg and is directed downward (South) with speed of 100 m/s. The second part of 20 Kg mass is directed 37° above the horizontal North of east with speed of 100 m/s. What is the velocity of the third part in (m/s)?



Solution:

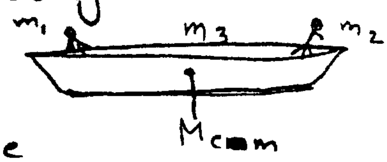
$$m_1 v_1 + m_2 v_2 + m_3 v_3 = 0$$

$$v_3 = -\frac{m_1 v_1 + m_2 v_2}{m_3}$$

$$= -\frac{12(-100)\hat{j} + 20\left[(100)(.8)\hat{i} + 100(.6)\hat{j}\right]}{8}$$

$$= -\frac{-1200\hat{j} + 1600\hat{i} + 1200\hat{j}}{8} = \boxed{-200\hat{i}} \quad \#$$

(10) A boat at rest of mass 25 Kg has two persons on it, a man of mass 80 Kg and a woman of mass 60 Kg. Suddenly and simultaneously the man dives towards the west with a speed of 2.5 m/s (relative to the water) and the woman towards the east with the same speed (also relative to the water). Calculate the velocity (speed and direction) of the boat immediately after the dive.



Solution: Consider East is positive.

Since $F_{ext.} = 0$, v_{com} is constant or zero, and since velocity of center of mass is 0 then find $v_{cm} = 0$

$$\therefore \quad v_{cm,f} = \frac{m_1 v_{1f} + m_2 v_{2f} + m_3 v_{3f}}{M} = 0$$

Solving for v_{3f} we find

$$v_{3f} = -\frac{m_1 v_{1f} + m_2 v_{2f}}{m_3} = -\frac{80(-2.5) + 60(2.5)}{25}$$

$$= -\left(\frac{-200 + 150}{25}\right) = +2 \text{ m/s} \quad \text{Towards the East}$$