1 An object moves under the action of an applied force $\vec{F}$:
$\vec{F} = [(2x)\hat{i} + (4y^2)\hat{j}]$ N where x and y are in meters.
If the object is displaced from an initial position $(2\hat{i} + 3\hat{j})$ m to a final position $(4\hat{i} + 7\hat{j})$ m how much work (in Joule) is done by the force during this displacement?

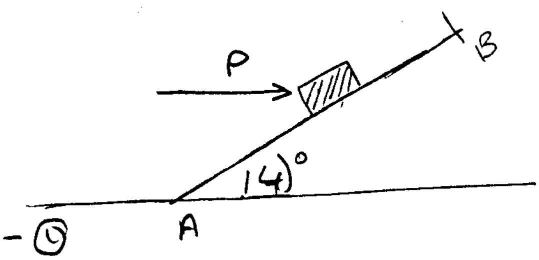
Solution:

$$W_x = \int_{x_i}^{x_f} F(x)\,dx = \int_2^4 2x\,dx = [x^2]_2^4 = [(4)^2 - (2)^2] = 12\ J$$

$$W_y = \int_{y_i}^{y_f} F(x)\,dy = \int_3^7 4y^2\,dy = \left[\tfrac{4}{3}y^3\right]_3^7 = \tfrac{4}{3}[343 - 27] = 421$$

$$\therefore\ W_{net} = W_x + W_y = 12 + 421 \boxed{= 433\ J}\ \#$$

2 A 1.4 kg block is pushed up a frictionless $14°$ incline from point A to point B by a force P as shown in the figure. Point A and B are 1.2 m apart. If the kinetic energies of the block at A and B are 3 J and 4 J respectively, how much work (in Joule) is done by the force P on the block?



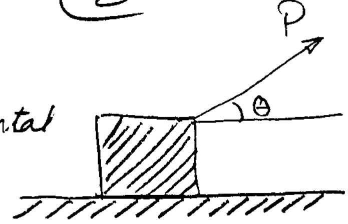
Solution $\quad \Sigma W = \Delta K$

$$W_g + W_P = K_f - K_i \quad \text{-----} \ (1)$$

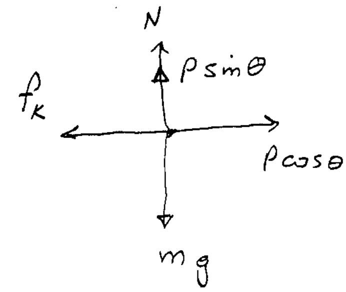
But $W_g = mgd\,(\cos 90 + 14) = -mgd \sin 14$

from equ (1). $\quad -mgd\sin\theta + W_P = K_f - K_i$

$$W_P = \Delta K + mgd \sin 14$$
$$= (4-3) + (1.4)(9.8)(1.2)\sin 14$$
$$\boxed{= 5\ J}\ \#$$



3 A force P pulls a block of mass m on a rough, horizontal surface where $\mu_k = 0.4$. If the block moves a distance $s = 20$ m, find the work done by the friction force. $m = 5$ kg, $P = 30$ N, and $\theta = 37°$.



Solution

$$W_f = -\mu_k N d$$
$$= -\mu_k (mg - P\sin\theta) d$$
$$= -(.4)[5(10) - (30 \sin 37)] 20$$
$$= \boxed{-256 \text{ J}} \quad \#$$

4 Starting from rest at $t = 0$, a 4.0 kg block is pulled across a horizontal frictionless surface by a constant horizontal force having a magnitude of 12 N. Find the Power delivered by the force at $t = 5.0$ sec.

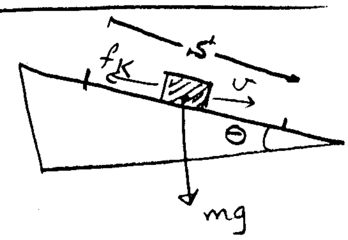
Solution given: mass = 4 kg $F = 12$ N $t = 5$ sec

$$P = F v$$

We have F but we need v after 5 sec

$$v_f = v_0 + at = 0 + at = \left(\frac{F}{m}\right)t = \left(\frac{12}{4}\right)5 = 15 \text{ m/s}$$

$$\Rightarrow P = (12)(15) = 180 \text{ W}$$



5 A block of mass 50 kg slides down an incline of angle 37° with a constant speed v. as shown. The sliding distance is 4 m. What is the net Work done on the block? what is w_g?

