Chapter 10

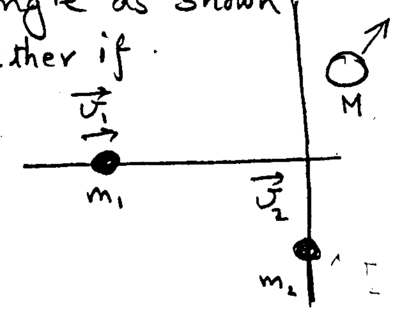
(1) A 1.2 Kg object moving with a speed of 8 m/s collides perpendicularly with a wall and rebound (ارتدّ) with a speed of 6 m/s in the opposite direction. If the object is in contact with the wall for 2 ms, what is the magnitude of the average force (in KN) on the object by the wall?

Solution:

$$\vec{J} = \Delta \vec{P} = \vec{P}_f - \vec{P}_i = m v_f - m v_i$$
$$= (1.2)(6) - (1.2)(-8)$$
$$= 16.8 \ \text{Kg.m/s}$$

also $\vec{J} = F_{avg} \Delta t \implies F_{avg} = \frac{J}{\Delta t} = \frac{16.8}{2 \times 10^{-3}} = \boxed{8.4 \text{ KN}}$ #

(2) Two particles approach each other at right angle as shown in the figure. They collide and stick (التصقا) together if $m_1 = 3\text{Kg}$, $m_2 = 1\text{Kg}$, $v_1 = 4 \text{ m/s}$, $v_2 = 8 \text{ m/s}$, find the speed V after collision (in m/s)



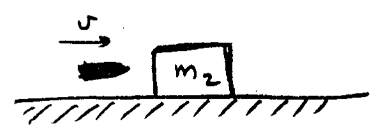
Solution:

$$M \vec{V}_f = m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i}$$

$$\vec{V}_f = \frac{3(4)\hat{i} + 1(8)\hat{j}}{3+1} = 3\hat{i} + 2\hat{j}$$

$$|V| = \sqrt{(3)^2 + (2)^2} = \boxed{3.6 \text{ m/s}} \ \#$$

(3) A bullet of mass m_1 and initial speed v_1, hits a block of wood of mass m_2 at rest. The bullet is stopped in the block. If $m_1 = 20\text{gm}$, $m_2 = 980 \text{ mg}$ and $v_1 = 200 \text{ m/s}$, find the final energy of the system.



Solution:

$$m_1 v_{1i} = (m_1 + m_2) V$$

$$V = \frac{m_1 v_{1i}}{m_1 + m_2} = 4 \text{ m/s}$$

$m_1 = .02$ Kg
$m_2 = .98$ Kg
$m_1 + m_2 = 1$ Kg

Energy is all Kinetic $= \frac{1}{2}(m_1 + m_2) V^2 = \frac{1}{2}(1)(4)^2$
$$= \boxed{8 \text{ J}} \ \#$$

(4) A block of mass 1 Kg rests on a horizontal frictionless surface. It is connected to an unstretched spring whose other end is fixed (see fig). The spring constant K = 500 N/m. A 4-Kg block moving with a speed of 5 m/s collides with the 1 kg mass and the two blocks stick together after the collision. Find the maximum compression (in m) of the spring.

Solution:

$$\vec{P}_{before} = \vec{P}_{after}$$

$$m_1 v_1 = (m_1 + m_2) V \Rightarrow V = \frac{m_1 v_1}{m_1 + m_2} = \frac{(4)(5)}{4+1} = 4 \text{ m/s}$$

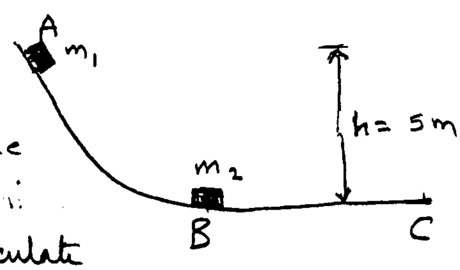
also we know that $E_i = E_f$ just at the first movement of the spring

$$\Rightarrow \frac{1}{2}(m_1+m_2) V^2 = \frac{1}{2} K x^2$$

$$\Rightarrow x = \sqrt{\frac{m_1+m_2}{K}}\, V = \sqrt{\frac{5}{500}}\,(4) = \boxed{0.4 \text{ m}} \quad \#$$

(5) Two blocks m_1 and m_2 are placed on a track ABC as shown in the figure.

