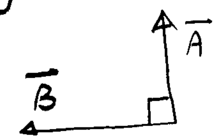
Solutions of selected Problems on chapter 3 phy. 101

(1) $\vec{A}$ and $\vec{B}$ are two vectors as shown in the figure. Let $A = 4$ and $B = 3$. Find the magnitude of the vector $C = A - B$.



a) 1 b) 3 c) 4 d) 5 e) 7 f) other

Solution:

$\vec{A} = 4\hat{j}$ $B = -3\hat{i}$

$\vec{A} - \vec{B} = 4\hat{i} - (-3\hat{i}) = 3\hat{i} + 4\hat{j}$

$|C| = \sqrt{(3)^2 + (4)^2} = 5$ #

(2) Vector $\vec{A}$ has magnitude 10 and makes an angle 37 with respect to positive x-axis. Vector $\vec{B}$ is $(-4\hat{i} - 3\hat{j})$. The angle between the vectors is.

a) $0°$ b) $90°$ c) $180°$ d) $37°$ e) $53°$ f) $164°$

$\vec{A} = A\cos\theta\,\hat{i} + A\sin\theta\,\hat{j} = 10(.8)\hat{i} + 10(.6)\hat{j}$

$= 8\hat{i} + 6\hat{j}$

$\vec{B} = -4\hat{i} - 3\hat{j}$

$\theta = \cos^{-1}\frac{A.B}{AB} = \cos^{-1}\frac{-50}{50} = \cos^{-1} -1 = 180°$ #

(3) Vector $\vec{A} = 5\hat{i} - 5\hat{j}$ $\vec{B} = 5\hat{i} + 5\hat{j}$

The magnitude of vector $\vec{C}$ which result from those two vectors and is perpendicular to both of them is.

a) zero b) 50 c) 7 d) 25 e) 35 f) Other

$\vec{C} = \vec{A} \times \vec{B} = (5\hat{i} - 5\hat{j}) \times (5\hat{i} + 5\hat{j})$

$= 25\hat{K} + (-25)(-\hat{K})$

$= 50K$ $|c| = 50$ #

or $C = AB\sin\theta$

$= \sqrt{50} \cdot \sqrt{50}\,(1) = 50$ #

Selected problems. chapter 3

(4): If $\vec{A} = 2\hat{i} - 3\hat{j}$ and $\vec{B} = -4\hat{i} + 5\hat{k}$. find the magnitude of the vector $\vec{C} = 2\vec{A} - \vec{B}$.

Solution
$$\vec{C} = 2\vec{A} - \vec{B}$$
$$= (4\hat{i} - 6\hat{j}) - (-4\hat{i} + 5\hat{k})$$
$$= 8\hat{i} - 6\hat{j} - 5\hat{k}$$
$$C = \sqrt{(8)^2 + (6)^2 + (5)^2} = \sqrt{125} = 11.18 \text{ units}$$

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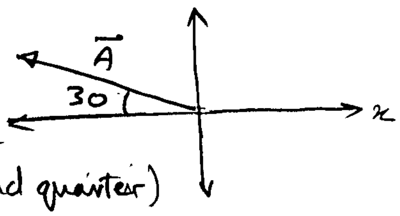
(5) If $\vec{A}$ is a vector of 25 units magnitude, making an angle of 30° with the negative x-axis, and $\vec{B} = 6\hat{i} - 3\hat{j}$, find the direction of the resultant vector $\vec{A} + \vec{B}$.

Solution let determine the x, y components of $\vec{A}$ if $\vec{A}$ as shown



$$\vec{A} = -25\cos 30\,\hat{i} + 25\sin 30\,\hat{j}$$
$$= -21.65\hat{i} + 12.5\hat{j}$$
$$\vec{B} = 6\hat{i} - 3\hat{j}$$
$\therefore\ \vec{A} + \vec{B} = -15.65\hat{i} + 9.5\hat{j}$ (in the second quarter)

$\therefore\ \theta = \tan^{-1}\frac{9.5}{-15.65} = -31.26°$ i.e $\theta = 180° - 31.26° = \underline{\underline{148.74°}}$ $\#_1$