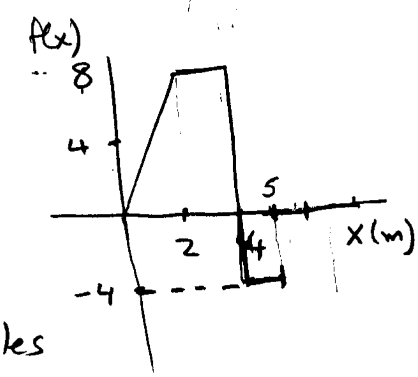
Solution: If the block is moving with constant speed then $\Delta K = K_f - K_i = $ zero.

$$\therefore W_{net} = \boxed{\text{zero}} \quad \#$$

But work done by gravity $W_g = mg s(\cos(90 - \theta))$

$$W_g = (50)(10)(4)(.6) = \boxed{1200 \text{ J}}$$

[6] Find the work done by the variable force F(x) shown for the displacement x = 0 To x = 5 m.



Solution:

Work done is the area under the curve.

There are three areas : triangle + 2 rectangles

$$= \frac{1}{2}(2)(8) + (2)(8) + (1)(-4)$$

$$= 8 + 16 - 4 = \boxed{20\ J}$$

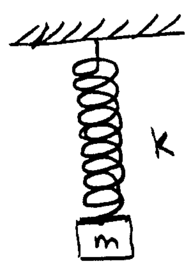
[7] Find the work done by a spring force as the spring is streched from an initial deformation $x_i = -20$ cm to a final deformation $x_f = 10$ cm. The force constant $k = 6 \times 10^4$ N/m.

Solution:

$$W = \frac{1}{2} k x_i^2 - \frac{1}{2} k \cdot x_f^2$$

$$= \frac{1}{2}(6 \times 10^4)\left[(-.2)^2 - (.1)^2\right] = \boxed{900\ J}$$

[8] A block of mass 40 kg is attached to a fixed spring as shown in figure. The spring is extended by 5 cm. When the mass is doubled (i.e. 80 kg) the work done by spring during its full extension is (in J)



Solution: first we have to find K.

$$-Kx = -mg \Rightarrow K = \frac{(40)(10)}{.05} = 8000\ \text{N/m}$$

if we doubled the mass m = 80 kg

$$\therefore\ x' = \frac{F}{K} = \frac{(80)(10)}{8000} = .1\ m$$

$$\Rightarrow W = -\frac{1}{2} K x'^2 = -\frac{1}{2}(8000)(.1)^2 = \boxed{-40\ J}$$

9 A car of mass 3000 kg accelerate from rest to 16 m/s in 1.6 min. The average power of the engine in hp is (1 hp = 750 W)

Solution:

$$\bar{P}_{avg} = \frac{W}{t} = \frac{\frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_i^2}^{\text{zero}}}{t} = \frac{\frac{1}{2}(3000)(16)^2}{1.6 \times 60} = 4000\,W$$

$$\text{in hp} \Longrightarrow \frac{4000}{750} = \boxed{5.33\ hp}$$

10 The position of a 2 kg object moving along the x-direction is given by:

$x = 4 + 8t - t^2$, x in meter and t in sec.

What is the average power delivered by the net force on the object between $t = 0$ and $t = 5$ sec (in W)? (Hint: use the work-energy theorem)

Solution:

$$P = \frac{W}{t} = \frac{\Delta K}{t} = \frac{K_f - K_i}{t}$$

$$= \frac{\frac{1}{2}m(V^2 - V_0^2)}{t}$$

$$= \frac{\frac{1}{2}(2)\left[(-2)^2 - (8)^2\right]}{5} = -12\ \text{Watt}$$

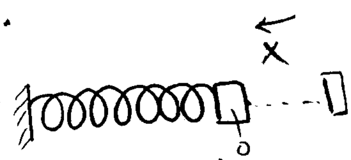
$$= \boxed{12\ W}$$

$v = 8 - 2t$

$v(0) = 8\,m/s$

$v(5) = -2\,m/s$

11 A 2 kg block sliding on a frictionless horizontal surface is attached to one end of a horizontal spring ($K = 600\,N/m$) which has its other end fixed. The speed of the block when the spring is extended 20 cm is equal to 3 m/s. What is the maximum speed (in m/s) of this block as it oscillates?



Solution

first – the maximum velocity will be at equilibrium point (at $x = 0$)

second we know that $W = \Delta K = K_f - K_i$

$$\Longrightarrow \frac{1}{2}kx^2 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$\Longrightarrow \frac{1}{2}(600)(.2)^2 = \frac{1}{2}(2)v_f^2 - \frac{1}{2}(2)(3)^2$$

$$\Longrightarrow 12 = v_f^2 - 9$$

$$v_f^2 = 12 + 9 = 21 \Longrightarrow v_f = \boxed{4.58\ m/s}$$

12 The only force acting on a 2 Kg body as it moves out of the positive x-axis is given by $F_x = (4x)$ N. where x is in m. If the kinetic energy of the body at $x = 2$ m is 20 J, what is its Kinetic energy (in J) at $x = 3$ m?

