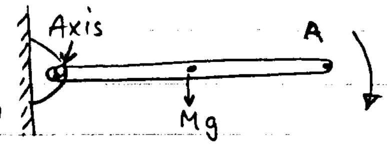
آخر تعديل ٢٠٠٢/٧/٥

① A uniform rod of Length L and mass M is free to rotate about a horizontal, frictionless pivot ([illegible]) (axis), as shown. The rod starts from rest in the horizontal position and rotates in a vertical plane. Find the initial tangential acceleration of the free end A of the rod. For this case $I_{com} = \frac{1}{12} M L^2$. Hint: First find α due to the torque generated by the weight of the rod. You do not need the values of M and L.



Solution:

At the free end at point A the tangential acceleration a_t

$$a_t = L\alpha \quad \text{(where } \alpha \text{ is the angular acceleration)}$$

but we know that $\alpha = \frac{\text{Torque}}{\text{Inertia}} = \frac{\tau}{I}$ - - - - - ①

- also we know that torque due to the gravitational force at the center mass is $\tau = Fr = Mg\left(\frac{1}{2}L\right)$
- and the inirtia $I = I_{com} + Mh^2$

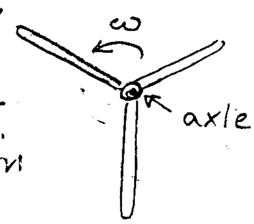
$$= \frac{1}{12} M L^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3} M L^2$$

from ①

$$\alpha = \frac{\tau}{I} = \frac{\cancel{M} g \cancel{L}}{2} \times \frac{3}{\cancel{M} L^{\cancel{2}}} = \frac{3g}{2L}$$

$$\Rightarrow \quad a_t = L \cdot \frac{3g}{2L} = \frac{3}{2} g = 1.5 g = \boxed{15 \ m/s^2} \quad \#$$

② The rotor of a helicopter consists of three blades, as shown in the figure. The rotor is rotating at 5 rev/s. The engine is switched off and the rotor comes to rest after 100 s, under the effect of friction with the axle. Calcute the frictional torque. The total moment of inertia I of the blades round the axle is 1500 kg.m^2



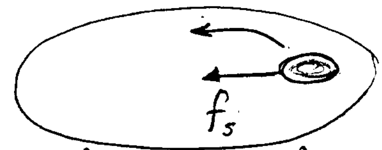
Solution:

$$\tau = I\alpha$$
$$\omega = \omega_o + \alpha t$$
$$0 = (5)(2\pi) + \alpha(100)$$
$$\alpha = -\frac{(5)(2\pi)}{100} = -\frac{\pi}{10} \ rad/s^2$$
$$\tau = -(1500)\left(\frac{\pi}{10}\right) = -150\pi \ Nm$$
$$\boxed{= -471.2 \ Nm} \quad \#$$

1 rev = 2π
5 rev = x
$\therefore x = 5 \times 2\pi$
$= 10\pi$

(3) A coin is placed on a disk rotating about a fixed axis, at a distance of .6 m from the axis. When the disk rotates with a uniform angular speed of .6 rev/s the coin is just about to slide off the disk. What is the value of the coefficient of static friction between the coin and the disk?

Solution



In this case the only force acting on the coin which makes it rotates is f_s (the friction force)

$$\therefore \quad f_s = -\frac{mv^2}{R}$$

$$-\mu_s N = -\frac{mv^2}{R}$$

$$\mu_s \cancel{m} g = \frac{\cancel{m} v^2}{R}$$

$$\mu_s = \frac{v^2}{gR} \qquad \text{but } v = R\omega$$

$$\Longrightarrow \quad \mu_s = \frac{R^2\omega^2}{gR}$$

$$\mu_s = \frac{R\omega^2}{g} = \frac{0.6\left[(.6)2\pi\right]^2}{10} = \boxed{.85} \quad \#$$

(4) At $t=0$, a wheel rotates from rest about a fixed axis with a constant angular acceleration of 2 rad/s². What is the magnitude of the total linear acceleration at $t = 2$ s for a point at 50 cm from the axis of rotation.

