① A particle starts from $2\hat{i}+4\hat{j}$ (m) with initial velocity $\hat{i}-3\hat{j}$ (m/s). What must be the acceleration in such a way the particle passes the origin after 2 seconds (in m/s²)?

a) $-\hat{i}+3\hat{j}$ b) $5\hat{j}$ c) $-2\hat{i}+\hat{j}$ d) $\hat{i}-\hat{j}$ e) $-2\hat{i}+5\hat{j}$ f) other

Solution $\vec{r}_1 = 2\hat{i}+4\hat{j}$ and $\vec{v} = \hat{i}-3\hat{j}$

$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ But r_2 = zero (at origin)

$\therefore \Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j}$ or $r_2 - r_1$

$= -2\hat{i} - 4\hat{j}$

$\bar{v} = \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{-2\hat{i}-4\hat{j}}{2} = -\hat{i} - 2\hat{j}$

$$\Delta\vec{r}_1 = \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

$-2\hat{i} - 4\hat{j} = (\hat{i}-3\hat{j})(t) + \frac{1}{2}\vec{a}\,t^2$ at $t = 2$

$-2\hat{i} - 4\hat{j} = 2\hat{i} - 6\hat{j} + \frac{1}{2}\vec{a}\,(4)$

$-2\hat{i} - 4\hat{j} - 2\hat{i} + 6\hat{j} = 2\vec{a}$ divide all by 2

$-\hat{i} - 2\hat{j} - \hat{i} + 3\hat{j} = \vec{a}$

$\vec{a} = -2\hat{i} + \hat{j}$ //

② A plane flies (تطير) horizontally with a constant speed of 1000 Km/h at an altitude of 5 Km from ground. A particle a is dropped. After two seconds another particle b is dropped from the plane. When the two particles a and b strike the ground. what is their separation distance?

a) 556 m b) 2000 m c) 1112 m d) 1000 m e) 10000 m other

$v = 1000\left(\dfrac{1000}{3600}\right) = 278$ m/s

$x = vt$ (horizontal distance)

$= 278\,(2) = 556$ m //

(3) The position of a particle is given by $\vec{r} = 2t\hat{i} + (20t - 2t^2)\hat{j}$, in the SI system. Find the position $\vec{r}$ of the particle when its velocity $\vec{v}$ and acceleration $\vec{a}$ are perpendicular to each other.

Solution:

velocity $\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{i} + (20 - 4t)\hat{j}$

Acceleration $\vec{a} = \frac{d\vec{v}}{dt} = -4\hat{j}$

$\Rightarrow \vec{a} = a_x\hat{i} + a_y\hat{j} = 0\hat{i} - 4\hat{j}$

When $\vec{v} \perp \vec{a}$ this means that

$\vec{v} \cdot \vec{a} = v\,a \cos 90^\circ = \text{Zero}$

But $\vec{v} \cdot \vec{a} = v_x a_x + v_y a_y$

$\Longrightarrow (2)(0) + (20 - 4t)(-4) = 0$

$\Longrightarrow -80 + 16t = 0$

$t = 5$ sec

$\Longrightarrow$ $\vec{v}$ and $\vec{a}$ are perpendicular

$\therefore$ at 5 seconds

$\vec{r}(5) = 2(5)\hat{i} + (20(5) - 2(5)^2)\hat{j}$

$= 10\hat{i} + (100 - 50)\hat{j}$

$= 10\hat{i} + 50\hat{j}$ #

(4) A ball is thrown, from the ground, with a speed of 30 m/s at an angle of 37° above the horizontal. It lands on the roof of a building at a point 24 m horizontally from the throwing point. How high is the roof?

Note : $\cos 37^\circ = .8$ - $\sin 37^\circ = .6$.

