① The speed of an object, moving in a straight line, decreases at a constant rate from v_o to zero in 4 sec. If, during that time, the object moves 8 m, find the value of v_o (in m/s).

Solution

given a is constant $v_o = ?$ $v = $ zero $t = 4$ sec

$x - x_o = 8\,m$

using $x - x_o = \frac{1}{2}(v_o + v)t$ when a is missing

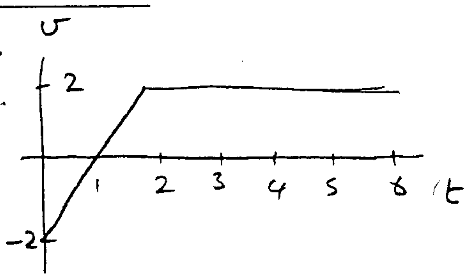
$8 = \frac{1}{2}(v_o + 0)(4)$

$8 = 2(v_o + 0)$

$8 = 2v_o$

$v_o = 4\ m/s$ #

② The velocity of a particle moving along the x-axis varies with time as shown in the figure. If its initial position at $t = 0$ is $x_o = 4\,m$, find its position at $t = 5$ sec.



Solution

given from figure that

$v(0) = -2$, $v(1) = 0$, $v(2) = 2$, $v(t>2) = 2$

also we know that $x_o = 4\,m$

x is needed ?

Δx = area under the curve.

$\Delta x = \frac{1}{2}(-2)1 + \frac{1}{2}(2)1 + 2(3) = 6\ m$

$\Delta x = x - x_o$

$6 = x - 4$

$x = 10\ m$ #

③ The motion of a particle in one direction is given by $x = 4t^3 + 3t^2 - 5$ (m/s). Find the average acceleration (in m/s²) between $t_1 = 1$ sec and $t_2 = 4$ sec.

Solution $v = \frac{dx}{dt} = 12t^2 + 6t$

$v(1) = 12 + 6 = 18\ m/s$

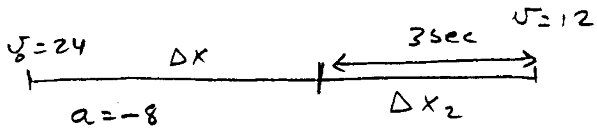
$v(4) = 12(16) + 6(4) = 216\ m/s$

$\bar{a} = a_{avg} = \frac{216 - 18}{4 - 1} = 66\ m/s^2$ #

(4) A car starts motion with an initial velocity of 24 m/s and decelerates at a rate of 8 m/s^2 until it stops. It then accelerates in the same direction until its velocity reaches 12 m/s after 3 sec. Find the average velocity (in m/s) during this trip.

Solution:

Known $\bar{v} = \frac{\Delta x}{\Delta t}$ ($v_{average}$)



for first distance

$$v = v_0 + at$$

$$0 = 24 - 8(t)$$

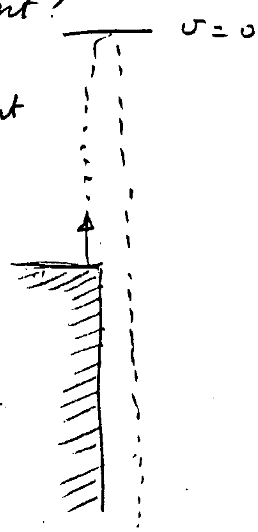
$$\therefore t = 3 \text{ sec.}$$

$$\Delta x_1 = \tfrac{1}{2}(24+0)(3) = 36 m$$

$$\Delta x_2 = \tfrac{1}{2}(0+12)(3) = 18 m$$

$$\Rightarrow \bar{v} = \frac{36+18}{3+3} = \frac{54}{6} = 9 \text{ m/s}$$

(5) A stone is projected vertically upward from the edge of a cliff with a speed of 12 m/s. If the stone travels a distance of 19.4 m. what is its displacement (in m) at that moment?