Solution:

$$\omega = \omega_0 + \alpha t = 0 + \alpha t = \alpha t$$

$$a_r = r\omega^2 = r\alpha^2 t^2 = (.5)(2)^2(2)^2 = 8 \text{ m/s}$$

$$a_t = r\alpha = (.5)(2) = 1$$

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(1)^2 + (8)^2} = \boxed{8.1 \text{ m/s}^2}$$

(5) A Torque of 30 N.m applied to a wheel and produces an angular acceleration of 20 rad/s². If the wheel starts from rest, find the rotational Kinetic energy of the wheel after 2 seconds.

Solution

$$\tau = I\alpha \Longrightarrow I = \frac{\tau}{\alpha} = \frac{30}{20} = 1.5 \text{ Nm.s}^2$$

But $K = \frac{1}{2} I \omega^2$ so we need ω.

$$\omega = \omega_0 + \alpha t = 0 + 20(2) = 40 \text{ rad/s}$$

$$\therefore \quad K = \tfrac{1}{2}(1.5)(40)^2 = \boxed{1200 \text{ J}} \quad \#$$

(6) A wheel rotates with a constant angular acceleration α around a fixed axis. Find the number of revolutions the wheel covers in a time of t in seconds. Take $\alpha = 4\ rad/s^2$, $\omega_0 = 2\ rad/s$, and $t = 6\ s$.

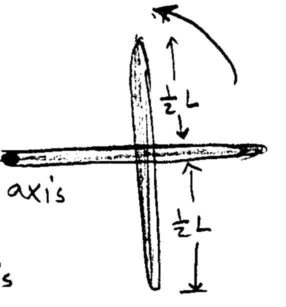
Solution:

We know that $\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$ $\quad (\theta_0 = 0)$

$$\Rightarrow \theta = 2(6) + \frac{1}{2}(4)(6)^2 = 84 \text{ rad}$$

number of revolutions is $\frac{84}{2\pi} = \boxed{13.4 \text{ rev}}$ #

(7) A system consists of 2 uniform rods, each of length L and mass M and normal to each other in the middle, as shown. The system rotates around an axis (normal to the rods), with an angular velocity $\omega = 3$ rad/s. Find the kinetic energy of the system. Hint: Use the parallel-axis Theorem and $I_{com} = \frac{1}{12}ML^2$ for a rod to evaluate the total I for the system.



Solution:

$$I = \sum m_i r_i^2 = I_1 + I_2 \text{ for the two rods.}$$

using the parallel Theorem to calculate I for the horizontal rod.

$$I_1 = I_{com} + Mh^2$$
$$= \frac{1}{12}ML^2 + M\left(\frac{1}{2}L\right)^2 = \frac{1}{12}ML^2 + \frac{1}{4}ML^2 = ML^2\left(\frac{1}{12} + \frac{1}{4}\right)$$
$$= ML^2\left(\frac{4+1}{12}\right) = \frac{1}{3}ML^2$$

using the same theorem to calculate I for the vertical rod.

$$I_2 = I_{com} + Mh^2$$
$$= \frac{1}{12}ML^2 + M\left(\frac{1}{2}L\right)^2 = \frac{1}{3}ML^2$$
$$\Rightarrow I = I_1 + I_2 = \frac{1}{3}ML^2 + \frac{1}{3}ML^2 = \frac{2}{3}ML^2$$

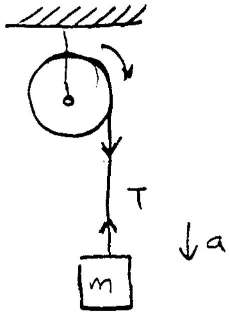
$$E = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{3}ML^2\right)(\omega^2)$$
$$= \frac{1}{2}\left(\frac{2}{3}(2.5)(2)^2\right)(3)^2 = \boxed{30\text{ J}}$$ #

(7) When a torque of 50 N.m is applied to a wheel, it produced an angular acceleration of 30 rad/s². If the wheel starts motion from rest, find the rotational kinetic energy (in KJ) after 5 seconds.