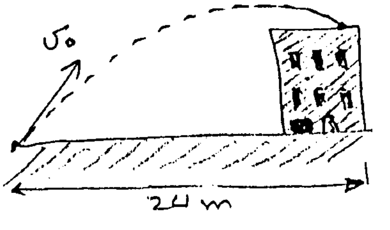
Solution: $V_{x0} = 30\cos 37 = 24$ m/s

$V_{y0} = 30 \sin 37 = \boxed{18 \text{ m/s}}$

Horizontally: $x = V_{x0}t \Rightarrow t = \frac{x}{V_{x0}} = \frac{24}{24} = 1$ s

vertically: $y = V_{y0}t - \frac{1}{2}gt^2$

$= 18(1) - \frac{1}{2}(10)(1)^2 = 13$ m #



(5) A man runs at a speed of 9.1 m/s around a circular track with a centripetal acceleration of magnitude 3.8 m/s^2. How long does he take to go completely around the track?

a) 15 s   b) 7.5 s   c) 30. s   d) 21.5 s   e) other

Solution: $\therefore a = \frac{v^2}{r} \Rightarrow r = \frac{v^2}{a} = \frac{(9.1)^2}{3.8} = 21.79$ m

$\therefore t = \frac{2\pi r}{v} = \frac{2\pi(21.79)}{9.1} = 15.s$ #

where $2\pi r$ is the circumference of a circle

(6) Raindrops are falling with a vertical of −10 m/s. If a car is moving horizontally at a speed of 25 m/s, what is (in m/s) the speed of the raindrops relative to the car?

a) 15   b) 26.9   c) 35   d) 46.9   e) other

Soln: $\vec{v}_{RC} = \vec{v}_{RG} + \vec{v}_{GC} = \vec{v}_{RG} - \vec{v}_{CG} = -10\hat{j} - 25\hat{i}$

where $\vec{v}_{RC}$ is velocity of Rain relative to car.

$\vec{v}_{RG}$ is velocity of Rain relative to ground.

$\vec{v}_{CG}$ is velocity of car relative to ground

$\Rightarrow \vec{v}_{RC} = \sqrt{(-10)^2 + (-25)^2} = 26.9$ m/s

(7) The speed of a particle moving in a circle of radius 2 m increases at a constant rate of 4.4 m/s^2. At the instant when the magnitude of the total acceleration is 6 m/s, what is the speed (in m/s) of the particle?

a) 5.5   (b) 3.5   c) 3.55   d) 2.85   e) other

Solution: $a_c^2 = a^2 - a_t^2 = 6^2 - (4.4)^2 \Rightarrow a_c = 4.08$ m/s

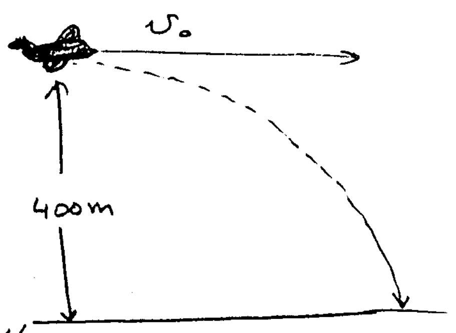
$\therefore a_c = \frac{v^2}{r} \Rightarrow v = \sqrt{a_c r} = \sqrt{4.08 \times 8} = 2.86$ m/s

where a_t is the tangential acceleration and a_c is the centripetal acc.

(8) An airplane flies horizontally with a speed of 300 m/s at an altitude of 400 m. What horizontal distance (in km) from a target must the pilot release a bomb so as to hit the target?

a) 3 b) 2.4 c) 3.3 d) 2.7 e) 1.7 f) other

Solution: $y = v_{y_0} t - \frac{1}{2} g t^2$



$-400 = 0 - \frac{1}{2}(10) t^2$

$\Rightarrow t = 8.94\ s$

$\therefore x = v_{x_0} t$

$= 300(8.94) \simeq 2.7\ km$ #

(9) A particle starts motion from a position $r_0 = 2\hat{i} + 3\hat{j}$ with initial velocity $v_0 = 3\hat{i} - 4\hat{j}$ and acceleration $a = 2\hat{j}$. What is the position of the particle (in m) after 5 s?

a) $24\hat{i} + 17\hat{j}$ b) $15\hat{i} + 21\hat{j}$ c) $17\hat{i} + 8\hat{j}$ d) 41 e) $13\hat{i} + 19\hat{j}$ / other

