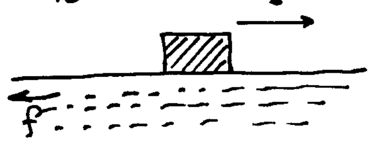
1 A sled of mass m is moving on a frozen pond with an initial velocity v_0. Find the distance travelled by the sled till its velocity is reduced to half its initial value, i.e $\frac{1}{2}v_0$. Take $v_0 = 8\,m/s$ and $\mu_k = .2$. Hint: Use the W-K theorem.



Solution:

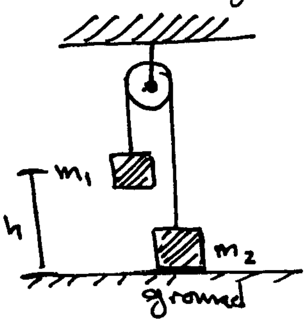
$$W_f = K_f - K_i \qquad (\text{from } W_{net} = \Delta K)$$

$$-\mu N \cdot d = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2 \qquad (N = mg)$$

$$d = \frac{v_i^2 - v_f^2}{2\mu g} = \frac{(8)^2 - (4)^2}{2(0.2)(10)} = \boxed{12\ m}$$

(you also can use $\Delta E_{mec} + \Delta E_{th} = 0$ for isolated system)

2 In the Atwood machine shown $m_2 = 2\,Kg$, $m_1 = 4\,Kg$, and the system starts from rest. When released, m_1 reaches the ground with a speed $v = 5\,m/s$. Find the initial height h of m_1 as shown:



Solution: for isolated system and conservative forces.

$$E_{mec_1} = E_{mec_2} \Rightarrow K_1 + U_1 = K_2 + U_2$$

consider two incidents 1, 2

(initial) ① when the system just started to move so:

m_1 has potential energy due to gravity and no Kinetic energy.

m_2 has no potential energy and no Kinetic energy.

(final) ② when m_1 hits the ground with v_f with Kinetic energy but no potential

m_2 will have potential energy and Kinetic energy.

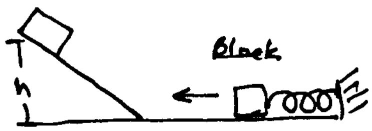
$$\Rightarrow \Big[\underbrace{(m_1 g h + 0)}_{\text{for } m_1} + \underbrace{(0 + 0)}_{\text{for } m_2}\Big]_i = \Big[\underbrace{(0 + \tfrac{1}{2} m_1 v^2)}_{\text{for } m_1} + \underbrace{(m_2 g h + \tfrac{1}{2} m_2 v^2)}_{\text{for } m_2}\Big]_f$$

$$g h (m_1 - m_2) = \tfrac{1}{2}(m_1 + m_2) v^2$$

$$h = \frac{(m_1 + m_2) v^2}{2g(m_1 - m_2)} = \frac{(4+2)(5)^2}{2(10)(4-2)} = \boxed{3.75\ m}$$

3 A block of mass m is pressed along a spring of constant K by a distance x and then released from rest. The block then moves along a horizontal plane, followed by an incline, as shown. The work done by friction during the journey is $-100\,J$. The block moves up the incline to a height h. Let $x = .5\,m$, and $h = 2\,m$ and $m = 5\,kg$.

Find the spring constant k.



Solution:

$$W_f = E_2 - E_1$$

where W_f is work done by friction

$$= mgh - \tfrac{1}{2} K x^2$$

for isolated system

$$\Delta E_{mec} + \Delta E_{th} = 0$$

$$\Delta E_{mec} = -\Delta E_{th}$$

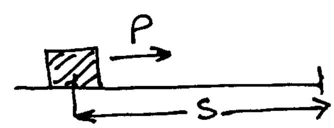
$$\Delta E_{mec} = W_{friction}$$

$$K = \frac{2(mgh - W_f)}{x^2}$$

$$= \frac{2[(5)(2)(10) - (-100)]}{(.5)^2} = 1600 \ N/m$$

$$= \boxed{1.6 \times 10^3 \ N/m}$$

4 A block of mass $m = 5\,kg$ slides along rough horizontal surface under the action of a force $P = 60\,N$. Take $\mu_k = .4$ and the initial velocity to be zero. Find the velocity of the block after a displacement of $10\,m$. Use energy-work method.



