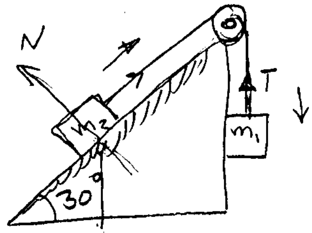
[1] Two blocks are connected over a pulley as shown. If the coefficient of Kinetic friction between the block and the incline is .3, what is the magnitude of the acceleration (in m/s^2) as the suspended block falls? ($m_1 = 3m_2$)



Solution:

for $m_1 \Rightarrow \quad m_1 g - T = m_1 a \Rightarrow T = m_1 g - m_1 a \quad \text{------} \quad (1)$

for $m_2 \Rightarrow \quad T - m_2 g \sin 30 - \mu_k m_2 g \cos\theta = m_2 a \quad \text{--} \quad (2)$

Substitute T into equation (2)

$$m_1 g - m_1 a - m_2 g \sin 30 - \mu_k m_2 g \cos\theta = m_2 a$$

$$m_1 g - m_2 g \sin 30^\circ - \mu_k m_2 g \cos\theta = m_2 a + m_1 a$$

$$m_1 g - m_2 g \sin 30^\circ - \mu_k m_2 g \cos\theta = a(m_1 + m_2)$$

But $m_1 = 3m_2 \Rightarrow \quad 3m_2 g - m_2 g \sin 30^\circ - \mu_k m_2 g \cos 30^\circ = a(4m_2)$

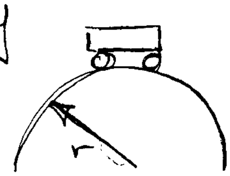
divide by $m_2 \Rightarrow \quad 3g - g \sin 30^\circ - \mu_k g \cos 30^\circ = 4a$

$$3(10) - 10(.5) - (.3)(10)(.866) = 4a$$

$$30 - 5 - 2.6 = 4a$$

$$\boxed{a = 5.6 \ m/s^2} \ \#$$

[2] A car goes over a circular hill of .5 km radius at a constant speed of 60 Km/hr. What is the force (in N) of the seat of the car on the 70 kg driver at the top of the hill?



Solution

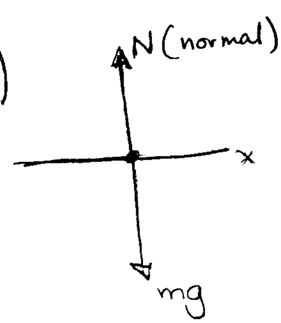
at the car $\quad N - mg = -\dfrac{mv^2}{r}$ (minus sign because acceleration is inward)

$$N = mg - \frac{mv^2}{r}$$

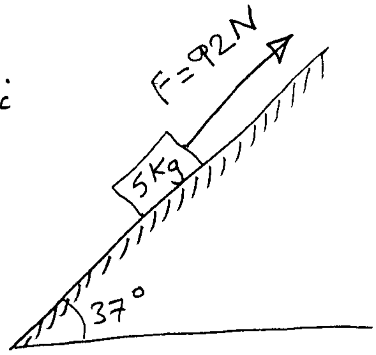
$$= m\left(g - \frac{v^2}{r}\right)$$

$$= 70\left(10 - \left(\frac{60 \times 1000}{3600}\right)^2 \cdot \frac{1}{500}\right)$$

$$\boxed{N = 661 \ N} \ \#$$



[3] In figure calculate the acceleration if the surface is rough and the coefficient of kinetic friction is .3 (in m/s^2)