Solution

$$\tau = I\alpha \Rightarrow I = \frac{\tau}{\alpha} = \frac{50}{30} = 1.67\ Kg \cdot m^2$$
$$\omega = \omega_0 + \alpha t \Rightarrow \omega = 0 + 30(5) = 150\ rad/s$$
$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(1.67)(150)^2 = \boxed{18.75\ K.J}$$ #

(8) A mass m is attached to a string wounded around a wheel of radius R and moment of inertia I, as shown. suppose that $M = 2$ Kg and $R = 3$ m. Also suppose that m falls down with an acceleration $a = 5\ m/s^2$. Find the moment of inertia of the wheel.
Hint: First, you may find the tension in the cord.



Solution:

forces acting on the block are

$$F_{net} = ma$$

$$mg - T = ma \quad ------- \ (1)$$

for the wheel $\quad \tau = I\alpha = I\left(\frac{a}{R}\right) \quad$ (where $\alpha = \frac{a}{R}$)

also $\quad \tau = F_t r = TR \quad$ (where $F_t = T$)

$$\Rightarrow \quad TR = I\left(\frac{a}{R}\right)$$

$$T = I\frac{a}{R^2}$$

from (1) $\quad mg - I\frac{a}{R^2} = ma$

$$I\frac{a}{R^2} = mg - ma$$

$$I = \frac{mR^2}{a}(g-a)$$

$$= \frac{2(3)^2}{5}(10-5) = \boxed{18\ Kg \cdot m^2} \ \#$$

(9) A wheel starting from rest, turns through 8 revolutions in a time interval of 17 seconds. Assuming constant angular acceleration, what is the angular speed (in rad/s) at the end of this time interval?

Solution

$$\theta = \frac{1}{2}(\omega_0 + \omega)t \Rightarrow (8)(2\pi) = \frac{1}{2}(0+\omega)t$$

$$\omega = \frac{(2)(8)(2\pi)}{17} = \boxed{5.9\ rad/s} \ \#$$

another method:

$$\omega = \omega_0 + \alpha t \quad \& \quad \Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

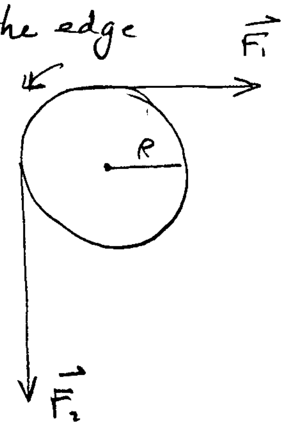
$$8(2\pi) = 0 + \frac{1}{2}\alpha(17)^2$$

$$\alpha = \frac{2(8)(2\pi)}{(17)^2} = .35\ rad/s^2$$

$$\therefore \ \omega = 0 + (.35)(17)$$

$$= \boxed{5.9\ rad/s} \ \#$$

(10) If $F_1 = 3N$ and $F_2 = 4N$ are two forces acting at the edge of a solid cylinder of radius $R = 20$ cm and mass $M = 2$ Kg (see the figure). then the cylinder will rotate with angular acceleration (in rad/s²).
Consider $I_{com} = \frac{1}{2}MR^2$ for the cylinder.



Solution

$$\tau = I\alpha = F_t R$$
$$\tau_{net} = (4)(.2) - (3)(.2) = .2 \text{ N.m} \quad (\text{where } F_2 \text{ is positive } \& F_1 \text{ is } -)$$
$$I = \frac{1}{2}MR^2 = \frac{1}{2}(2)(.2)^2 = .04 \text{ kg.m}^2$$
$$\Rightarrow \alpha = \frac{\tau}{I} = \frac{.2}{.04} = \boxed{5 \text{ rad/s}^2} \#$$

(11) A turntable of a record player operates at 78 rev/min, turns through 40 revolutions after being switched off before coming to rest. What is the (assumed) constant angular acceleration (in rad/s²) of the turntable as it slows?