Solution

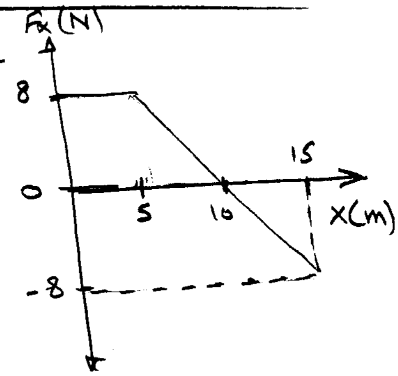
$$W = \int_{x_i}^{x_f} f(x)\,dx = \int_{x_1}^{x_f} 4x\,dx = 4\int_2^3 x\,dx$$

$$= 4\left[\left(\frac{x^2}{2}\right)\right]_2^3 = 4\left(\frac{9}{2} - \frac{4}{2}\right) = 10\ J$$

$$W = \Delta K = K_f - K_i$$

$$10 = K_f - 20 \Rightarrow K_f = 30\ J$$

13 For the variable for F_x, find the work done in the interval $x = 0$ To 15 m.



Solution

Work is area under curve

$$W = W_1 + W_2 + W_3$$

$$= (5)(8) + \tfrac{1}{2}(5)(8) + \tfrac{1}{2}(5)(-8) = \boxed{40\ J}$$

14 A 3 Kg block is dragged (pulled) over a rough horizontal surface by a constant force of 16 N acting at an angle 37° above the horizontal. The speed of the block increases from 4 m/s to 6 m/s in a displacement of 5 m. What work was done by the friction force during this displacement? (Take $\cos 37° = .8$ and $\sin 37° = .6$).

Solution

$$\Sigma W = \Delta K = K_f - K_i \quad \text{------} \quad (1)$$

$$= \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2$$

$$= \tfrac{1}{2}(3)(6)^2 - \tfrac{1}{2}(3)(4)^2 = 30\ J$$

also we know that $\Sigma W = W_{friction} + W_{force} = 30\ J$ .... (2)

But $W_{force} = F d \cos\theta = (16)(5)(\cos 37) = 64\ J$

from eqn (2) $\Rightarrow W_{friction} = 30 - 64 = \boxed{-34\ J}$

15 A 5000 Kg rocket acquired a speed of 7200 Km/h in 1 minute and 40 s after launch. What is the average power expended during this time interval, neglecting air resistance?

Solution:

average Power $\bar{P} = \frac{W}{\Delta t}$

$W = \Delta K = K_f - K_i$

$= \frac{1}{2} m v_f^2 - 0 = \frac{1}{2}(5000)\left(\frac{7200 \times 1000}{3600}\right)^2$

$= \boxed{1 \times 10^{10}\ J}$

$\bar{P}_{avg} = \frac{W}{\Delta t} = \frac{1 \times 10^{10}}{(60+40)} = 1 \times 10^{8}\ W$

$= \boxed{1 \times 10^{5}\ KW}$

16 The position of a 2 kg object is given by $x = t^3 - 4t$ in SI system. Find the power P delivered to the particle at the instant $t = 2\,s$.

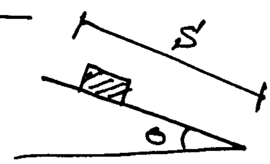
Solution $P = F v = m a v$

But v at 2. $v(2) = \frac{dx}{dt} = 3t^2 - 4 = 3(4) - 4 = 8\ m/s$

$a(2) = \frac{dv}{dt} = 6t = 6(2) = 12\ m/s^2$

$P = 2(12)(8) = \boxed{192\ W}$

17 A block of mass $m = 4$ Kg slides a distance $S = 8$ m down a smooth incline before reaching the bottom (figure). Find the total work done on the block. Take $\theta = 37°$. $\sin(37) = .6$ & $\cos(37) = .8$



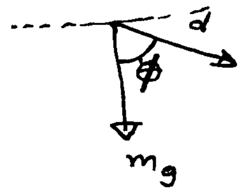
Solution $W = F d \cos\theta$

The only force acting on the block is the gravity.

$\phi = 90° - 37°$

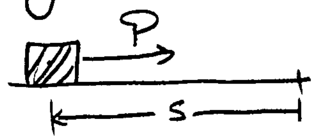
$\therefore W = m g d (\cos 90° - 37°)$

$= (4)(10)(8)(.6) = \boxed{192\ J}$



18 A block of mass $m = 5\,kg$ slides along a rough horizontal surface under the action of a force $P = 60\,N$. Take $\mu_k = .4$ and the initial velocity to be zero. Find the velocity of the block after a displacement of 10 m.