Solution: $\Delta\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a} t^2$

$\vec{r} - \vec{r}_0 = (3\hat{i} - 4\hat{j})5 + \frac{1}{2}(2\hat{j})(5)^2$

$\vec{r} - 2\hat{i} - 3\hat{j} = 15\hat{i} - 20\hat{j} + 25\hat{j}$

$\vec{r} = 17\hat{i} + 8\hat{j}$

(10) A particle starts from origin at $t = 0$ with a velocity of $(16\hat{i} - 12\hat{j})$ m/s and moves in the xy plane with constant acceleration of $a = (3\hat{i} - 6\hat{j})$ m/s². What is the speed (in m/s) of the particle at $t = 2.0$ s?

a) 52 b) 39 c) 46 d) 33 e) 43 f) other

Solution: $\vec{v} = \vec{v}_0 + a t$

$= (16\hat{i} - 12\hat{j}) + (3\hat{i} - 6\hat{j})2$

$= 22\hat{i} - 24\hat{j}$

$|v| = \sqrt{(22)^2 + (24)^2} = 32.5 \simeq 33\ m/s$ #

(11) At an altitude of 20 m, a particle of 1.5 kg mass attached to a 2 m حبل cord rotates in a horizontal circle with constant speed. The cord breaks and the particle flies (as a projectile). It strikes the ground 10 m from the center of rotation. The acceleration of the particle during its rotation is

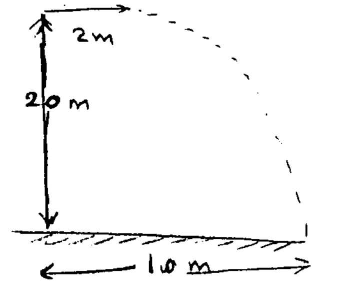
a) $10\ m/s^2$ b) zero c) $4.5\ m/s^2$ d) $8\ m/s^2$ e) $12.5\ m/s^2$ other

Solution: $y = v_{y_0} t - \frac{1}{2} g t^2$ (y component is zero)

$-20 = 0 - \frac{1}{2}(10)t^2 \Rightarrow t = 2\ sec$

$v_x = \frac{x}{t} = \frac{10}{2} = 5\ m/s$

$a_r = \frac{v_x^2}{r} = \frac{(5)^2}{2} = 12.5\ m/s^2$ #



(12) A اطفائي fireman 50.0 m away from a burning building directs a stream of water from a fire hose at an angle of 30.0 degree above the horizontal. If the velocity of the stream is 40.0 m/s, at what height will the stream of water strike the building?

a) 18.4 m b) 20.0 m c) 38.2 m d) 49.9 m e) other

Solution: $x = (v_0 \cos\theta)t \Rightarrow t = \frac{50}{40 \cos 30^\circ} = 1.44\ s$

Vertical: $y = (v_0 \sin\theta)t - \frac{1}{2} g t^2$

$= (40 \sin 30^\circ)(1.44) - \frac{1}{2}(10)(1.44)^2$

$= 18.45\ m$ #

you also can use the Trajectory equation

(13) If the speed of a projectile when it reaches its maximum height is one half its initial speed, what is (in degrees) the initial projection angle?

a) 10° b) 30° c) 45° d) 60°

Solution: At maximum height, speed $= v_{0x} = v_0 \cos\theta$ only.

also $v_0 \cos\theta = \frac{1}{2} v_0$

$\therefore\ \theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$ #

(14) A particle starts from rest at the top of a smooth incline of length S and angle θ. Take $S = 3.5$ m and $\theta = 45°$, find the speed of the particle at the bottom of the incline in (m/s).

(a) 49.5 (b) 7.04 (c) 8.4 (d) 3.76 (e) 2.8 f) None of others

Solution: we take the Y component of v only.

$v_y^2 = v_{y0}^2 - 2gS \Rightarrow v = \sqrt{2g\sin\theta\,(S)}$

$\therefore V_y = \sqrt{2(10)(3.5)(.707)} = 7.04$ m/s

$v_{y0} = 0$
$a = g\sin\theta$
$S = 3.5$
$h = S\sin\theta$

(15) A projectile is fired with a speed $v_0 = 200$ m/s and at an angle of maximum range R. Find the time of flight of the projectile.