Solution we will calculate the maximum height

$$v^2 = v_0^2 - 2g\Delta y$$

$$0 = 144 - 20\,\Delta y$$

$$\Delta y = \frac{144}{20} = 7.2 m$$

Therefore the stone went up to 7.2 m then back down for another 7.2 m for total distance $7.2 + 7.2 = 14.4$ meters
this will make its displacement = zero.
– now the stone continued to fall down to make total distance = 19.4

$$\therefore 19.4 - 14.4 = 5 \text{ meters}$$

$\therefore$ the displacement = −5 meter (downward)

(6) An airplane moves in a straight line from Kuwait to Dhahran with a constant acceleration of 1.3×10^4 Km/h^2. What is its acceleration in m/s^2?

$$1.3 \times 10^4 \frac{Km}{h^2} = 1.3 \times 10^4 \frac{10^3}{(3600)^2} = 1 \; m/s^2$$

(7) A stone is released from a balloon that is descending at a constant speed of 10 m/s. Neglecting air resistance, after 2 sec the speed of the stone is:

Solution:

The movement of the balloon downward will give the stone initial velocity $v_0 = -10$ m/s

$\therefore \; v = v_0 - gt$

$= -10 - (10)(2) = -30$ m/s

(8) The position of a moving car is given by $x = -12t + 4t^2$ in m and s units. Find the acceleration of the car when its velocity is zero.

Solution:

$v = \frac{dx}{dt} = -12 + 8t$

when $v = 0 \Rightarrow t = \frac{12}{8} = 1.5$ sec.

$\therefore \; a = \frac{d^2x}{dt^2} = 8$ m/s^2

$\therefore$ acceleration is always constant $= 8$ m/s^2

(9) The velocity of a particle moving along the x-axis is given for $t > 0$ by $v(t) = (32t - 2t^3)$ m/s. What is the acceleration in m/s^2 of the particle when it achieves its maximum displacement in the positive x direction?

Solution: Maximum position at $v = 0$

$\Rightarrow 32t - 2t^3 = (32 - 2t^2)t = 0$

or $2t^2 = 32 \Rightarrow t^2 = 16 \Rightarrow t = 4$ sec

$\therefore \; a = \frac{dv}{dt} = 32 - 6t^2$

$\Rightarrow a(4) = 32 - 6(4)^2 = 32 - 96$

$= -64$ m/s^2

(10) A ball is thrown vertically upward with an initial velocity of 20 m/s. Two seconds later, another ball is thrown also upward with the same initial velocity. Find the height at which the two balls meet.

Solution

for ball 1, height is given as:

$$y_1 = v_0 t_1 - \frac{1}{2} g t_1^2$$

$$y_1 = 20 t_1 - 5 t_1^2 \quad \text{------} \quad (1)$$

for ball 2, height is given as:

$$y_2 = v_0 t_2 - \frac{1}{2} g t_2^2$$

where $t_2 = t_1 - 2$ ------ (2)

$$\therefore y_2 = 20(t_1 - 2) - 5(t_1 - 2)^2$$

$$= 20 t_1 - 40 - 5(t_1^2 - 4t_1 + 4)$$

$$= 20 t_1 - 40 - 5t_1^2 + 20 t_1 - 20$$

$$y_2 = -5t_1^2 + 40 t_1 - 60 \quad \text{------} \quad (3)$$

But we know that the two balls will meet

$$\therefore y_2 = y_1$$

$$\Rightarrow 20 t_1 - \cancel{5t_1^2} = -\cancel{5t_1^2} + 40 t_1 - 60$$

$$20 t_1 = 60$$

$$t_1 = 3 \text{ s}$$

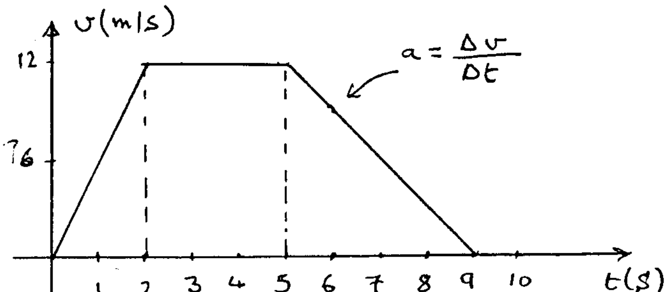
from equ (2), $t_2 = t_1 - 1 = 3 - 2 = 1$ s #

from equation (1)

$$y_1 = y_2 = (20)(3) - \frac{1}{2}(10)(3)^2$$

$$= 60 - 45 = 15 \text{ m} \quad \#$$

(11) The diagram shown represent the straight-line motion of a car. Determine the acceleration at $t = 6$ sec.