If, the mass m_1 is released from A to make a head-on-elastic collision with the other mass $m_2 = 10$ Kg at B, initially at rest, Calculate the maximum height (in m) to which m_1 will rise after collision



Solution:

first let us calculate v_{1f} and its direction

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \Longrightarrow v_{1f} = \frac{5-10}{5+10} v_{1i} = \frac{-5}{15} v_{1i} \quad \ldots (1)$$

But we don't know v_{1i}. To find v_{1i} use $E_i = E_f$

$\Rightarrow \quad K_1 + U_1 = K_2 + U_2$ Between Points A and B.

$0 + \cancel{m_1} g h = \frac{1}{2} \cancel{m_1} v_{1i}^2$ just before collision

$$\Rightarrow \quad v_{1i} = \sqrt{2gh} = \sqrt{2(10)(5)} = 10 \text{ m/s}$$

from equation (1) $\quad v_{1f} = \frac{-5}{15}(10) = -3.33$ m/s

To find maximum hight m_1 will reach then use $E_i = E_f$ again

$$\frac{1}{2} m_1 v_{1f}^2 = m_1 g h' \Rightarrow 20h' = v_{1f}^2 = \boxed{.56 \text{ m}} \quad \#$$

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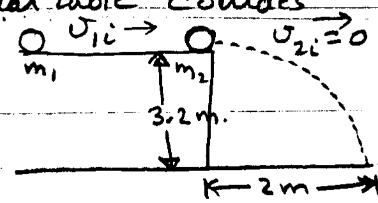
(6) A ball of mass $m_1 = .8\ Kg$ moving on a horizontal table collides head-on with another ball of mass $m_2 = 1.2\ Kg$ at rest. After collision (elastic) m_2 is projected off the table surface and lands $2m$ away from the table base. If the table surface is $3.2\ m$ above the ground, what is the speed of m_1 before collision?



Solution: we can use elastic collision equation

$$v_{2f} = \frac{2m_1}{m_1+m_2}\, v_{1i} \quad ------ \ (1)$$

But we need to know v_{2f} which we can calculate from using the free-fall equations or Trajectory equation for m_2.

→ from free-fall:

$$\Delta y = -\tfrac{1}{2}gt^2 \Rightarrow -3.2 = -\tfrac{1}{2}(10)t^2 \Rightarrow t = 0.8\,s$$

$$\Delta x = v_0 t \Rightarrow 2 = v_0(0.8) \Rightarrow v_0 = 2.5\ m/s \ \text{is}\ v_{2f}$$

→ trajectory equation:

$$\Delta y = (\tan\theta_0)x - \frac{g x^2}{2(v_0\cos\theta_0)^2} \Rightarrow -3.2 = -\frac{10(2)^2}{2(v_0)^2} \Rightarrow v_0 = 2.5\ m/s$$

we can see that v_{2f} is the v_0 of m_2.

from equ. (1) $\quad 2.5 = \dfrac{2(0.8)}{0.8+1.2}\, v_{1i} \Rightarrow \boxed{v_{1i} = 3.1\ m/s}$ #

(7) The two blocks shown have masses $m_1 = 60g$ and $m_2 = 120g$. Initially they are moving towards each other at the same speed of $3\ m/s$. After collision the blocks stick together. Neglecting friction with the horizontal surface, find their final speed.

Solution:

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)\, v_f$$

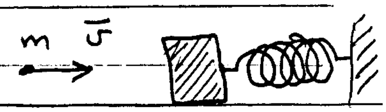
$$(.06)(3) + (.12)(-3) = (.06 + .12)\, v_f$$

$$.18 - .36 = .18\, v_f$$

$$-.18 = .18\, v_f$$

$$v_f = \boxed{-1\ m/s} \ \#$$

⑧ A bullet of mass 25g, speed 400 m/s, strikes and embeds itself in a block of wood whose mass is M = 975 g. Initially the wooden block was at rest on the horizontal frictionless surface next to, but not pressing on, a spring of force constant K = 40,000 N/m. Determine the maximum compression of the spring.