Note: $R = \frac{v_0^2}{g}\sin(2\theta_0)$

(a) 28.3 S (b) 20 S (c) 14.1 S (d) 35.3 S (e) 53.7 S f) None above.

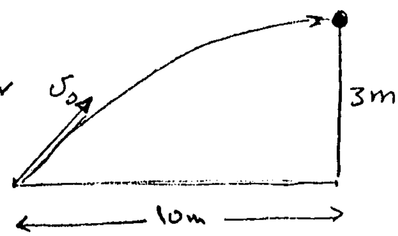
Solution: $\therefore t = \frac{2V_0\cos 45}{g}$

$\therefore t = \frac{2(200)(.707)}{10} = 28.3$ S

$y = v_{y0}t - \frac{1}{2}gt^2$
$\Rightarrow 0 = V_0\cos\theta t - \frac{1}{2}gt^2$
$\Rightarrow t = \frac{2v_0\cos\theta}{g}$

(18) A ball is kicked off from the ground at a point 10 m far from the base of a building. 1 sec. later, the ball hits a window 3 m above the ground level (see figure). With what speed V_0 (in m/s) was the ball kicked off?

(a) 2 (b) 8 (c) 10 (d) 13 (e) 18 (f) other



$y = v_{y0}t - \frac{1}{2}gt^2 \Rightarrow 3 = v_{y0}(1) - \frac{1}{2}(10)(1)^2$

$V_{y0} = 3 + 5 = 8$ m/s

$x = V_{x0}t \Rightarrow U_{x0} = 10$ m/s

$v = \sqrt{V_{x0}^2 + v_{y0}^2} = \sqrt{(8)^2 + (10)^2} = 12.8 \simeq 13$ m/s

(17) An Airplane flying with no wind, takes 2 hours between two cities which are 1000 Km apart. How long (in hours) would the same trip take against a constant wind blowing at the speed of 100 Km/h?

(a) 2.2 (b) 2.5 (c) 2.7 (d) 3.5 (e) other

Solution: Speed relation to the ground $= \frac{1000}{2} - 100 = 400$ Km/h

$$\text{Time of flight} = \frac{1000\,\text{Km}}{400\,\text{km/h}} = 2.5 \text{ hrs.} \#$$

(18) Starting at the origin, a particle moves with a constant acceleration of ($10\hat{i}$ m/s) and initial velocity of ($-3\hat{i} + 4\hat{j}$ m/s). At what value of y in meters does the particle return to the y-axis? [that is $x = 0$ again].

(a) 3.6 (b) 2.4 (c) 1.2 (d) 0.5 (e) other

Horizontal: $x = U_{x0} t + \frac{1}{2} a_x t^2$

$0 = -3t + \frac{1}{2}(10)t^2 \Longrightarrow t = 0.6\,s$

Vertical: $y = U_{y0} t + \frac{1}{2} a_y t^2$

$= 4(0.6) + 0 = 2.4\,m$

$a = 10\hat{i}$ m/s²
$\vec{v}_0 = -3\hat{i} + 4\hat{j}$ m/s
$x = 0$
$y = ?$

(19) At an altitude of 1280 m, an airplane was moving horizontally at 720 Km/h when its engines went suddenly dead. At what speed (in m/s) did the airplane crash against the ground.