Solution:

$$\omega^2 = \omega_0^2 + 2\alpha(\Delta\theta) \quad \text{------} \quad (1)$$

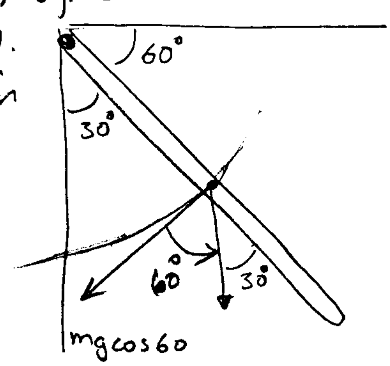
But $\omega_0 = 78$ rpm $\Rightarrow 78\,\frac{(2\pi)}{60} = 8.168$ rad/s

and $\Delta\theta = 40$ rev. $\Rightarrow 40(2\pi) = 251.33$

from (1) $0 = (8.168)^2 + 2\alpha(251.33)$.

$$\alpha = \boxed{-0.13 \text{ rad/s}^2} \#$$

(12) A uniform rod (mass $m = 2.0$ kg, length $\ell = .6$ m) is free to rotate about a frictionless pivot (fixed) at one end. What is the magnitude of the angular acceleration (in rad/s²) of the rod at the instant is 60° below the horizontal?
Hint: The moment of inertia of the rod about its end at O is $\frac{1}{3}m\ell^2$.



Solution:

$\tau = I\alpha$ also $\tau = F_t r = mg\cos(60)\,\ell/2$

$$(mg\cos 60)\frac{1}{2}\ell = \left(\frac{1}{3}m\ell^2\right)\alpha$$
$$\alpha = \frac{3g\cos 60}{2\ell} = \frac{3(10)(.5)}{2(0.6)} = \boxed{12.5 \text{ rad/s}^2} \#$$

(13) A motor accelerates from rest to 30,000 rev/min in 5 minutes. Find the number of revolutions the rotor turned in this time interval.

Solution:

$$\theta = \frac{1}{2}(\omega_o + \omega)t$$

$$\theta = \frac{1}{\cancel{2}}(0 + \overset{15000}{\cancel{30000}})(5) = 75000 \text{ rev} = \boxed{7.5 \times 10^4 \text{ rev}}$$

#

(14) Calculate the magnitude of the acceleration of a point on the rim of a bicycle wheel which is rotating uniformly at an angular velocity of 5 rad/s. Assume the wheel has a diameter of 1.2 m.

Solution:

$$S = \theta R$$

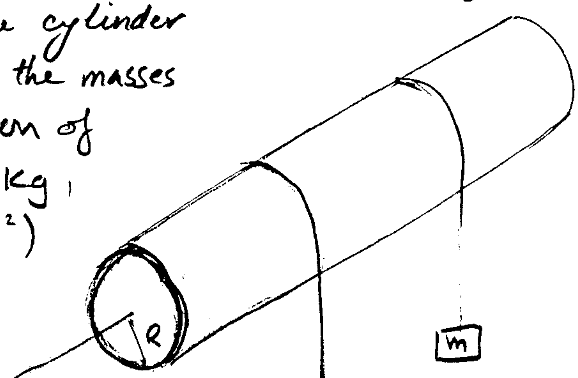
and $\quad v_t = \omega R$

also $\quad a_t = R\alpha = 0$

$$a_r = \frac{v^2}{R} = \frac{\omega^2 R^2}{R} = \omega^2 R$$

$$= (25)\left(\frac{1.2}{2}\right) = \boxed{15 \ m/s^2} \quad \#$$

(15) Two equal masses each with mass m are attached to a cylinder of mass M by strings as shown. If the cylinder rotates around its central axis, as the masses fall down, what is the acceleration of the two masses (in m/s^2)? $M = 2$ kg, $m = 200$ g (Hint: $I_{cylinder} = 1/2 MR^2$)