Solution:

$$P_i = P_f$$
$$m_1 v_{1i} = (m_1 + m_2)\, v_f$$
$$(.025)(400) = (.025 + .975)\, v_f$$
$$10 = 1\; v_f$$
$$\therefore v_f = 10 \text{ m/s}$$

now using energy conservation.

$$\tfrac{1}{2} m v^2 = \tfrac{1}{2} K x^2$$
$$(1)(100) = 40000\, x^2$$
$$x^2 = \frac{1}{400} \Rightarrow x = .05 \text{ m} \Rightarrow \boxed{x = 5\text{ cm}}\ \#$$

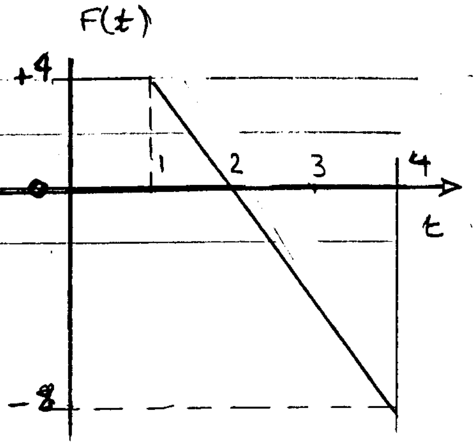
⑨ A 2kg block of wood is placed at rest on a smooth horizontal surface. A 50g bullet moving at 300 m/s strickes the block and emerges at the other side with a speed of 200 m/s. If it takee the bullet 0.5 seconds to transverse the block, what would be the average acceleration of the block during the impact?

Solution:

$$P_i = P_f$$
$$(.05)(300) = 2v_f + (.05)(200)$$
$$2v_f = (.05)(300) - (.05)(200)$$
$$= (.05)(100)$$
$$v_f = \frac{5}{2} = 2.5$$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{2.5}{.5} = 5 \text{ m/s}^2$$

(10) The only force acting on 2 kg object moving along the x-axis is shown in the figure. If the velocity at $t=0$ is $-2.0\ m/s$, what is the velocity at $t=4.0$ sec?



Solution

from the figure we can calculate the impulse, the area under the curve

$$J = (4) + \tfrac{1}{2}(4) + \tfrac{1}{2}(2)(-8) =$$
$$4 \quad + 2 \quad -8 \quad = -2\ \text{Kg.m/s}$$

But $J = P_f - P_i = m v_f - m v_i = m(v_f - v_i)$

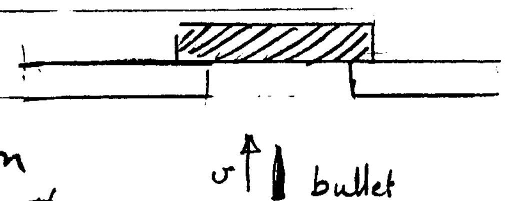
$$-2 = 2(v_f - (-2))$$
$$v_f = -1 - 2 = \boxed{-3\ m/s} \ \#$$

(11) A ball of $m_1 = 8$ Kg is moving to the right at a speed of 3 m/s. Another ball of $m_2 = 4$ kg is moving to the left at a speed of 3 m/s. The two balls undergo a one-dimentional head-on collision. Find the speed of the center of mass after collision?

Solution

$$P_i = P_f$$
$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_{com}$$
$$v_{com} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{8(3) + 4(-3)}{8+4}$$
$$v_{com} = \frac{24 - 12}{12} = \boxed{1\ m/s}$$

(12) A 10 g bullet moving 1000 m/s strikes and passes through (تخترق) a 2 kg wooden block initially at rest as shown in the figure. The bullet emerges from the block with a speed of 400 m/s. To what maximum height will the block rise above its initial position?



Solution:

$$m_1 v_{1i} + 0 = m_1 v_{1f} + m_b v_{bf}$$ (m_b is mass of the block)

$$(.01)(1000) = (.01)(400) + 2V = \underline{V = 3\ m/s}$$

also $\tfrac{1}{2} m_b v^2 = m_b g h \Rightarrow h = \frac{v^2}{2g} = \frac{(3)^2}{2(10)} = \boxed{\begin{array}{l} .45\ m \\ = 45\ cm \end{array}} \ \#$

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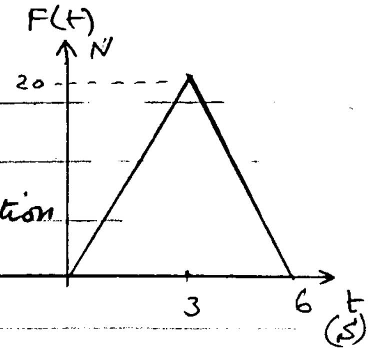
(13) The force acts on a 50 g ball varies with time as shown in figure. If the initial velocity of the particle is 20 m/s, what is the average acceleration of ball during this time?