Solution:

- The line between 5-9 s is straight this means that the acceleration is constant and changing in a constant rate.
- so if we know the slope then we know the acceleration at any point. ⇒ a at point 6 s is the slope value $\frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$

$$a = \frac{0 - 12}{9 - 5} = \frac{-12}{4} = -3 \text{ m/s}^2 \quad \#$$

(12) Two cars A and B. Car A is moving with a constant velocity 50 m/s, while car B started to move with initial velocity 10 m/s and a constant acceleration of 4 m/s². Find the time (in seconds) required for car B to overcome car A.

Solution

For car A $\quad \Delta x = v t = 50t \quad$ --- ①

for car B $\quad \Delta x = v_0 t + \frac{1}{2} a t^2$

$\quad = 10t + 2t^2 \quad$ --- ②

The two cars will meet when B overcome A when Δx of both are equal. $\Delta x_1 = \Delta x_2$

i.e equ. ① equals equ. ②

$50t = 10t + 2t^2$

$40t = 2t^2$

$t = 20$ second #

(13) A particle is thrown downward with a speed of 4 m/s. It touches the ground after 4 seconds. From what height was the particle thrown.

Solution:

$v_{y_0} = -4\ m/s \quad \& \quad t = 4$ sec.

$y = v_{y_0} t - \frac{1}{2} g t^2$

$= -4(4) - \frac{1}{2}(10)(4)^2$

height ⇒ $= -96$ m #

(14) A helicopter is moving upward with velocity 5 m/s. A parachute is droped from the helicopter at a height of 500 m from the ground level. Find its height from the ground level after 4 seconds.

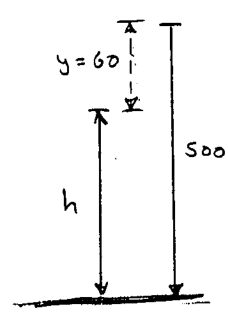
Solution: $v_{y_0} = 5\ m/s \quad h = 500m \quad t = 4s$

$y = v_{y_0} t - \frac{1}{2} g t^2$

$= 5(4) - \frac{1}{2}(10)(4)^2$

$y = -60$ m

$h = 500 - 60 = 440$ m #



Chapter 2

(15) A particle confined to motion along the x-axis moves with constant acceleration from $x_1 = 2\,m$ to $x_2 = 8\,m$ during 2.5 S time interval. The velocity of a particle at $x = 8.0\,m$ is 2.8 m/s. What is the acceleration (in m/s^2) during this time interval?

(a) .48 (b).32 (c).64 (d) .80 (e).57 f) other

$\boxed{\text{formula number 5}}$

$$x - x_0 = vt - \tfrac{1}{2}at^2$$

$$8 - 2 = (2.8)(2.5) - \tfrac{1}{2}a(2.5)^2$$

$$6 = 7 - 3.1\,a$$

$$a = 0.32\ m/s^2 \ \#$$

---

(16) A ball is thrown vertically from a height of 40 m above the ground and reaches the ground 4 Sec later. Find the initial velocity (in m/s).

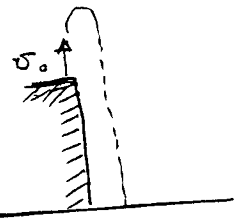
(a) 30 (b)-30 (c) 10 (d)-10 (e) 15 (f) others

$$y = v_{y_0}t - \tfrac{1}{2}gt^2$$

$$-40 = v_{y_0}(4) - \tfrac{1}{2}(10)(4)^2$$

$$v_{y_0} = \frac{80 - 40}{4} = 10\ m/s \ \#$$

upward



(17) The position x of a particle varies with time according to the equation $x = 2 + 2t - t^2$ with x in meter and t in seconds. Find the average speed (in m/s) of the particle during the first 4 seconds. [Hint: First find the time at which the particle changes direction, where $v = 0$].