Solution:

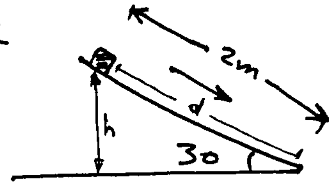
$$\Sigma W = \Delta K$$

$$(P)(d)(\cos 0^\circ) + \mu_k(mg)(d)(\cos 180^\circ) = \tfrac{1}{2} m v^2 - 0$$

$$v = \sqrt{\frac{(Pd - \mu_k mgd)2}{m}}$$

$$v = \sqrt{\frac{2[(60)(10) - (.4)(5)(10)(10)]}{5}} = \boxed{12.65 \ m/s}$$

5 A child slides down a smooth ramp of length 2 m with incline angle of 30°. If he starts from rest his speed at the bottom is (in m/s):



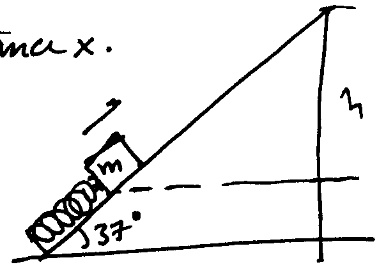
(h = d sin θ)

Solution

$$mgh = \tfrac{1}{2} m v^2$$

$$v = \sqrt{2gh} = \sqrt{2(10)(2)(.5)} = 4.5 \text{ m/s}$$

6 A block of mass m is placed in front of a compressed spring of spring constant K and compression distance x. The system is placed onto an incline rough surface of coefficient of Kinetic friction μ_k. Take $m = 5\text{kg}$, $K = 8000$ N/m, $x = 10$ cm, $\mu_k = .2$, $\theta = 37°$.



Find maximum height of the block if the spring is released?

Solution:

$$W_f = E_f - E_i$$

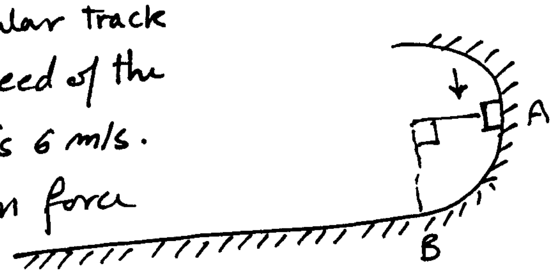
$$-\mu_k mg \cos\theta \, d = mgd \sin\theta - \tfrac{1}{2} k x^2$$

$$-(.2)(50)(.8)d = (50)(d)(0.6) - \tfrac{1}{2}(8000)(0.1)^2$$

$$d = \frac{40}{8+30} = 1.05 \text{ m}$$

$$h = d \sin\theta = \boxed{0.63 \text{ m}}$$

7 A 4Kg mass is projected down a rough circular track (radius = 2m) as shown in the figure. The speed of the mass at point A is 3 m/s. and at point B, it is 6 m/s. How much energy (in J) is dissipated by friction force as the mass goes from A to B?



Solution:

$$W_f = \Delta E = E_f - E_i$$

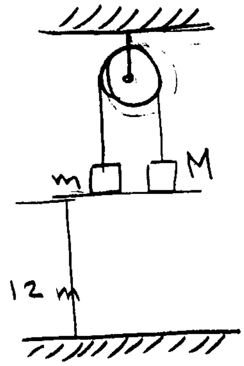
$$W_f = \tfrac{1}{2} m v_B^2 - (mgr + \tfrac{1}{2} m v_A^2)$$

$$W_f = \tfrac{1}{2}(4)(6)^2 - [4(10)(2) + \tfrac{1}{2}(4)(3)^2]$$

$$W_f = -26 \text{ J} \qquad W_f = -\Delta E_{th}$$

$$\boxed{\Delta E_{th} = 26 \text{ J}}$$

8 Two masses ($M = 3\,kg$, $m = 2\,kg$) are attached to a string that passes over a smooth pulley as shown. If the system starts from rest when both masses are 12 m above ground, determine their speed when M is 4 m above ground. Use energy method.