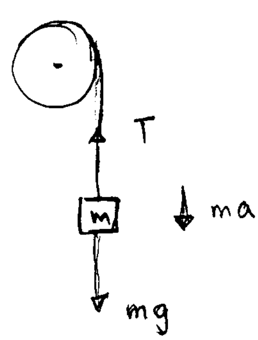


Solution:

$$\tau_{net} = I\alpha \quad ---- \quad (1)$$

$$2TR = \left(\frac{1}{2}MR^2\right)\left(\frac{a}{R}\right) \Rightarrow T = \frac{1}{4}Ma$$

also we can use the following equation for one m



$$mg - T = ma$$

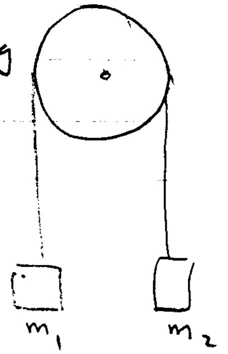
$$mg - \frac{1}{4}Ma = ma$$

$$mg = ma + \frac{1}{4}Ma = a\left(m + \frac{1}{4}M\right)$$

$$(.2)(10) = a\left(.2 + \frac{1}{4}(2)\right)$$

$$a = \boxed{2.86 \ m/s^2} \quad \#$$

(16) A pulley has a radius $R = .5$ m and moment of inertia $I = 2\ \text{kg m}^2$. The masses are $m_1 = 14$ kg and $m_2 = 2$ kg. Calculate the torque acting on the pulley (in N.m). Hint: $a = \left(\dfrac{m_1 - m_2}{m_1 + m_2 + \frac{I}{R^2}}\right) g$



Solution

for m_1 $\longrightarrow$ $m_1 g - T_1 = m_1 a$ ------ ①

for m_2 $\longrightarrow$ $T_2 - m_2 g = m_2 a$ ------ ②

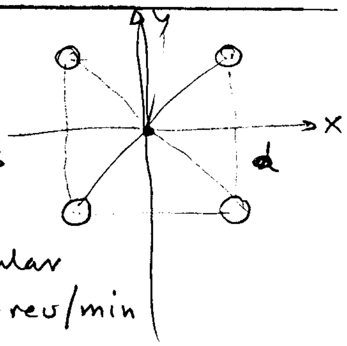
also we know that $\tau_{net} = I\alpha = I\dfrac{a}{R}$

$$(T_1 - T_2)R = I\frac{a}{R}$$

$$a = \frac{14-2}{14+2+\frac{2}{(0.5)^2}}\, 10 = 5\ \text{m/s}$$

$$\tau = I\frac{a}{R} = 2\left(\frac{5}{.5}\right) = \boxed{20\ \text{N.m}}$$ #

(17) Four equal masses, each of value m, are located at the corners of a square, of side d as shown which is held together by massless rods. The square rotates around an axis at the center normal to the square, with a constant angular speed ω. Take $m = 2$ kg, $d = 5$ m, and $\omega = 60$ rev/min and find the energy of the system.



Solution

$$I = \sum m_i r^2 = 4 m s^2$$

(where s is distance from each ball to the center of mass - axis)

$$s = \sqrt{\left(\tfrac{1}{2}d\right)^2 + \left(\tfrac{1}{2}d\right)^2} = \sqrt{\frac{d^2}{4} + \frac{d^2}{4}} = \sqrt{\frac{d^2}{2}}$$

$$\therefore I = 4m\left(\frac{d^2}{2}\right) = (4)(2)\left(\frac{5^2}{2}\right) = 100\ \text{kg.m}^2$$

$$\Rightarrow \omega = 60\ \frac{\text{rev}}{\text{min}} = \frac{60\,(2\pi)}{60} = 2\pi = 6.28\ \text{rad/s}$$

$$\Rightarrow K = \tfrac{1}{2} I \omega^2 = \tfrac{1}{2}(100)(6.28)^2 = \boxed{1974\ \text{J}}$$

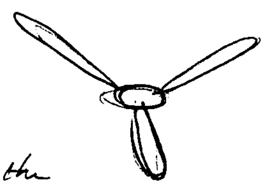
(18) Calculate the rotational Kinetic energy of the earth around its own axis. Assume the earth to be a perfect sphere, where $M = 6\times10^{24}$ Kg and $R = 6.4\times10^{6}$ m.
Hint: $I = \frac{2}{5}MR^2$ and 1 day = 24 h.