$v_0 = 720 \left(\frac{1000}{3600}\right) = 200.16$ m/s

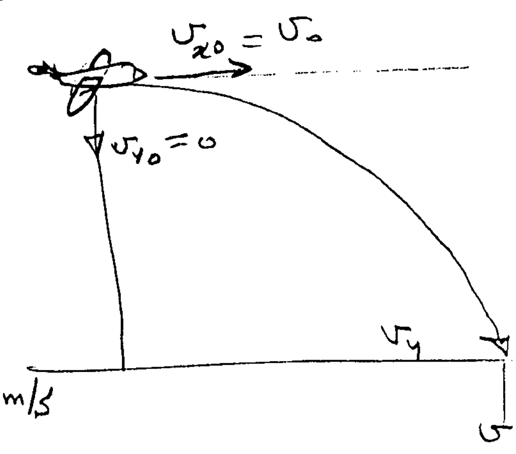
$v_{x0} = 200.16$ m/s, $v_{y0} = 0$

$v_y^2 = v_{y0}^2 - 2gh$

$= 0 - 2(10)(-1280) = 2.56 \times 10^4$

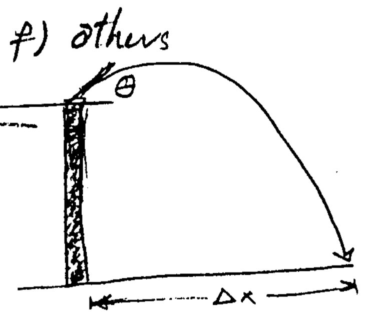
$v = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{(200.16)^2 + (2.56 \times 10^4)} = 256.2$ m/s



(20) A rock is projected from a point at the edge of the roof of a building with an initial velocity of 14 m/s at an angle of 53° above the horizontal. The rock strikes the ground a horizontal distance of 25 m from the base of the building. Find the height (in meters) of the building.

(a) 15.1 (b) 8.2 (c) 11.5 (d) −12 (e) 13.1 f) others



$v_{x_0} = v_0 \cos\theta = 14 \cos 53 = 8.4 \text{ m/s}$

$v_{y_0} = v_0 \sin\theta = 14 \sin 53 = 11.2 \text{ m/s}$

$t = \dfrac{x}{v_x} = \dfrac{25}{8.4} = 2.97\,s \sim 3\,s$

$y = v_{y_0} t - \frac{1}{2} g t^2 = (11.2)(3) - \frac{1}{2}(10)(3)^2 = -11.4 \text{ m}$

$\therefore h = 11.4 \text{ m}$ //

(21) A boat heading due north crosses a wide river with a speed of 10 Km/h relative to the water. The river has a uniform speed of 5.0 Km/h due east relative to Earth. Determine the velocity (in Km/h) of the boat relative to a stationary ground observer.

(a) 8.7 (b) 15 (c) 5 (d) 11 (e) 12 f) other

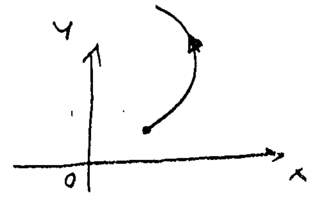
$\vec{V}_{bw} = 10\hat{j}$ Km/h

$\vec{V}_{wg} = 5\hat{i}$ Km/h

$\vec{V}_{bg} = \vec{V}_{bw} + \vec{V}_{wg} = 10\hat{j} + 5\hat{i}$

$|\vec{V}_{bg}| = \sqrt{(10)^2 + (5)^2} = 11.2$ m/s

(22) A particle is observed first when its position is at $4\hat{i} + 6\hat{j}$ and its velocity is $8\hat{i}$ m/s, and moves subsequently with a constant acceleration $\vec{a} = -2\hat{i} + 3\hat{j}$. All units are in SI system. Find the coordinates of the particle when its x coordinate is maximum. Hint: x is at maximum when $\frac{dx}{dt} = v_x = 0$



$(x, y) =$ (a) (20, 30) m (b) (20, 0) m (c) (20, 24) m (d) (40, 20) m (e) (30, 40) m (f) (30, 20) m

x	y
$x_0 = 4$	$y_0 = 6$
$v_{x_0} = 8$	$v_{y_0} = 0$
$a_x = -2$	$a_y = 3$

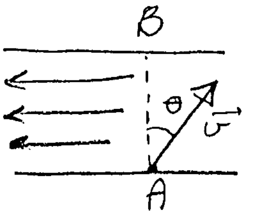
Solution $v_x = v_{x_0} + at \Rightarrow t = \dfrac{v_{x_0}}{a_x}$ (at $v_x = 0$) $= \dfrac{-8}{-2} = 4\,s$

also $\vec{r} - \vec{r}_0 = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \Rightarrow \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$

$\vec{r} = (4\hat{i} + 6\hat{j}) + (8(4)\hat{i}) + \frac{1}{2}(-2\hat{i} + 3\hat{j})(4)^2$

$\vec{r} = 4\hat{i} + 6\hat{j} + 32\hat{i} - 16\hat{i} + 24\hat{j} = \underline{20\hat{i} + 30\hat{j}}$

(23) A man in a sail-boat wishes to cross a river from point A on one shore of the river to a point B on the other shore, where point B is exactly opposite to A (see the figure). The maximum sailing speed of the boat is 5 Km/h and the river flows to the left at a speed of 3 Km/h. At what angle θ up the river (figure) should he sail to make the trip in the shortest time?