Solution

$$E_i = E_f$$

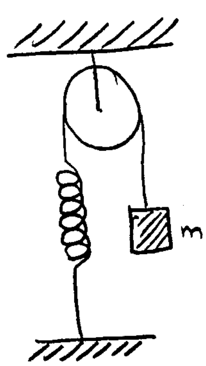
$$(m+M)gh = \tfrac{1}{2}(m+M)v^2 + mgh_1 + Mgh_2$$

$$5(10)(12) = \tfrac{1}{2}(5)v^2 + 2(10)(20) + 3(10)(4)$$

$$600 = 2.5v^2 + 400 + 120$$

$$v = \sqrt{\frac{600 - 400 - 120}{2.5}} = \boxed{5.66\ m/s}$$

9 A 20 kg mass is fastened to a light spring of constant of 380 N/m which is attached to a string that passes over a frictionless pulley. The mass is released from rest when the spring is unstretched. Determine the speed of the mass after it has dropped .4 m.



Solution

$$E_i = E_f$$

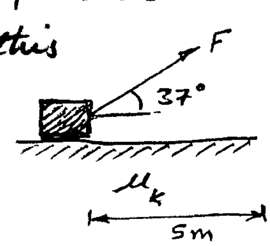
$$mgh_1 + 0 = \tfrac{1}{2}Kx^2 + \tfrac{1}{2}mv^2 + mg(h_1 - .4)$$

$$\tfrac{1}{2}mv^2 = mg(.4) - \tfrac{1}{2}Kx^2$$

$$\tfrac{1}{2}(20)v^2 = (20)(10)(.4) - \tfrac{1}{2}(380)(.4)^2$$

$$10v^2 = 80 - 30.4 \quad = \boxed{2.23\ m/s}$$

10 A 3 kg block is dragged over a rough horizontal surface by a constant force of 16 N acting at an angle of 37° above the horizontal. The speed of the block increases from 4 m/s to 6 m/s in a displacement of 5 m. What work was done by the friction force during this displacement? (Take $\cos 37 = .8$ and $\sin 37 = .6$)



Solution

$$\Sigma W = \Delta K$$

$$= \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$= \tfrac{1}{2}(3)(6)^2 - \tfrac{1}{2}(3)(4)^2 = 30\ J$$

$$W_f = F\cos\theta\, d = (16)(.8)(5) = 64\ J$$

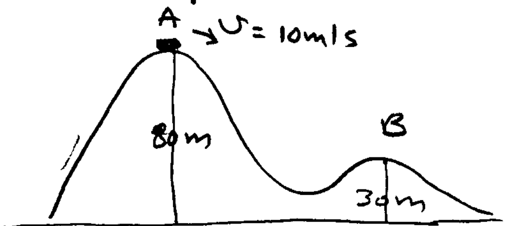
$$W_f = W - W_F = 30 - 64 = \boxed{-34\ J}$$

11 A 5,000 Kg rocket acquires a speed of 7200 Km/h in 1 minute and 40s after launch. What is the average power expended during this time interval, neglecting air resistance?

Solution

$$W = \Delta K$$
$$= \frac{1}{2} m v^2 - 0$$
$$= \frac{1}{2}(5000)\left(\frac{7200 \times 1000}{3600}\right) = 1 \times 10^{10}\ J$$
$$\bar{P} = \frac{W}{\Delta t} = \frac{1 \times 10^{10}}{(60+40)} = 1 \times 10^{8}\ W = \boxed{1 \times 10^{5}\ KW}$$

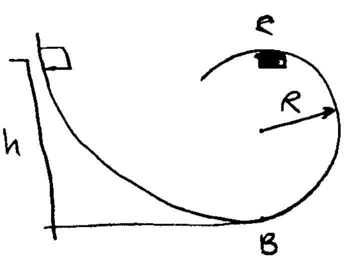
12 A 10 Kg block moves on a rough surface between two peaks as shown in the figure. The block started from point A with a speed of 10 m/s and reached point B with zero speed. Find the work done by friction.



Solution

$$W = \Delta E$$
$$= (mgh_2 + 0) - (mgh_1 + \frac{1}{2} m v^2)$$
$$= [10(10)(30)] - [10(10)(80) + \frac{1}{2}(10)(10)^2]$$
$$= 3000 - (8000 + 500)$$
$$= \boxed{-5500\ J}$$

13 At what height h above the bottom of the shown loop (point A) should a block be released from rest so that it is on the verge of losing contact with the loop at the top point, C? The loop is assumed to be frictionless. R = 2m.