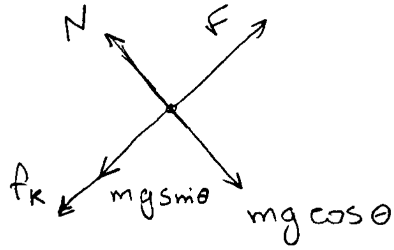


Solution:

$$F - mg\sin\theta - f_k = ma$$

$$a = \frac{92 - 5(10)(.6) - (.3)(5)(10)(.8)}{5}$$

$$\simeq \boxed{10\ m/s^2}$$



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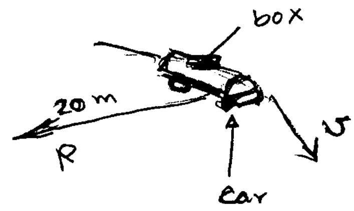
[4] A box of mass 2 kg is placed on the surface of a car. The car negotiates a curved road of radius 20 m with velocity of 8 m/s. What must be the coefficient of static friction between the box and the surface in order that the box does not move.

Solution:



$$-f_s = -ma$$

$$\mu_s N = ma$$

$$\mu_s \cancel{m}g = \cancel{m}\frac{v^2}{R}$$

$$\mu_s = \frac{v^2}{gR} = \boxed{.32}$$ at least

---

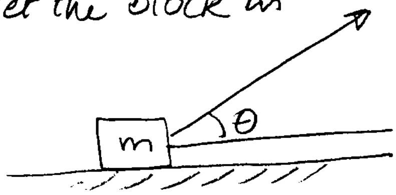
[5] A student stands 3 m away from the center of a rotating turntable. If the coefficient of static friction between the student and the table is .60, what is the speed (in m/s) at which the student will start to slip?

Solution: $-\mu_s \cancel{m}g = -\cancel{m}\frac{v^2}{R} \Rightarrow \mu_s = \frac{v^2}{gR}$

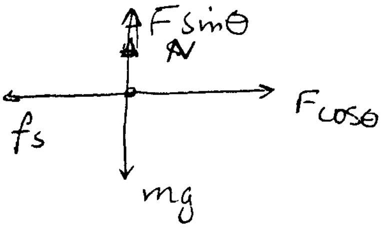
$$\Rightarrow v = \sqrt{\mu_s R g}$$

$$v = \sqrt{(.6)(3)(10)} = \boxed{4.2\ m/s}$$

6 In the figure, the coefficient of static friction between the block and the surface is .2. Take $m = 2$ kg and $\theta = 60°$. Find the value of F required to set the block in motion. (in Newton)



Solution:



y-axis: $N + F\sin\theta = mg$

$N = mg - F\sin\theta$

x-axis: $F\cos\theta - f_s = 0$

$F\cos\theta - \mu_s N = 0$

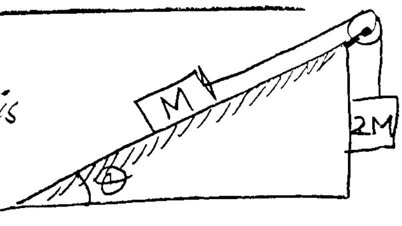
$F\cos\theta - \mu_s(mg - F\sin\theta)$

$F\cos\theta - \mu_s mg + \mu_s F\sin\theta$

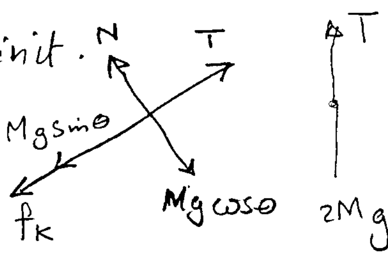
$F(\cos\theta + \mu_s \sin\theta) = \mu_s mg$

$F = \dfrac{\mu_s mg}{\cos\theta + \mu_s \sin\theta} = \dfrac{(.2)(2)(10)}{\cos 60 + .2\sin 60} = \boxed{5.9\,N}$

In the figure shown, the coefficient of kinetic friction between the block and the incline is .40 and $\theta = 40°$. What is the magnitude of the acceleration (in m/s²) of the suspended block as it falls? Disregard any pulley mass or friction in it.