Solution:

$$J = F_{avg}\,\Delta t$$

from figure J = area under curve = $(\frac{1}{2})(6)(20) = \underline{60\ kg.m/s}$

also from figure $\Delta t = 6\ sec$

$$\therefore F_{avg} = \frac{60}{6} = 10\ N$$

$$\Rightarrow a_{avg} = \frac{F_{avg}}{m} = \frac{10}{(.050)} = \boxed{200\ m/s^2}\ \#$$

Another method:

$$J = \Delta P = P_f - P_i = m v_f - m v_i$$

$$\frac{J}{m} = v_f - v_i$$

$$\frac{60}{.05} = v_f - 20 \Rightarrow v_f = 1220\ m/s$$

$$a_{avg} = \frac{1220 - 20}{6} = \boxed{200\ m/s^2}\ \#$$

(14) An explosion breaks an object at rest into two pieces, one of which has 1.5 time the mass of the other (i.e $m_1 = 1.5 m_2$). If the total energy released in this explosion was 4500, how much Kinetic energy (K_1, K_2) in (KJ) did each piece release during explosion?

Solution:

$$P_i = P_f$$

$$0 = 1.5\, m_2 v_1 + m_2 v_2$$

$$0 = 1.5\, v_1 + v_2$$

$$v_2 = -1.5\, v_1$$

$K_2 = \frac{1}{2} m_2 v_2^2$ and $K_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2}(1.5 m_2)\left(\frac{v_2}{1.5}\right)^2$

$$\Rightarrow K_1 = \frac{1}{1.5}\left(\frac{1}{2} m_2 v_2^2\right) = \frac{1}{1.5} K_2$$

$$\therefore \boxed{K_2 = 1.5\, K_1}$$

$$E = K_1 + K_2 = K_1 + 1.5 K_1 = 4500\,J$$

$$\Rightarrow 2.5\, K_1 = 4500\,J \Rightarrow \boxed{K_1 = 1800\,J}$$

$$K_2 = 1.5\, K_1 = (1.5)(1800) = \boxed{2700\ J}\ \#$$

(15)- The only force acting on a 2 kg object as it moves along the x-axis is given by $F = 4t$, where t is measured in s. If the speed of the object at $t = 0$ is $v_1 = 3\,m/s$, at what value of t will velocity $v_2 = 8\,m/s$?

Solution

The impulse due to the force.

$$J = \int_{t_i}^{t_f} F(t)\,dt = \int_{t_i}^{t_f} 4t\,dt = 2t^2\Big]_0^{t_f} = 2t_f^2$$

also
$$J = \Delta P = P_f - P_i = m(v_f - v_i) = 2(8-3)$$
$$= 10\,kg.m/s$$

$$2t_f^2 = 10 \Rightarrow \boxed{t = 2.24\ sec}$$

(16) A mass m_1 of velocity $\vec{v}_{1i}$ collides with another mass m_2 at rest. Assume the masses stick together after collision. Let $m_1 = 3\,kg$, $m_2 = 6\,kg$, and $v_{1i} = 6\,m/s$. Calculate the energy lost in the collision if any.

Solution:

$$K_i = \tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2}(3)(6)^2 = \underline{54\,J}$$

from $P_i = P_f$

$$V_f = \frac{m_1 v_{1i}}{m_1 + m_2} = \frac{3\times 6}{3+6} = 2\,m/s$$

$$\therefore K_f = \tfrac{1}{2}(m_1 + m_2)v^2 = \tfrac{1}{2}(9)(2)^2 = 18\,J$$

$\Rightarrow$ The lost energy $= K_i - K_f = \boxed{54 - 18 = 36\,J}$ #

(17) A mass $m_1 = 3\,kg$ and of velocity $v_{1i} = 6\,m/s$ collides with another mass $m_2 = 6\,kg$ at rest. Assume that the collision is elastic, Calculate the velocity of the center of mass after collision.

Hint: You may use conservation of momentum and properties of center of mass.

Solution:
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{--- (1)}$$

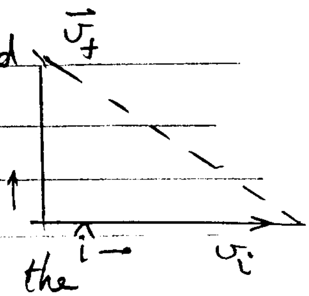
also
$$M v_{com} = m_1 v_{1f} + m_2 v_{2f} \quad \text{--- (2)}$$

from (1): $3\times 6 + 0 = 3v_{1f} + 6v_{2f}$ --- (3)

from (2): $9\cdot v_{com} = 3v_{1f} + 6v_{2f}$ --- (4)

from (3) and (4): $18 = 9\,v_{com} \Rightarrow v_{com} = \boxed{2\,m/s}$ #

(18) A car of mass 900 Kg, and moving east with a speed 40 m/s, collides with a wall. After the collision, the car rebounds (ارتد) with a speed of 30 m/s in the north direction. The collision lasted for 5 sec. Find the magnitude of the average force exerted on the car during the collision.