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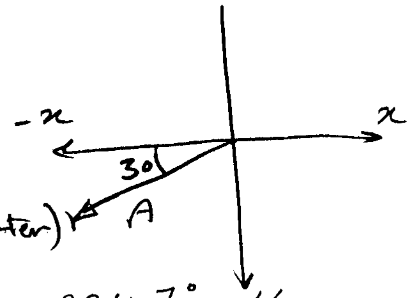
if $\vec{A}$ makes an angle 30° with negative x-axis as shown



$\therefore\ \vec{A} = -21.65\hat{i} - 12.5\hat{j}$

and $\vec{B} = 6\hat{i} - 3\hat{j}$

$\vec{A} + \vec{B} = -15.65\hat{i} - 15.5\hat{j}$ (in the third quarter)

$\Rightarrow \theta = 44.7°$ $\therefore\ \theta = 180° + 44.7 = \underline{\underline{224.7°}}$ $\#_2$

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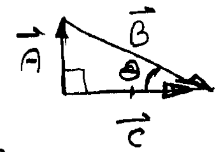
(6) If a vecter $\vec{A}$ is added to another vector $\vec{B}$ which has 5 units magnitude a third vector $\vec{C}$ results that is perpendicular to $\vec{A}$ and has a magnitude that is twice that of $\vec{A}$, Find the magnitude of $\vec{A}$.

Solution: $\vec{C} = \vec{A} + \vec{B}$ & $|C| = 2|A|$ & $|B| = 5$ units

if we draw a triangle to show $\vec{A}$ and $\vec{B}$ where $\vec{C} \perp$ to $\vec{A}$ as follow

we know that $\tan\theta = \frac{|A|}{|C|} = \frac{A}{2A} = \frac{1}{2}$



$\therefore\ \theta = \tan^{-1}\frac{1}{2} = 26.56°$

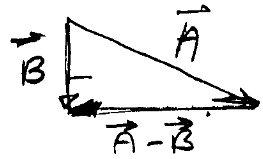
But we know that $\sin\theta = \frac{|A|}{|B|} \Rightarrow |A| = 5\sin\theta$
$= 2.236$ units #

(7) Given two vectors $\vec{A} = c\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} - \hat{k}$. Find the constant c such that $(\vec{A} - \vec{B})$ is perpendicular to $\vec{B}$.

Solution:

$$\vec{C} = \vec{A} - \vec{B} = c\hat{i} - 2\hat{j} + \hat{k} - (\hat{i} + 2\hat{j} - \hat{k})$$

$$\vec{C} = (c-1)\hat{i} - 4\hat{j} + 2\hat{k}$$



also $\vec{C} \cdot \vec{B} = 0$ because $\vec{C} \perp \vec{B}$

$$\Rightarrow [(c-1)\hat{i} - 4\hat{j} + 2\hat{k}] \cdot (\hat{i} + 2\hat{j} - \hat{k}) = 0$$

$$c-1 \quad - 8 \quad - 2 \quad = 0$$

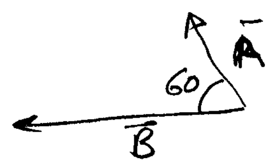
$$(c-1) = +10$$

$$c = 11 \quad \#$$

(8) If $\vec{A} = \hat{i} - 2\hat{j} + 5\hat{k}$ and another vector $\vec{B}$ has 7 units magnitude. When the two vectors are drawn starting from the same point, the angle between them is $60°$; Find the scalar (dot) product between these vectors.

Solution:

$$|A| = \sqrt{(1)^2 + (2)^2 + (5)^2} = 5.48 \text{ units}$$



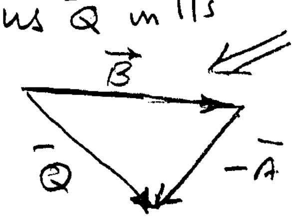
$$A \cdot B = AB\cos\theta$$
$$= (5.48)(7)(.5) = 19.17 \quad \#$$

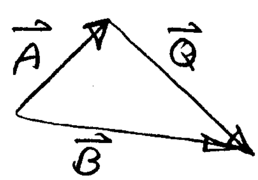
(9). In the figure, the vector Q is given by