Solution:



$2Mg - T = 2Ma$ for the suspended block

$T - Mg\sin\theta - f_k = Ma$ for the other block

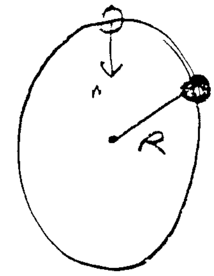
$\Rightarrow 2Mg - Mg\sin\theta - \mu_k Mg\cos\theta = 3Ma$ divid all by M

$a = \dfrac{2g - g\sin\theta - \mu_k g\cos\theta}{3}$

$= \dfrac{20 - 6.43 - 3.06}{3} = \boxed{3.5\,m/s^2}$ #

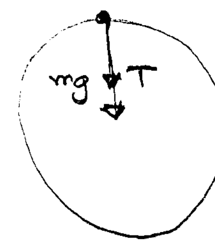
Chapter 6

[8] A .2 kg object attached to the end of a string swings in a vertical circle (radius = 80 cm). At the top of the circle the speed of the object is 4.5 m/s. What is the magnitude of the tension in the string (in Newton) at this position



Solution:

$$-mg - T = m\left(-\frac{v^2}{R}\right)$$



$$T = m\frac{v^2}{R} - mg$$

$$= m\left(\frac{v^2}{R} - g\right)$$

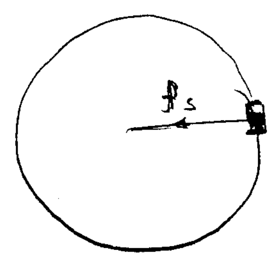
$$= .2\left(\frac{(4.5)^2}{.8} - 10\right) = \boxed{3.06\ N} \#$$

[9] A race car Travels at a speed of 40 m/s around an unbanked circular track of radius 200 m. What is the magnitude of the net force on the 80-kg driver of this car (in Newton)?

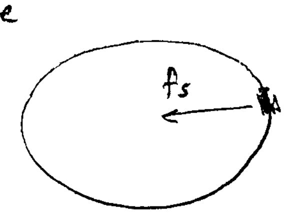
Solution:

$$-f_{s,max} = m\left(-\frac{v^2}{R}\right)$$

$$= 80\ \frac{(40)^2}{200} = \boxed{640\ N}$$



[10] A car of mass m moves uniformly around a horizontal, circular track of radius r. The maximum allowed speed before slipping is v_m. Consider $r = 80$ m and $v_m = 20$ m/s. Find the friction coefficient μ_s of the track.



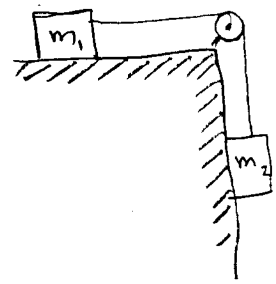
Solution:

$$-f_s = m\left(\frac{-v^2}{R}\right)$$

$$\mu_s \cancel{m} g = \cancel{m}\left(\frac{v^2}{R}\right)$$

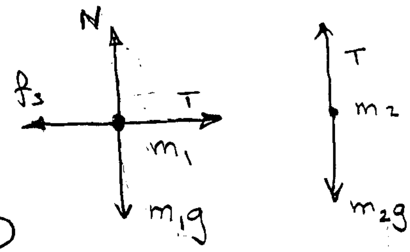
$$\mu_s = \frac{(20)^2}{(80)(10)} = \boxed{0.5}$$

11 Two blocks of masses m_1 and m_2 are attached to a string, which runs over a smooth pulley, as shown. The plane on which m_1 lies is horizontal and rough. $m_1 = 10$ Kg and the maximum m_2 value before sliding of the system is $m_2 = 6$ Kg. Find the friction coefficient μ_s of the plane.