Solution:

$$\omega = \frac{1}{24(3600)}(2\pi) = 7.27\times10^{-5}\ \text{rad/s}$$

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}\right)(6\times10^{24})(6.4\times10^{6})^2(7.27\times10^{-5})^2$$
$$= \boxed{2.6\times10^{29}\ \text{J}}\ \#$$

(19) Problem 38 - Page 242 - Halliday 6th Ed.



Solution:

(a) To calculate the inertia of the blades we can use the Parallel-axis theorm for one blade only first then add the other (assume each blade is a small rod)

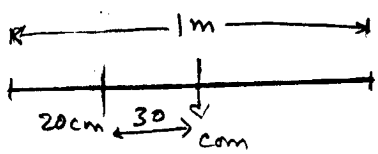
$$I_{\text{one blade}} = I_{com} + Mh^2$$
$$= \frac{1}{12}ML^2 + M\left(\frac{1}{2}L\right)^2$$
$$= \frac{1}{12}ML^2 + \frac{1}{4}ML^2 = \frac{1}{3}ML^2$$

for the three blades $3\left(\frac{1}{3}M\right)L^2 = ML^2 =$
$$= (240)(5.2)^2 = \boxed{6.49\times10^{3}\ \text{Kg.m}^2}\ \#$$

(b) Total Kinetic Energy

$$K = \frac{1}{2}I\omega^2 \qquad \left(\text{where } \omega = \frac{(350)(2\pi)}{60} = 36.65\ \text{rad/s}\right)$$
$$\Rightarrow K = \left(\frac{1}{2}\right)(6.49\times10^{3})(36.65)^2$$
$$= 4.36\times10^{6}\ \text{J} = \boxed{4.36\ \text{MJ}}\ \#$$

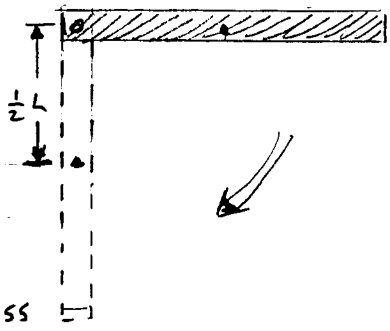
(20) Problem 39 - Page 242 - Halliday 6th Ed



Solution:

$$I = I_{com} + Mh^2$$
$$= \frac{1}{12}ML^2 + M(0.3)^2$$
$$= \frac{1}{12}(.56)(1)^2 + (.56)(0.3)^2 = .097\ \text{Kg.m}^2$$
$$= \boxed{9.7\times10^{-2}\ \text{Kg m}^2}\ \#$$

(E1) A uniform rod of length L and Mass M is free to rotate on a frictionless pin through one end as shown. The rod is released from rest in the horizontal position. (a) What is the angular speed of the rod at its Lowest point.



Solution

- when the rod is horizontal all its energy is potential, if we consider its center of mass then $U_g = Mg(\frac{1}{2}L) = \frac{1}{2}MgL$
- when it is released then all energy will become rotational energy at the lowest point

$$\Rightarrow \quad Mg\frac{L}{2} = \frac{1}{2}I\omega^2 \qquad (\text{where } I = \frac{1}{3}ML^2)$$

$$\Rightarrow \quad \frac{1}{2}MgL = \frac{1}{2}(\frac{1}{3}ML^2)\omega^2$$

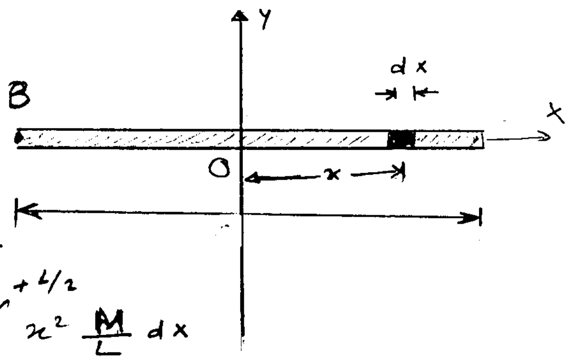
$$\Rightarrow \quad \omega = \sqrt{\frac{3g}{L}}$$

(b) Determine the Linear speed of the center of mass and The linear speed of edge of the rod at the lowest point.