A) $\vec{A} + \vec{B}$ b) $\vec{A} - \vec{B}$ c) $\vec{B} - \vec{A}$ d) $-B-A$ e) Other

we can take $-A$ $-\vec{A}$

then take $\vec{B} + (-\vec{A})$ gives us $\vec{Q}$ in its direction.





$$\vec{B} = \vec{A} + \vec{Q}$$
$$\vec{Q} = \vec{B} - \vec{A}$$

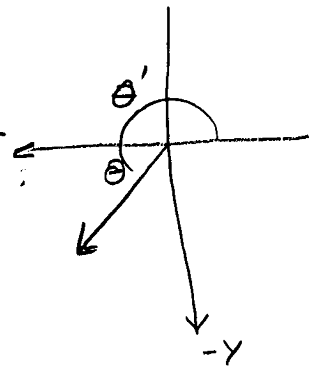
(10) Find the angle (in degrees) between the vector $\vec{A} = -2\hat{i} - 3\hat{j}$ and the positive x-axis

a) 33.7 b) 41.8 c) 48.2 d) 56.3 e) 236

Solution:

$$\tan\theta = \frac{-3}{-2} = 1.5$$

$$\therefore \theta = \tan^{-1} 1.5 = 56.31^\circ$$



But this angle is between $\vec{A}$ and the negative x-axis, so we have to find all

$$\theta' = 56.31^\circ + 180^\circ = 236^\circ \quad \#$$

(11) An airplane makes the displacements $\vec{A} = 40\hat{i} + 10\hat{j} + 70\hat{k}$ m and $\vec{B} = -10\hat{i} + 30\hat{j} + 50\hat{k}$ m consecutively. Find the magnitude of the resultant displacement $\vec{A} + \vec{B}$.

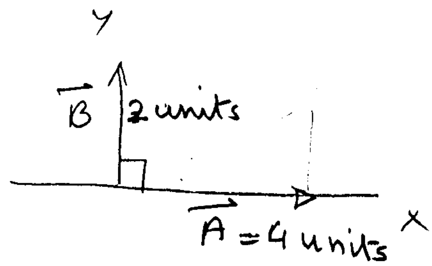
a) 140 m b) 81 m c) 59 m d) 130 m e) 150 m

$$\vec{R} = \vec{A} + \vec{B} = (40\hat{i} + 10\hat{j} + 70\hat{k}) + (-10\hat{i} + 30\hat{j} + 50\hat{k})$$
$$= 30\hat{i} + 40\hat{j} + 120\hat{k}$$

$$R = \sqrt{(30)^2 + (40)^2 + (120)^2} = 130 \text{ m} \quad \#$$

(12) In the figure, find the angle between the X-axis and the vector $\vec{C} = 2\vec{B} - \frac{1}{2}\vec{A}$. Hint: You may use the unit vectors $\hat{i}$ and $\hat{j}$

a) $\theta = 243^\circ$ b) $\theta = 117^\circ$ c) $\theta = 154^\circ$ d) $\theta = 135^\circ$ e) $\theta = -63^\circ$ f) $\theta = 63^\circ$

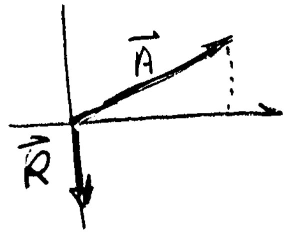


$$\vec{C} = 2\vec{B} - \tfrac{1}{2}\vec{A} = -\tfrac{1}{2}\vec{A} + 2\vec{B}$$
$$= -\tfrac{1}{2}(4)\hat{i} + 2(2)\hat{j}$$
$$= -2\hat{i} + 4\hat{j}$$

$$\theta = \tan^{-1}\left(\frac{4}{-2}\right) = -63^\circ$$

$$\therefore \theta' = 180 - 63 = \underline{117^\circ}$$

(13) $\vec{A} = 4\hat{i} + 3\hat{j}$ and the resultant vector $\vec{R} = \vec{A} + \vec{B} = -3\hat{j}$ as shown. Find the vector $\vec{B}$.