Solution:

for $m_2 \Rightarrow \quad T - m_2 g = 0 \Rightarrow T = m_2 g$ ①



for m_1:

x-axis: $T - f_s = 0 \Rightarrow T = f_s$ ----- ②

y-axis: $N - m_1 g = 0 \Rightarrow N = m_1 g$ ----- ③

using equation ② to find μ_s

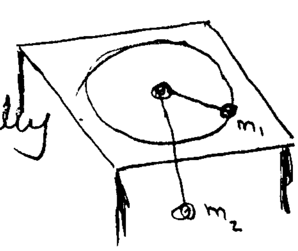
$$\mu_s N = T$$

from equ ① $\mu_s N = m_2 g$

and from equ ③ $\mu_s m_1 g = m_2 g \Rightarrow \mu_s = \frac{m_2 g}{m_1 g} = \frac{m_2}{m_1}$

$$= \frac{6}{10} = \boxed{0.6} \ \#$$

12 A mass m_1 at the end of a string, rotates on a horizontal, frictionless surface, making a circle of radius r. The string passes through a hole, and a mass $m_2 = 0.8\, m_1$ hangs vertically at the bottom end of the string. The mass m_2 is at equilibrium. Find the acceleration of the rotating mass m_1.



Solution:

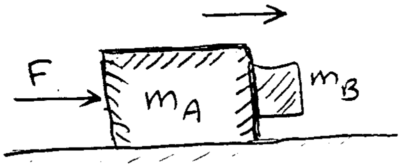
$T = m_1 \frac{v^2}{r}$ for m_1

$T = m_2 g$ for m_2

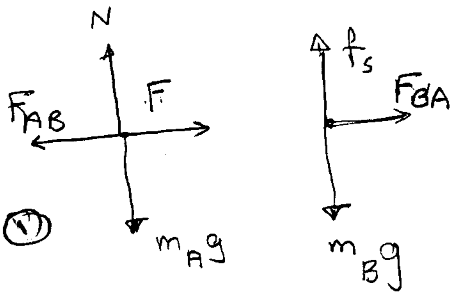
$\therefore \quad m_1 \frac{v^2}{r} = m_2 g \Rightarrow \frac{v^2}{r} = \frac{m_2}{m_1} g$

But $a = \frac{v^2}{r} = \frac{m_2}{m_1} g = \frac{.8 m_1}{m_1} g = \boxed{8\ m/s^2} \ \#$

13 Blocks of mass m_A and m_B are as shown. The system moves and the applied force F has the minimum value required for m_B not to slide down. The friction coefficient μ_s between the vertical surfaces of m_A and m_B is .5. Find the acceleration a of the blocks.



Solution



We know that $F_{BA} = F_{AB}$ but opposite

for m_B

(y-component) $f_s = m_B g$ .... (1)

$\mu_s N_B = m_B g$ But $N_B = F_{BA}$

$\therefore \mu_s F_{BA} = m_B g$ ----- (2)

(x-component) $F_{BA} = m_B a$ ----- (3)

for m_A (x-axis) $F - F_{AB} = m_A a$ ----- (4)

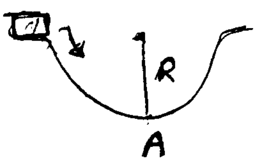
(y-axis) $N_A = m_A g$

From equations (2) and (3)

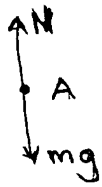
$\mu_s \cancel{m_B} a = \cancel{m_B} g$

$\Rightarrow a = \frac{g}{\mu_s} = \frac{10}{.5} = \boxed{20 \ m/s^2}$ #

14 A loaded roller-coaster car of total mass m = 800 kg. drives in a vertical circle of radius r = 14.4 m. If the maximum allowed normal force of the track is N = 5mg, find the maximum allowed speed v at the bottom point A.