Solution$_1$:

$$\Delta\vec{P} = \bar{F}_{avg}\,\Delta t$$

$$F_{avg} = \frac{900(\vec{v}_f - \vec{v}_i)}{\Delta t}$$

$$= \frac{\cancel{900}^{180}(30\hat{j} - 40\hat{i})}{\cancel{5}}$$

$$= 5400\hat{j} - 7200\hat{i}$$

$$F_{avg} = \sqrt{(5400)^2 + (7200)^2} = \boxed{9000\,N}$$

Solution$_2$:

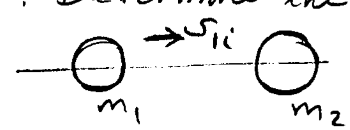
$$\Delta P_x = \bar{F}_x\,\Delta t$$

$$\bar{F}_x = \frac{(900)(40)}{5} = (8)(900)\,\hat{i}$$

$$\bar{F}_y = \frac{(900)(30)}{5} = (6)(900)\,\hat{j}$$

$$\bar{F} = \sqrt{F_x^2 + F_y^2} = \boxed{9000\,N}$$

(19) A 1.8 Kg mass sliding on a frictionless surface has a velocity of 3 m/s east when it undergoes a one-dimentional elastic collision with an unknown mass that is initially at rest. After collision the velocity of 1.8 Kg mass is 2 m/s west. Determine the unknown mass.



Solution:

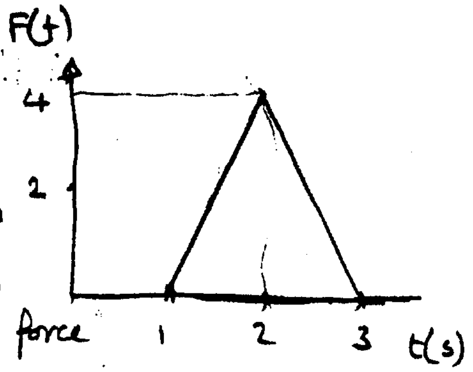
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}\,v_{1i}$$

$$-2 = \frac{1.8 - m_2}{1.8 + m_2}\cdot 3$$

$$-2(1.8 + m_2) = 5.4 - 3m_2$$

$$m_2 = 5.4 + 3.6 = \boxed{9\,Kg} \;\#$$

(20) A 5 Kg object can move along the x-axis. A force F acts on it in the positive x-direction; a graph of the F as a function of time is shown. What is the change in the speed of the object over the time the force is applied? (in m/s).



Solution: $\Delta \vec{p} = \int^{3} F(t)\,dt$

This is the area under the curve $= m\Delta v$

$$\frac{1}{2}(2)(4) = 5\,\Delta v$$

$$\Delta v = \boxed{.8\ m/s}$$ #

(21) A projectile of mass m_1 and initial velocity v_{1i} collides head-on with a target at rest whose mass $m_2 = 2m_1$, and the collision is elastic. Find the fraction of energy transmitted from the projectile to the target.

Note: $v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$ $\quad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$ $\quad f = \frac{K_i - K_f}{K_i}$

Solution:

energy transmitted from the projectile m_1 to the target m_2 is K_{2f}

$$K_2 = K_{1i} - K_{1f}$$

$\therefore$ $f = \frac{K_i - K_f}{K_i}$ is the fraction required $\Rightarrow \frac{K_{2f}}{K_{1i}}$

$$= \frac{\frac{1}{2} m_2 v_{2f}^2}{\frac{1}{2} m_1 v_{1i}^2} = \frac{m_2}{m_1} \frac{v_{2f}^2}{v_{1i}^2} = \frac{m_2}{m_1}\left(\frac{v_{2f}}{v_{1i}}\right)^2$$

now using the second equation above

$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$ divide both sides by v_{1i}

$$\Rightarrow \frac{v_{2f}}{v_{1i}} = \frac{2m_1}{m_1 + m_2} \quad \text{.......} \quad (2)$$

Sub (2) into (1) $\quad \frac{m_2}{m_1}\left(\frac{2m_1}{m_1 + m_2}\right)^2 = \frac{m_2}{m_1} \frac{4m_1^2}{(m_1 + m_2)^2} = \frac{2m_1(4m_1^2)}{m_1 \cdot (3m_1)^2}$

$$= \frac{8m_1 \cdot m_1^2}{m_1 \cdot 9m_1^2} = \frac{8m_1^3}{9m_1^3} = \frac{8}{9} = .89\%$$