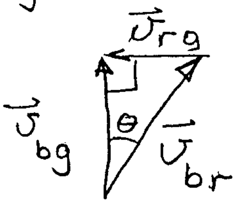
Solution:

using the following subscripts b : boat & r = river & g : Ground

$\Rightarrow$ $\vec{v}_{bg}$: velocity of boat relative to ground.

$\vec{v}_{rg}$: velocity of river relative to ground.

$\vec{v}_{br}$: velocity of boat relative to river.



$$\Rightarrow \quad \vec{v}_{bg} = \vec{v}_{br} + \vec{v}_{rg} \qquad \left(\vec{v}_{31} = \vec{v}_{32} + \vec{v}_{21} \right)$$

The directions of $\vec{v}_{bg}$ and $\vec{v}_{rg}$ are specified as shown.

from the right-angle triangle $\sin\theta = \dfrac{v_{rg}}{v_{br}} = \dfrac{3}{5}$

$$\therefore \quad \theta = \sin^{-1}\left(\tfrac{3}{5}\right) = 37^\circ \quad //$$

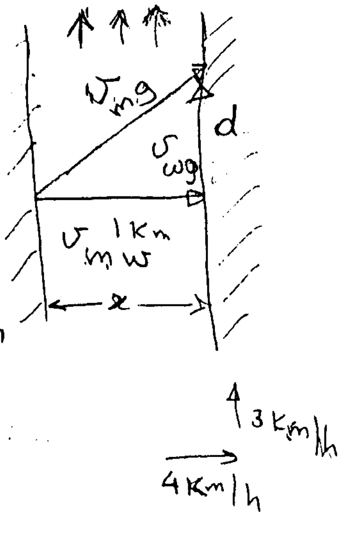
(24) A river flows due north with a velocity of 3 Km/h. A man rows a boat across the river, his velocity relative to the water being 4 Km/h due east. If the river is 1 Km wide, how far north of his starting point will he reach the opposite bank?

Solution: The distance d is required as shown in fig.

– we know that time is the same to cover the distance x or distance d $\Rightarrow t_1 = t_2 = \frac{x}{v}$

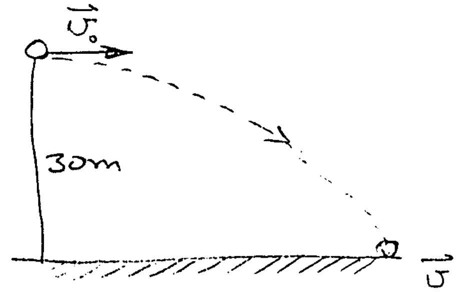
$$\therefore \quad t = \frac{x}{v_{mw}} = \frac{d}{v_{wg}}$$

$$\Rightarrow \quad d = \frac{v_{wg}}{v_{mw}}(x) = \frac{3}{4} \times 1 = 0.75 \text{ Km}$$



(25) A ball is thrown horizontally from the top of a building at a height of 30 m and hits the ground with a final speed v that is 4 times its initial speed v_0. What was the initial speed (in m/s)?

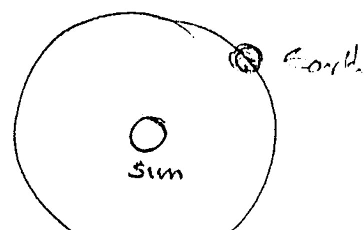
Solution:



$$v_f^2 = v_x^2 + v_y^2 \qquad \text{but} \quad v_f = 4v_0$$
$$(4v_0)^2 = v_0 + v_y^2$$
$$15 v_0^2 = v_y^2 = -2g\,\Delta y$$
$$15 v_0^2 = (-2)(10)(-30) = 600$$
$$v_0^2 = 40 \longrightarrow v_0 = 6.32 \ \text{m/s}$$

(26) What distance (in meters) does the Earth travel in two months period in its orbit about the Sun with a radius of 1.5×10^{11} m? (Hint: The Earth completes one revolution in 12 months, also assume a circular orbit).