Solution: $-N + mg = m(-\frac{v^2}{R})$ (consider − sign towards the center)

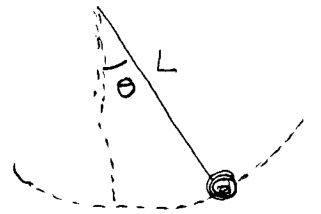


$-5\cancel{m}g + \cancel{m}g = \cancel{m}\left(\frac{-v^2}{14.4}\right)$

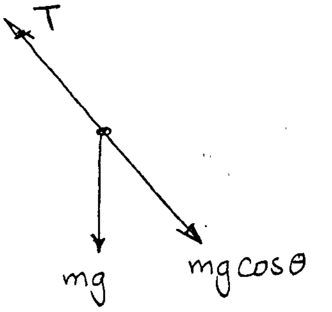
$-4g = \frac{-v^2}{14.4} \Rightarrow v^2 = 576$

$\boxed{v = 24 \ m/s}$

[15] A pendulum of length L and ball mass m swings back and forth in a vertical circle. Find the Tension T in the string when it makes angle θ with the vertical and its mass has a speed v, as shown. Take $L = 2m$ $\theta = 37°$, $v = 4$ m/s, and $m = 4$ kg.



Solution



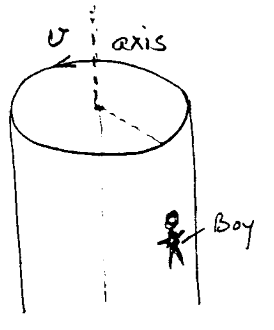
$$-T + mg\cos\theta = m\left(\frac{-v^2}{R}\right)$$

$$T = mg\cos\theta + m\left(\frac{v^2}{R}\right)$$

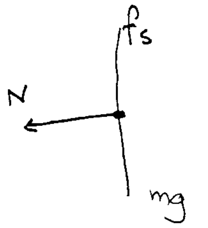
$$= (4)(10)\cos 37° + (4)\left(\frac{(4)^2}{2}\right)$$

$$= 31.95 + 32 \simeq \boxed{64\ N} \quad \#$$

[16] A boy is standing against the wall of a vertical cylinder of radius r. The cylinder is spinning around its axis. The friction coefficient between the person and the wall is μ_s. Let $r = 1.8$ m, $\mu_s = .5$. Find the minimum speed of rotation v of the wall so that the boy does not fall.



Solution:



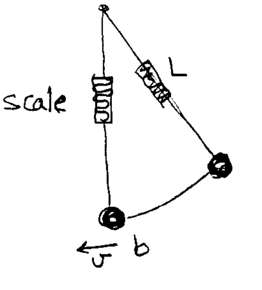
$$-N = m\frac{-v^2}{R} \Rightarrow N = m\frac{v^2}{R}$$

$$f_s = mg \Rightarrow \mu_s N = mg$$

$$\mu_s \frac{\cancel{m} v^2}{R} = \cancel{m} g$$

$$v^2 = \sqrt{\frac{Rg}{\mu_s}} = \sqrt{\frac{(1.8)(10)}{(.5)}} = 6 \text{ m/s}$$

[17] A spring (ميزان) scale is attached to a pendulum of length $L = 2m$ to read the Tension in the cord, as shown. If the mass of the pendulum $m = 2$ kg, find the reading on the scale at the bottom point b, when the speed of the mass is 8 m/s.



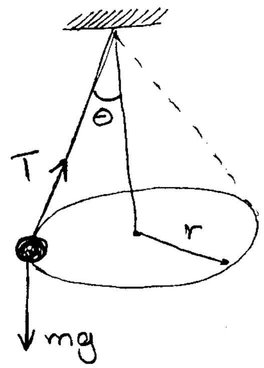
Solution:

$$-T + mg = m\left(\frac{-v^2}{R}\right) \quad \text{(at point } b\text{)}$$

$$T = mg + \frac{mv^2}{R}$$

$$= 2(10) + \frac{2(8)^2}{2} = \boxed{84\ N}$$

18 A small body of mass m is suspended from a string of length L. The body revolves in a horizontal circle of radius r with constant speed v, as in the figure. Find the speed of the body and period of revolution T_p.