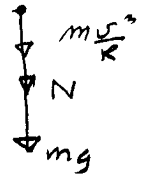


Solution at point C: $N = 0$

$$\Rightarrow mg = m\frac{v^2}{R}$$
$$v^2 = Rg$$

using Energy-Theorem: $E_1 = E_2$ at point C

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \cancel{m}g(2R) \Rightarrow gh = \frac{1}{2}Rg + g(2R)$$
$$h = \frac{1}{2}R + 2R$$
$$= 2.5R = 2.5(2) = \boxed{5\ m}$$



14 A block of mass 2.5 kg is resting on a vertical spring. The block is pushed down, compressing the spring .5m from its relaxation position and released. The force constant of the spring is 980 N/m. How high above the compressed position of the spring will the block rise?

Solution

$$E_1 = E_2$$

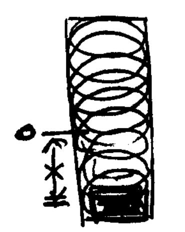
$$K_1 + U_1 = K_2 + U_2$$

$$mgh = \frac{1}{2}kx^2$$

$$h = \frac{kx^2}{2mg} = \frac{980(.5)^2}{2(2.5)(9.8)} = \boxed{5m}$$

15 A vertical spring is enclosed inside a cylinder with rough inner surface as shown in the figure. A 5kg object is attached to the spring's lower end and left to fall from rest. When the spring reaches equilibrium, it is extended by .12 m. What is the work done by friction?

$K = 416.7$



Solution:

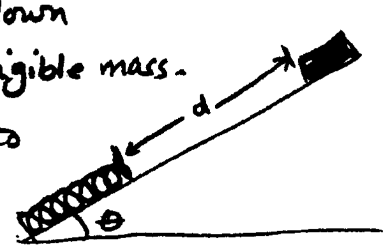
$$W_f = E_f - E_i$$

$$= \frac{1}{2}Kx^2 - mgh$$

$$= \frac{1}{2}(416.7)(.12)^2 - 5(10)(.12)$$

$$\boxed{\approx -3\,J}$$

16 A 4Kg block starts at rest and slides a distance d down a frictionless 30° incline where it runs into a spring of negligible mass. The block slides an additional 21 cm before it is brought to rest momentarily by compressing the spring, whose force constant K = 430 N/m. Find the value of d (in cm).



Solution

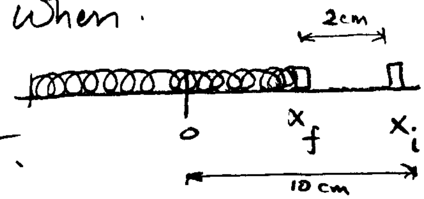
$$W = \frac{1}{2}kx^2$$

$$mg(d+x)\sin\theta = \frac{1}{2}kx^2$$

$$d = \frac{\frac{1}{2}kx^2}{mg\sin\theta} - x$$

$$= \frac{\frac{1}{2}(430)(.21)^2}{4(10)(.5)} - .21 = \boxed{26.4\text{ cm}}$$

17 A 25 Kg block on a horizontal surface is attached to a light spring ($K = 8K$ N/m). The block is pulled 10 cm to the right from its equilibrium position and released from rest. When the block has moved 2 cm towards its equilibrium position, its Kinetic energy is 12 J.



Find the energy dissipated as heat (in J) by frictional force as the block moves the 2cm.

Solution:

$$W_f = E_f - E_i$$

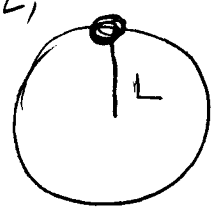
$$= \left(\tfrac{1}{2} k x_f^2 + \tfrac{1}{2} m v_f^2\right) - \left(\tfrac{1}{2} k x_i^2 + \tfrac{1}{2} m v_i^2\right)$$

$$= \left(\tfrac{1}{2}(8000)(.08)^2 + 12\right) - \tfrac{1}{2}(8000(.1)^2 + 0$$

$$= (25.6 + 12) - 40$$

$$E = -2.4 \text{ J}$$

18 A mass m, attached to a massless rod of Length L, rotates in a vertical circle, as shown. The mass is given an initial speed v_0 at the top. Take $L = .4$ m, $v_0 = 3$ m/s. Calculate its speed when it reaches the bottom. Neglect friction.



Solution:

$$E_f = E_i$$

$$\tfrac{1}{2} m v_f^2 + 0 = \tfrac{1}{2} m v_i^2 + mg(2L)$$

$$v_f = \sqrt{v_i^2 + 4gL} = \sqrt{(3)^2 + 4(10)(.4)} = 5 \text{ m/s}$$