(a) −2 (b) −1.5 (c) 1.5 (d) 2 (e) 2.5 (f) other

$$v = \frac{dx}{dt} = 2 - 2t \quad \text{at } v = 0 \quad t = 1\ s$$

$$x_1 = x(0) = 2\ m$$

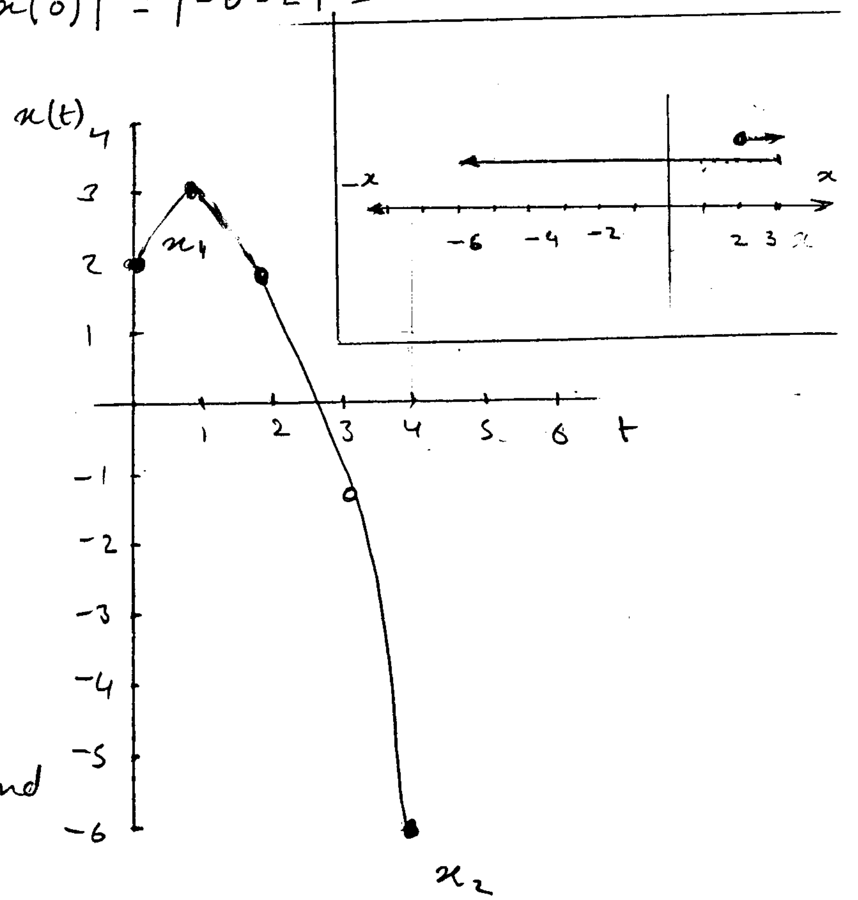
$$x_2 = x(4) = 2 + 2(4) - (4)^2 = -6\ m$$

$$d = |x(4) - x(0)| = |-6 - 2| = 8\ m$$

as distance is required not velocity then we need total distance

so $x(0) = 2m$
$x(1) = 3\ m$
$x(2) = 2m$
$x(3) = -1\ m$
$x(4) = -6\ m$

∴ total distance is
$2 + 8 = 10m$



the particle stopped and changed its direction at $t = 1$ sec

$$\Rightarrow S_{avg} = \frac{10}{4} = 2.5\ m/s$$

(18) (a) A particle moves in the x-direction according to the equation $x = 4t - 2t^2$

x is in meters and t is in seconds.

What is the average velocity between $t=0$ and $t=2s$

(a) zero (b) 2 m/s (c) 4 m/s (d) 8 m/s

(e) -8 m/s (f) other.

Solution:

$x(0) = 4(0) - 2(0)^2 = 0$ also $x(2) = 4(2) - 2(2)^2 = 0$

$v_{avg} = \bar{v} = \dfrac{x_2 - x_1}{t_2 - t_1}$

$= \dfrac{0-0}{2} = 0$

$x(0) = 0$
$x(2) = 8 - 8$
$= 0$

(b) What is the instantaneous velocity at $t = 2$ sec.

(a) zero (b) 2 m/s (c) -4 m/s (d) 8 m/s

(e) -8 m/s (f) other

Solution: $v = \dfrac{dx}{dt} = 4 - 4t$

$= 4 - 4(2)$

$= -4$ m/s

(19) A car is moving with a constant acceleration, passes the distance between two checkpoints 70 m apart in 5 sec. Its velocity at the first checkpoint is 8 m/s. Assuming that the car continues with the same acceleration after the second check point, calculate the distance it travels in the next 5 seconds interval after this point.