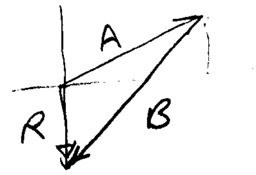


a) $4\hat{i}$ b) $3\hat{j}$ c) $4\hat{i} - 3\hat{j}$ d) $-4\hat{i} - 6\hat{j}$

e) $-3j$ f) None of the others

Solution:

We can draw B on the figure $\Rightarrow$

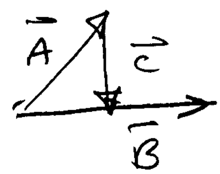


also $\vec{B} = \vec{R} - \vec{A}$

$= 0\hat{i} - 3\hat{j} - (4\hat{i} + 3\hat{j})$

$= -4\hat{i} - 6\hat{j}$ #

(14) Express the vector $\vec{C}$ in terms of $\vec{A}$ and $\vec{B}$. The tip of $\vec{C}$ is at the middle of $\vec{B}$



a) $\vec{C} = \vec{A} + \vec{B}$ b) $\vec{C} = \vec{A} + \frac{1}{2}\vec{B}$

c) $\vec{C} = \vec{A} - \vec{B}$ d) $\vec{C} = \vec{B} - \vec{A}$ (e) $\vec{C} = \frac{1}{2}\vec{B} - \vec{A}$

f) None of the others

from figure we can write the following vector equation

$\vec{A} + \vec{C} = \frac{1}{2}\vec{B}$ because tip of $\vec{A}$ is connected to the tail of $\vec{C}$

$\therefore \vec{C} = \frac{1}{2}\vec{B} - \vec{A}$ #

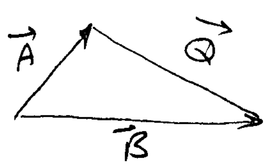
(15) In the figure, the vector Q is given by:

(a) $\vec{A} + \vec{B}$ (b) $\vec{A} - \vec{B}$ (c) $\vec{B} - \vec{A}$ (d) $-\vec{B} - \vec{A}$ (e) other

Solution

$\vec{B} = \vec{A} + \vec{Q}$

$\therefore \vec{Q} = \vec{B} - \vec{A}$ #

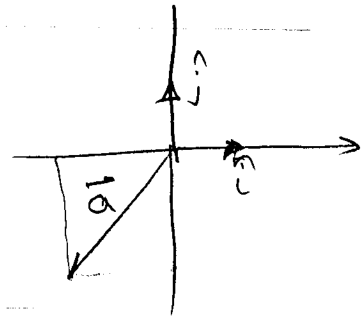


(16) Find the unit vector $\hat{a}$ in the direction of the vector $\vec{a} = -\hat{i} - 2\hat{j}$:

(a) $\hat{i} + 2\hat{j}$ (b) $\frac{1}{\sqrt{5}}(\hat{i}+2\hat{j})$ (c) $-\frac{1}{\sqrt{5}}(\hat{i}+2\hat{j})$ (d) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$

Solution

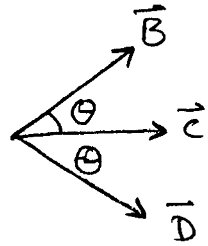
See the figure



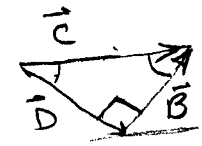
$$\hat{a} = \frac{\vec{a}}{|a|} = \frac{-\hat{i}-2\hat{j}}{[(-1)^2+(-2)^2]^{\frac{1}{2}}}$$

$$= -\frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}$$ //

(17) Three vectors $(\vec{B}, \vec{C}, \vec{D})$ all have the same magnitude. The angle θ between adjacent (متجاورة) vectors is $45°$ as shown. Show that the magnitude of $\vec{B}+\vec{D}$ is equal to $C\sqrt{2}$



or



$\therefore \vec{C} = \vec{B} + \vec{D}$

$|C| = \sqrt{2}\,C$

Solution:

$\because$ The angle between $\vec{B}$ and $\vec{D}$ is $90°$

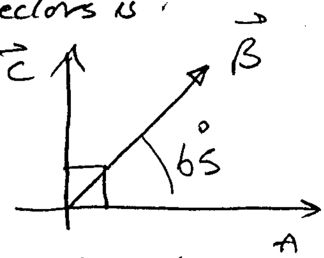
and $\because |D| = |B| = |C|$

$\therefore$ The magnitude of $\vec{B}+\vec{D} = \sqrt{|B^2| + |D^2|}$

$= \sqrt{|C|^2 + |C^2|} = \sqrt{2C^2} = C\sqrt{2}$ //

(18) Two vectors $\vec{A}$ and $\vec{B}$ have magnitude of $A = 10$ and $B = 15$. The angle between them is $65°$. The component of vector $\vec{B}$ along the line perpendicular to vector $\vec{A}$, in the plane of the vectors is:

(a) 0 (b) 4.2 (c) 6.3 (d) 9.1 (e) 14



Solution:

The component is:

$15\cos(90-65) = 15\cos 25 = 13.59 \simeq 14°$ //

or $15\sin(65) = 13.59° \simeq 14$

(19) If $\vec{A} = 2\hat{i} - 3\hat{j}$ and $\vec{A}\times\vec{B} = 14\hat{k}$. Find the vector $\vec{B}$ in unit vector notation if $B_y = 2B_x$ and $B_z = 0$.

Solution:

$\vec{A}\times\vec{B} = (2\hat{i} - 3\hat{j}) \times (B_x\hat{i} + B_y\hat{j}) = 2B_y\hat{k} + 3B_x\hat{k}$

$14\hat{k} = (2B_y\hat{k} + 3B_x\hat{k})$

$14\hat{k} = (2(2B_x) + 3B_x)\hat{k}$ because $B_y = 2B_x$

$\Rightarrow 7B_x = 14 \Rightarrow B_x = 2$

$\Rightarrow B_y = 4$

$\therefore \vec{B} = 2\hat{i} + 4\hat{j}$ //

(20) A vector $\vec{A}$ is added to $\vec{B} = 6\hat{i} - 8\hat{j}$. The resultant vector is in the positive x direction and has a magnitude equal to A. What is the magnitude of $\vec{A}$?

Solution:

$$\because \vec{A} + \vec{B} = A\hat{i}$$

$$A_x + 6 = A \Rightarrow A_x = A - 6$$

$$A_y - 8 = 0 \Rightarrow A_y = 8$$

$$A_x^2 + A_y^2 = A^2 \Rightarrow (A-6)^2 + (8)^2 = A^2$$

$$\cancel{A^2} + 36 - 12A + 64 = \cancel{A^2} \Rightarrow A = \frac{36+64}{12} = 8.3 \;\#$$

(21) Let $\vec{A} = 4\hat{i} + 5\hat{j}$, $\vec{B} = 2\hat{i} - 3\hat{j}$, $\vec{C} = 3\hat{i} + 5\hat{j}$ find $\vec{C}\cdot(\vec{A}\times\vec{B})$.

Solution:

$$\vec{A}\times\vec{B} = (4\hat{i} + 5\hat{j}) \times (2\hat{i} - 3\hat{j}) = -12\hat{k} - 10\hat{k} = -22\hat{k}$$

$$\vec{C}\cdot(\vec{A}\times\vec{B}) = (3\hat{i} + 5\hat{j})\cdot(-22\hat{k}) = 0 \;\#$$

(22) If $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{B} = -4\hat{i} + 5\hat{j} - \hat{k}$ find the angle (in degree) between $\vec{A}$ and $\vec{B}$.

Solution:

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$-8 - 15 - 4 = \left(\sqrt{4+9+16}\right)\left(\sqrt{16+25+1}\right)\cos\theta$$

$$\Rightarrow \theta = \cos^{-1}\frac{-27}{\sqrt{29}\sqrt{42}} = 140.7 \simeq 141^\circ \;\#$$

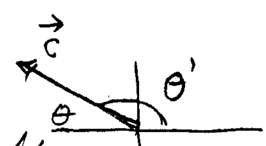
(23) Given $\vec{A} = 2\hat{i} + 3\hat{j}$, $\vec{B} = 4\hat{i} - \hat{j}$ and $\vec{C} = \vec{A} - 3\vec{B}$. What is the angle between $\vec{C}$ and the positive x-axis?

