Chapter 4

Motion equations for constant acceleration

One dimension

1. $v = v_o + at$
2. $x - x_o = v_o t + \frac{1}{2}at^2$
3. $v^2 = v_o^2 + 2a(x - x_o)$
4. $x - x_o = \frac{1}{2}(v + v_o)t$
5. $x - x_o = vt - \frac{1}{2}at^2$

Two dimensions

1. $(v_x\hat{i} + v_y\hat{j}) = (v_{ox}\hat{i} + v_{oy}\hat{j}) + (a_x\hat{i} + a_y\hat{j})t$
2. $[(x\hat{i} + y\hat{j}) - (x_o\hat{i} + y_o\hat{j})] = (v_{ox}\hat{i} + v_{oy}\hat{j})t + \frac{1}{2}(a_x\hat{i} + a_y\hat{j})t^2$
3. $(v_x\hat{i} + v_y\hat{j}).(v_x\hat{i} + v_y\hat{j}) = (v_{ox}\hat{i} + v_{oy}\hat{j}).(v_{ox}\hat{i} + v_{oy}\hat{j}) + 2(a_x\hat{i} + a_y\hat{j}).[(x\hat{i} + y\hat{j}) - (x_o\hat{i} + y_o\hat{j})]$
4. $[(x\hat{i} + y\hat{j}) - (x_o\hat{i} + y_o\hat{j})] = \frac{1}{2}[(v_x\hat{i} + v_y\hat{j}) + (v_{ox}\hat{i} + v_{oy}\hat{j})]t$
5. $[(x\hat{i} + y\hat{j}) - (x_o\hat{i} + y_o\hat{j})] = (v_x\hat{i} + v_y\hat{j})t - \frac{1}{2}(a_x\hat{i} + a_y\hat{j})t^2$

Three dimensions

1. $\vec{v} = \vec{v}_o + \vec{a}t$
2. $\vec{r} - \vec{r}_o = \vec{v}_o t + \frac{1}{2}\vec{a}t^2$
3. $v^2 = v_o^2 + 2\vec{a}.(\vec{r} - \vec{r}_o)$
4. $\vec{r} - \vec{r}_o = \frac{1}{2}(\vec{v} + \vec{v}_o)t$
5. $\vec{r} - \vec{r}_o = \vec{v}t - \frac{1}{2}\vec{a}t^2$

where

$\vec{r}_o = (x_o\hat{i} + y_o\hat{j} + z_o\hat{k})$

$\vec{r} = (x\hat{i} + y\hat{j} + z\hat{k})$

$\vec{v}_o = (v_{ox}\hat{i} + v_{oy}\hat{j} + v_{oz}\hat{k})$

$\vec{v} = (v_x\hat{i} + v_y\hat{j} + v_z\hat{k})$

$\vec{a} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k})$

Equation 2 in 3-D can be written in the following forms:

$\Delta\vec{r} = \vec{v}_o t + \frac{1}{2}\vec{a}t^2$ or $\vec{r} = \vec{r}_o + \vec{v}_o t + \frac{1}{2}\vec{a}t^2$

or

$$(x\hat{i} + y\hat{j} + z\hat{k}) = (x_o\hat{i} + y_o\hat{j} + z_o\hat{k}) + (v_{ox}\hat{i} + v_{oy}\hat{j} + v_{oz}\hat{k})t + \frac{1}{2}(a_x\hat{i} + a_y\hat{j} + a_z\hat{k})t^2$$