Solution:

$$x_1 = v_0 t + \frac{1}{2} a t^2$$

$$70 = 8(5) + \frac{1}{2} a (5)^2$$

$$a = 2.4 \text{ m/s}^2$$

$$\Rightarrow x = 8(5+5) + \frac{1}{2}(2.4)(10)^2$$

$$= 80 + 120 = 200 \text{ m}$$

$$x_2 = x - x_1 = 200 - 70 = 130 \text{ m}$$ #

OR:

$$v_1 = v_0 + at$$

$$= 8 + 2.4(5) = 20 \text{ m/s}$$

$$x_2 = v_1 t + \frac{1}{2} a t^2$$

$$= 20(5) + \frac{1}{2}(2.4)(5)^2$$

$$= 100 + 30 = 130 \text{ m}$$ #

(20) A car starts motion with an initial velocity of 24 m/s and decelerates at a rate of 8 m/s² until it stops. It then accelerates in the same direction until its velocity reaches 12 m/s after 3 sec. Find the average velocity (in m/s) during this trip.

answers:

a) 8.2 m/s b) 7 m/s c) 10.2 m/s d) 3.6 m/s

f) others

Solution:

$\Rightarrow \quad v^2 = v_0^2 + 2a\Delta x$

$0 = (24)^2 + 2(-8)\Delta x$

$\Delta x_1 = 36$

$\Rightarrow \quad v = v_0 + at$

$0 = 24 + (-8)t_1 \Rightarrow t_1 = 3$ Seconds

$\Rightarrow \quad \Delta x_2 = \left(\frac{v+v_0}{2}\right)t_2 \Rightarrow \Delta x_2 = \frac{12}{2}(3) = 18\ m$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{\Delta x_1 + \Delta x_2}{t_1 + t_2} = \frac{36+18}{3+3} = 9\ m/s$$

This problem solved before by another method

See page 2

(21) A man moves with constant velocity of 4 m/s for 5 seconds. He then accelerates to 12 m/s in 2 seconds in the same direction. The total distance covered by him during this motion is:

answer:

a) 16 m b) 20 m (c) 36 m d) 56 m e) 40 m

f) other

Solution:

* for the first distance he covered it with constant speed = v_{avg}.

$\therefore d_1 = vt \Rightarrow 4(5) = 20\,m$

* Second distance he accelerated

his $v_i = 4\,m/s$ and $v_f = 12\,m/s$

within intervale of time $\Delta t = 2$ seconds

$\therefore a = \frac{12-4}{2} = 4\,m/s^2$ where $a = \frac{\Delta v}{\Delta t}$

$\Delta x = x - x_0 = v_0 t + \frac{1}{2}at^2$

$d_2 = \Delta x = (4)(2) + \frac{1}{2}(4)(2)^2$

$= 8 + 8 = 16\,m$

total distance $= d_1 + d_2 = 20 + 16 = 36\,m$ #

note: Second part can be solved also as follows

$d_2 = \frac{1}{2}(v + v_0)t = \frac{1}{2}(4 + 12)2 = 16\,m$

Selected Problems on chapter 2

(22) A rock is thrown vertically downward from an unknown height above the ground with an initial speed of 20 m/s. It strikes the ground 5 seconds later.

Determine the initial height of the rock above the ground.

Answer:

a) 100 $\boxed{\text{b) 225}}$ c) 300 d) 50

Solution: $Y\,(\text{height}) = v_0 t - \frac{1}{2} g t^2$

$$= v_0 t - \tfrac{1}{2}(10)t^2$$

$$= v_0 t - 5t^2$$

$$= -20(5) - 5(5)^2$$

$$= -100 - 125$$

$$= -225 \text{ m}$$

(25) A ball thrown downwards from a height of 45 m, hits the ground with a velocity of -50 m/s.

Find its initial velocity.

(a) 0 m/s (b) -20 m/s (c) -40 m/s

(d) $+20$ m/s (f) others

Solution:

using $v^2 = v_0^2 - 2g\Delta y \Rightarrow v_0^2 = v^2 + 2g\Delta y$

$$= (-50)^2 + 2(10)(-45)$$

$$v^2 = +2500 - 900 = 1600$$

$$\Rightarrow v = \sqrt{1600} = -40 \text{ m/s (downwards)}$$