Solution:

$$3\vec{B} = 12\hat{i} - 3\hat{j}, \quad \vec{C} = (2-12)\hat{i} + (3+3)\hat{j}$$

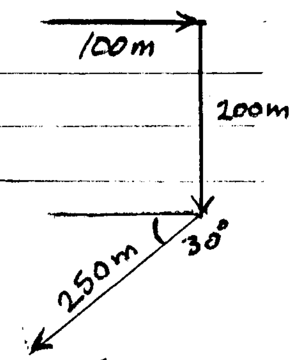
$$\Rightarrow \vec{C} = -10\hat{i} + 6\hat{j}$$

$$\theta = \tan^{-1}\frac{+6}{-10} \Rightarrow \theta = 31^\circ$$



from the figure $\theta' = 180 - 31 = 149^\circ \;\#$

(24) A person walks 100m towards east then 200m towards south then 250m 30° south of west as shown. What is the magnitude of the resultant displacement (in m). (Use Components method).



Solution

$$\vec{R} = \vec{A} + \vec{B} + \vec{C}$$
$$= 100\hat{i} - 200\hat{j} - (250\cos 30)\hat{i} - (250\sin 30)\hat{j}$$
$$\vec{R} = 116.6\hat{i} - 325\hat{j}$$
$$R = \sqrt{(116.6)^2 + (325)^2} = 345 \text{ m} \quad \#$$

(25) If $A.B = 8$, $A = 2$ and $B = 4$ find $|A \times B|$

Solution

$$\vec{A}.\vec{B} = AB\cos\theta$$
$$8 = 2(4)\cos\theta \Rightarrow \theta = \cos^{-1} 1 = 0$$
$$\therefore |\vec{A}\times\vec{B}| = AB\sin\theta = 2(4)(0) = 0 \quad \#$$

(26) Let $C = 6\hat{i} + 3\hat{j} + 2\hat{k}$. What is the angle (in degrees) between $\vec{C}$ and z-axis?

Solution Vector C has three components

The only component in the z axis is $2\hat{k}$ which will help us to calculate the θ

- to understand this problem try to draw it
- as you see in fig 2 θ is not the angle we need but we need ϕ
- $\phi = 90 - \theta$

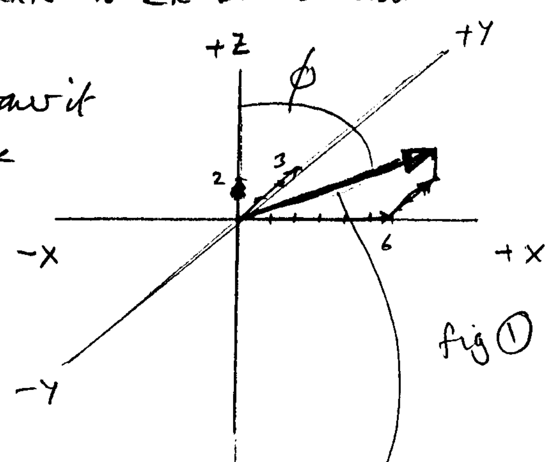


fig ①

$$\therefore \sin\theta = \frac{\text{المقابل}}{\text{الوتر}} = \frac{2}{|C|}$$

$$\theta = \sin^{-1}\frac{2}{\sqrt{6^2+3^2+2^2}} = 16.6°$$

$$\Rightarrow \phi = 90 - 16.6° = 73.4° \quad \#$$

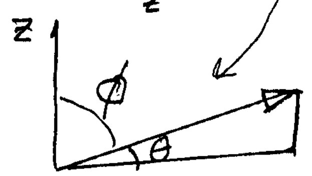


fig ②

OR simply

$$\theta_z = \cos^{-1}\frac{2}{|C|} = \cos^{-1}\frac{2}{\sqrt{6^2+3^2+2^2}} = 73.4° \quad \#$$