Solution

- $v_{com} = r\omega = \frac{L}{2}\omega = \boxed{\frac{1}{2}\sqrt{3g}}$ #
- v of the lowest point $= \boxed{2v_{cm} = \sqrt{3g}}$ #

(E2) Calculate the moment of inertia of a uniform rigid rod of length L and Mass M (as shown) about an axis perpendicular to the rod (the y axis) and passing through its center of mass.



Solution

- The small elemend has length dx and mass dm which is mass per unit length

$$dm = \frac{M}{L}dx$$

and we know that $I = \int_{-L/2}^{+L/2} x^2\, dm = \int_{-L/2}^{+L/2} x^2 \frac{M}{L}\, dx$

$$= \frac{M}{L}\left[\frac{x^3}{3}\right]_{-L/2}^{L/2} = \boxed{\frac{1}{12}ML^2} \;\#$$

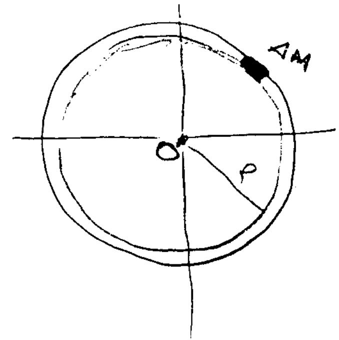
(b) Calculate the moment of inertia of the same rod at point B.

Solution

$$I = \int_0^L x^2 \frac{M}{L}\, dx = \frac{M}{L}\left[\frac{x^3}{3}\right]_0^L = \frac{M}{L}\frac{L^3}{3}$$

$$\boxed{= \frac{1}{3}ML^2} \;\#$$

(E3) Find the moment of inertia of a uniform hoop of mass M and radius R about an axis perpendicular to the plane of the hoop, through its center as shown.



Solution:

Take a small mass element of mass dm

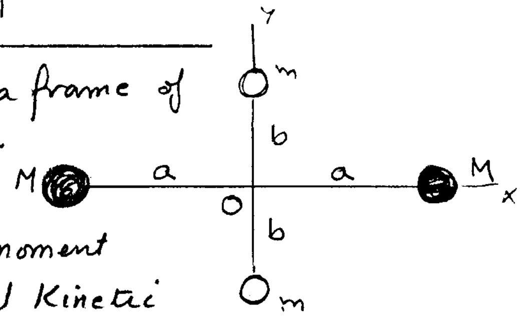
All mass elements are the same distance from O and $r = R$

$$\Rightarrow \therefore \quad I_z = \int r^2 dm = \int R^2 dm$$

But R^2 is constant

$$\Rightarrow \quad I_z = R^2 \int dm = \boxed{R^2 M}$$

(E4) Four point masses are fixed to the corners of a frame of negligible mass lying in the xy plane (as shown).



→ (a) If the rotation of the system occurs about the Y-axis with an angular speed ω, find the moment of inertia about the Y-axis and the rotational kinetic energy about this axis.

Solution:

– the two masses m do not contribute to I_y (because $r_i = 0$)

$$\Rightarrow \quad I_y = \sum m_i r_i^2 = Ma^2 + Ma^2 = 2Ma^2$$

⇒ The rotational Energy.

$$K_R = \tfrac{1}{2} I_y \omega^2 = \tfrac{1}{2}(2Ma^2)\omega^2$$
$$= Ma^2\omega^2 \quad \text{(where a is a distance)}$$

→ (b) Suppose the system rotates in the xy plane about an axis through O (the z axis). Calculate the moment of inertia about the z axis and the rotational energy.

Solution

$$I_z = \sum m_i r_i^2 = Ma^2 + Ma^2 + mb^2 + mb^2 = \boxed{2Ma^2 + 2mb^2}$$

$$\Rightarrow \quad K_R = \tfrac{1}{2} I_z \omega^2 = \tfrac{1}{2}(2Ma^2 + 2mb^2)\omega^2 = \boxed{(Ma^2 + mb^2)\omega^2}$$