Hint: Use energy-work-method



Solution

$$\sum W = \Delta K = W_P + W_{friction}$$

$$P(d)(\cos 0^\circ) + \mu_k(mg)d(\cos 180^\circ) = \frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_i^2}$$

$$v = \sqrt{\frac{(Pd - \mu_k mgd)2}{m}} = \sqrt{\frac{2[(60)(10) - (.4)(5)(10)(10)]}{5}} = \boxed{12.65\,m/s}$$

19 A car engine delivers a power of 200 hp to a car moving at a constant speed of 216 km/h. Find the force applied by the engine on the car. Take 1hp = 750 W.

Solution

$$P = \vec{F}\cdot\vec{v}$$

$$v = 216 \times \frac{1000}{3600} = 60\,m/s$$

$$\Rightarrow F = \frac{P}{v} = \frac{200(750)}{60} = \boxed{2.5\times10^3\,N}$$

20 The position of a 2 kg object moving along the x-direction is given by: $x = 4 + 8t - t^2$ x in meters and t in seconds.
What is the average power delivered by the net force on the object between t = 0 and t = 5 sec (in W)? (Hint use the work-energy Theorem)

Solution

$$v = \frac{dx}{dt} = 8 - 2t$$

$$v(0) = 8\,m/s$$

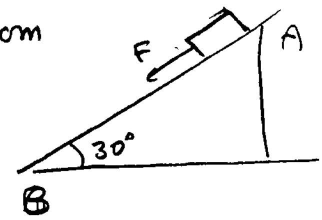
$$v(5) = 8 - 2(5) = -2\,m/s$$

$$\bar{P} = \frac{W}{\Delta t} = \frac{\frac{1}{2}m(v^2 - v_0^2)}{t}$$

$$= \frac{\frac{1}{2}(2)\cdot[(-2)^2 - (8)^2]}{5} = -12\,W$$

$$\boxed{= 12\,W}$$ #

21 · A 2Kg block slides down a frictionless incline from point A and B. A force F = 3 N acts on a block between A and B, as shown. Point A and B are 2 m apart. If the kinetic energy of the block at A is 10 J, what is the Kinetic energy of the block at B?



Solution

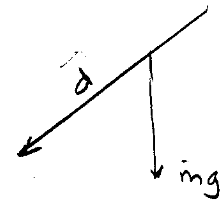
$$\Delta K = W_F + W_g = K_B - K_A$$

$$K_B = K_A + W_F + W_g$$

$$= K_A + Fd + mgd \sin 30$$

$$= 10 + 3\times 2 + (2)(10)(2)(\tfrac{1}{2})$$

$$= 10 + 6 + 20 = \boxed{36 J}$$

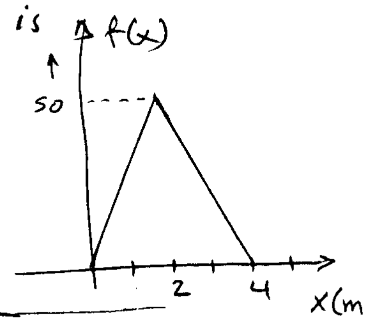


$\cos(90° - 30°) = \sin 30$

22 A particle of mass 5 kg is projected vertically upward. It returns back to the same point 4 seconds. The total Work (in J) done by the force of gravity is ??

solution $\boxed{\Sigma W_g = \text{zero}}$

23 A variable for F acts on a 2 kg particle. It varies with the position as shown in the figure. If the speed of the particle at (a) is 4 m/s. What must be its speed at (b)



Solution $W = \Delta K$

$$\tfrac{1}{2}(4)(50) = \tfrac{1}{2}(2)v_f^2 - \tfrac{1}{2}(2)(4)^2$$

$$v_f = 10.8 \text{ m/s}$$

24. A car of mass m and moving with a velocity v along a straight road has its engine turned off. It continues moving for a distance S before it stops. Take $m = 2\times 10^3$ kg. $v = 30$ m/s, and $S = 60$ m. Find the air resistance - friction force acting on the car, assuming it to be constant. Use The Work - energy theorm.

Solution: $W = \Delta K = K_f - K_i$ where $K_f = \text{zero}$

$$-FS = K_f - K_i \Rightarrow F = \frac{K_i}{S}$$

$$F = \frac{\tfrac{1}{2}(2\times 10^3)(30)^2}{60} = \boxed{1.5\times 10^4 \text{ J}}$$