Solution:

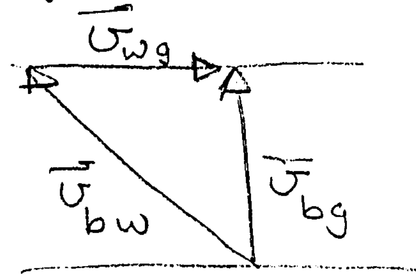


$$v_{earth} = \frac{2\pi r}{T}$$
$$= \frac{2\pi(1.5 \times 10^{11})}{12}$$
$$= 7.85 \times 10^{10} \ \text{m/month}$$

$\therefore$ d (distance in 2 months) $= 7.85 \times 10^{10} \times 2 = 1.57 \times 10^{11}$ m $\simeq 1.6 \times 10^{11}$ m

(27) A boat can move at 10 m/s in still water. If the boat is to travel directly across a river 1.6 Km wide (from A to B). How long (in minutes) will it take to cross the river if the water has a speed of 5 m/s?

Solution: $\because$ $\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg}$

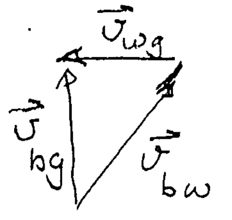


also $v_{bg} = \sqrt{v_{bw}^2 - v_{wg}^2} = \sqrt{100 - 25}$

$v_{bg} = \sqrt{75}$ m/s $\Rightarrow$ $t = \dfrac{d}{v_{Bg}} = \dfrac{1.6 \times 10^3}{\sqrt{75}} = 3.1$ min

or 184.8 sec.

(28) A boat moving due north relative to the ground crosses a 54 m wide river with a velocity of $(2i+3j)$ m/s relative to the water. The river has a uniform flow velocity of $-2i$ m/s. How long (in, s) does it take the boat to cross the river?



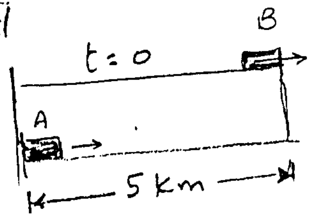
Solution:

$$V_{bw} = 2\hat{i} + 3\hat{j} \qquad V_{wg} = -2\hat{i}$$

$$V_{bg} = V_{bw} + V_{wg} = 2\hat{i} + 3\hat{j} - 2\hat{i} = 3\hat{j}$$

$$t = \frac{54}{3} = 18 \text{ Seconds}$$

(29) Cars A and B are moving in the same direction and with constant speeds V_A and V_A. Initially car B is ahead of A by 5 km. It is observed that car A catches up with B after 30 min. Find the relative velocity $\vec{v}$ of car A with respect to B. (V_{AB})



Solution:

$$\vec{V}_{AG} = \vec{V}_{AB} + \vec{V}_{BG}$$

$$\vec{V}_{AB} = \vec{V}_{AG} - \vec{V}_{BG}$$

as magnitude of $|\vec{V}_{AB}| \Rightarrow \frac{\Delta x}{\Delta t} = \frac{5 \text{ Km}}{.5 \text{ sec}} = \boxed{10 \text{ Km/h}}$ #

(30) A plane flies due East with speed of 540 Km/h (relative to still air). A strong wind is blowing due North with speed (as measured by an observer on the ground) of 50 m/s. The angle by which the plane shifts with respect to East is:

(a) 71.5° (b) 45° (c) 84.7° (d) 5.3° (e) 18.4° (f) other

Solution:

$$V_{pg} = V_{pa} + V_{ag}$$

$$= 150\hat{j} + 50\hat{i}$$

$$\theta = \tan^{-1}\left(\frac{150}{50}\right) = \boxed{71.5°} \quad \#$$