Solution

$$T\cos\theta = mg \quad \text{------} \quad (1)$$

$$T\sin\theta = ma_y = \frac{mv^2}{r}$$

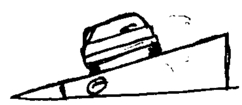
Dividing the two equations (1) & (2)

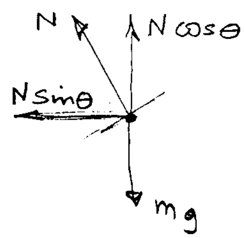
$$\tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg\tan\theta} = \boxed{\sqrt{Lg\sin\theta\tan\theta}} \quad (1)\#$$

where $r = L\sin\theta$

$$T_p = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{rg\tan\theta}} = 2\pi\sqrt{\frac{L\cos\theta}{g}} = \boxed{1.95\ \text{sec}} \quad \#_2$$

19 An engineer wishes to design a curved exit ramp for a toll road in such a way that a car will not have to rely on friction to round the curve without skidding. Suppose that a typical car rounds the curve with a speed of (13.4 m/s) and that the radius of the curve is 50 m. At what angle should the curve be banked?





Solution

If the street is not banked then the car will rely on friction. The banked street will replace the friction force by the N component in x-axis direction.

$$\Rightarrow \text{x-axis} \qquad n\sin\theta = \frac{mv^2}{r} \quad \text{-------} \quad (1)$$

$$\text{y-axis} \qquad n\cos\theta = mg \quad \text{-------} \quad (2)$$

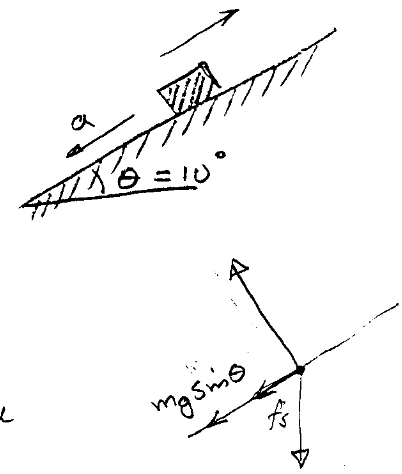
Dividing equation (1) and (2)

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left[\frac{(13.4)^2}{(50)(9.8)}\right] = \boxed{20.1^\circ} \quad \#$$

From final exams — Chapter 6

20 A 1.8 Kg block is projected up a rough 10° incline plane. As the block slides up the incline, its acceleration is 3.8 m/s² down the incline. What is the magnitude of the force of friction (in N) acting on the block?

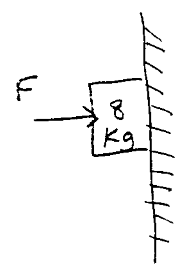


Solution: when the block is sliding up the incline its acceleration is down the incline because the block was stopping.

$\therefore -mg\sin\theta - f_s = -ma$ multiply by (−)

$$f_s = ma - mg\sin\theta$$
$$= (1.8)(3.8) - (1.8)(10)(0.174) = \boxed{3.7\ N}$$ #

21 Find the horizontal force which keeps the 8-kg block from sliding down. Take the coefficient of static friction between the block and the wall is 0.4.



Solution:

$$f_s = mg \Rightarrow \mu_s N = mg$$
$$F = N$$
$$\Rightarrow F = \frac{mg}{\mu_s} = \frac{80}{.4} = \boxed{200\,N}$$ #

22 If the coefficient of static friction between the tires of a car and the road is .10, at what maximum speed can the car round a level curve of radius 100 m without slipping?

Solution

$$f_s = \frac{mv^2}{r}$$
$$\mu_s mg = m\frac{v^2}{r} \Rightarrow v = \sqrt{\mu_s g r}$$
$$v = \sqrt{(.1)(10)(100)} = \boxed{10\ m/s}$$ #